Estimating Square Roots

Key Terms:

    a. Square Roots: Factor that multiplies by itself to give the designated product.

Not all numbers are perfect squares. This does not mean they don't have square roots though. It just means that not all numbers have roots that are natural numbers. Some numbers have roots that will appear as decimals.

Example: Use a calculator to find $\sqrt{50}$. When you do, a decimal answer of
approximately 7.07 is what shows up on the screen.

How do we estimate a square root? The process is actually very simple once you have a strong understanding of where the perfect squares exist on a number line. We will start first with a small example and then move on to bigger numbers.

Example A:

Estimate the value of $\sqrt{6}$ to the nearest hundredth.

We are looking for the number, that when multiplied by itself, gives us 6.

Step 1: Identify the perfect squares that come before 6 and after 6.

$$1 \text{ X } 1 = 1$$

$$2 \text{ X } 2 = 4$$

$$3 \text{ X } 3 = 9$$
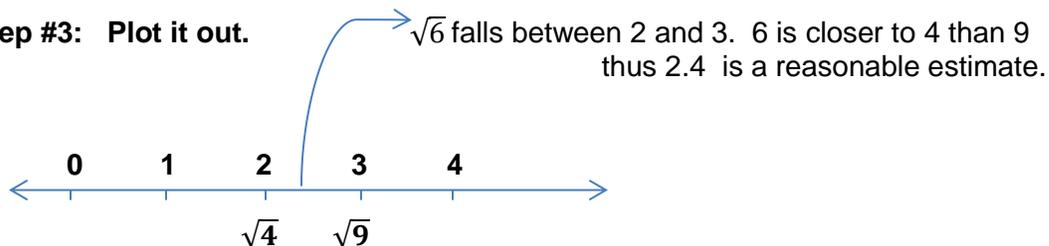
In this case, 6 falls between 4 and 9.

Step #2: Identify the square roots of the two perfect squares found in Step 1.

$$\sqrt{4} = 2$$

$$\sqrt{9} = 3$$

We now know that $\sqrt{6}$ falls somewhere between 2 and 3 on a number line.

Step #3: Plot it out. $\sqrt{6}$ falls between 2 and 3. 6 is closer to 4 than 9 thus 2.4 is a reasonable estimate.

Step #4: Check it out.

2.4 X 2.4 = 5.76   This is low.

2.5 X 2.5 = 6.25   This is high.

We know that the final answer is somewhere between 2.4 and 2.5 thus we must now move to the hundredths to zero in on our target.

6 falls almost directly in the middle of the square of 2.4 and 2.5 so let's try 2.45.

2.45 x 2.45 = 6.0025

This estimate is the most accurate estimate one can make when rounding to the nearest hundredth.

$$\sqrt{6} \doteq \mathbf{2.45}$$

Sample Questions

1. Between which two natural numbers does $\sqrt{77}$ fall between?

2. Estimate the value of $\sqrt{58}$ to the nearest hundredth.

Sample Questions

1. Between which two natural numbers does $\sqrt{77}$ fall between?

    $8 \times 8 = 64$

    $9 \times 9 = 81$

    $$\sqrt{77} \textbf{ falls between 8 and 9}.$$

2. Estimate the value of $\sqrt{58}$ to the nearest hundredth.

58 falls between the perfect squares 49 and 64. It is closer to 64 than it is to 49.

    $\sqrt{49} = 7$ and $\sqrt{64} = 8$

    Try 7.6

    $7.6 \times 7.6 = 57.76$ Too low

    $7.7 \times 7.7 = 59.29$ Too High

58 is closer to the first option (7.6 X 7.6). Now try the hundredths.

    $7.62 \times 7.62 = 58.06$.

This is very close thus we can say that $\sqrt{58} \doteq 7.62$.