

Example 1**Multiplying Two Fractions**

Multiply.

NOTE We multiply fractions in this way *not* because it is easy, but because it works!

$$\frac{2}{3} \times \frac{4}{5} = \frac{2 \times 4}{3 \times 5} = \frac{8}{15}$$

$$\frac{5}{8} \times \frac{7}{9} = \frac{5 \times 7}{8 \times 9} = \frac{35}{72}$$

**CHECK YOURSELF 1**

Multiply.

$$(a) \frac{7}{8} \times \frac{3}{10}$$

$$(b) \frac{5}{7} \times \frac{3}{4}$$

Step 3 indicates that the product of fractions should always be simplified to lowest terms. Consider the following.

Example 2**Multiplying Two Fractions**

Multiply and write the result in lowest terms.

$$\frac{3}{4} \times \frac{2}{9} = \frac{3 \times 2}{4 \times 9} = \frac{6}{36} = \frac{1}{6}$$

Noting that $\frac{6}{36}$ is not in simplest form, we divide numerator and denominator by 6 to write the product in lowest terms.

**CHECK YOURSELF 2**

Multiply and write the result in lowest terms.

$$\frac{5}{7} \times \frac{3}{10}$$

To find the product of a fraction and a whole number, write the whole number as a fraction (the whole number divided by 1) and apply the multiplication rule as before. Example 3 illustrates this approach.

Example 3**Multiplying a Whole Number and a Fraction**

Do the indicated multiplication.

Remember that $5 = \frac{5}{1}$.

$$(a) 5 \times \frac{3}{4} = \frac{5}{1} \times \frac{3}{4} = \frac{5 \times 3}{1 \times 4} = \frac{15}{4} = 3\frac{3}{4}$$

NOTE We have written the resulting improper fraction as a mixed number.

$$\begin{aligned}
 \text{(b)} \quad \frac{5}{12} \times 6 &= \frac{5}{12} \times \frac{6}{1} \\
 &= \frac{5 \times 6}{12 \times 1} \\
 &= \frac{30}{12} = 2\frac{6}{12} \\
 &= 2\frac{1}{2}
 \end{aligned}$$

NOTE Write the product as a mixed number, then reduce the fractional portion to simplest form.



CHECK YOURSELF 3

Multiply.

$$\text{(a)} \quad \frac{3}{16} \times 8$$

$$\text{(b)} \quad 4 \times \frac{5}{7}$$

When mixed numbers are involved in multiplication, the problem requires an additional step. First, change any mixed numbers to improper fractions. Then apply our multiplication rule for fractions.

Example 4

Multiplying a Mixed Number and a Fraction

$$1\frac{1}{2} \times \frac{3}{4} = \frac{3}{2} \times \frac{3}{4}$$

Change the mixed number to an improper fraction.

Here $1\frac{1}{2} = \frac{3}{2}$.

$$= \frac{3 \times 3}{2 \times 4}$$

Multiply as before.

$$= \frac{9}{8} = 1\frac{1}{8}$$

The product is usually written in mixed-number form.



CHECK YOURSELF 4

Multiply.

$$\frac{5}{8} \times 3\frac{1}{2}$$

If two mixed numbers are involved, change both of the mixed numbers to improper fractions. Our next example illustrates.

Example 5**Multiplying Two Mixed Numbers**

Multiply.

$$\begin{aligned} 3\frac{2}{3} \times 2\frac{1}{2} &= \frac{11}{3} \times \frac{5}{2} \\ &= \frac{11 \times 5}{3 \times 2} = \frac{55}{6} = 9\frac{1}{6} \end{aligned}$$

Change the mixed numbers to improper fractions.

**CAUTION****Be Careful!** Students sometimes think of

$$3\frac{2}{3} \times 2\frac{1}{2} \text{ as } (3 \times 2) + \left(\frac{2}{3} \times \frac{1}{2}\right)$$

This is *not* the correct multiplication pattern. You must first change the mixed numbers to improper fractions.**CHECK YOURSELF 5**

Multiply.

$$2\frac{1}{3} \times 3\frac{1}{2}$$

When multiplying fractions, it is usually easier to simplify, that is, remove any common factors in the numerator and denominator, *before multiplying*. Remember that to simplify means to *divide* by the same common factor.**Example 6****Simplifying Before Multiplying Two Fractions**

Simplify and then multiply.

$$\begin{aligned} \frac{3}{5} \times \frac{4}{9} &= \frac{\overset{1}{\cancel{3}} \times 4}{5 \times \underset{3}{\cancel{9}}} \\ &= \frac{1 \times 4}{5 \times 3} \\ &= \frac{4}{15} \end{aligned}$$

To simplify, we divide the *numerator* and *denominator* by the common factor 3. Remember that $\overset{1}{\cancel{3}}$ means $3 \div 3 = 1$, and $\underset{3}{\cancel{9}}$ means $9 \div 3 = 3$.**NOTE** Once again we are applying the fundamental principle to divide the numerator and denominator by 3.**NOTE** Because we divide by any common factors before we multiply, the resulting product is in simplest form.**CHECK YOURSELF 6**

Simplify and then multiply.

$$\frac{7}{8} \times \frac{5}{21}$$

Our work in Example 6 leads to the following general rule about simplifying fractions in multiplication.

Rules and Properties: Simplifying Fractions Before Multiplying

In multiplying two or more fractions, we can divide any factor of the numerator and any factor of the denominator by the same nonzero number to simplify the product.

When mixed numbers are involved, the process is similar. Consider Example 7.

Example 7

Simplifying Before Multiplying Two Mixed Numbers

Multiply.

$$\begin{aligned}
 2\frac{2}{3} \times 2\frac{1}{4} &= \frac{8}{3} \times \frac{9}{4} && \text{First, convert the mixed numbers} \\
 &&& \text{to improper fractions.} \\
 &= \frac{\cancel{2} \times \cancel{3}}{\cancel{3} \times \cancel{4}} && \text{To simplify, divide by the} \\
 &&& \text{common factors of 3 and 4.} \\
 &= \frac{2 \times 3}{1 \times 1} && \text{Multiply as before.} \\
 &= \frac{6}{1} = 6
 \end{aligned}$$



CHECK YOURSELF 7

Simplify and then multiply.

$$3\frac{1}{3} \times 2\frac{2}{5}$$

The ideas of our previous examples will also allow us to find the product of more than two fractions.

Example 8

Simplifying Before Multiplying Three Numbers

Simplify and then multiply.

$$\begin{aligned}
 \frac{2}{3} \times 1\frac{4}{5} \times \frac{5}{8} &= \frac{2}{3} \times \frac{9}{5} \times \frac{5}{8} && \text{Write any mixed or whole numbers} \\
 &&& \text{as improper fractions.} \\
 &= \frac{\cancel{2} \times \cancel{3} \times \cancel{5}}{\cancel{3} \times \cancel{5} \times \cancel{8}} && \text{To simplify, divide by the common factors} \\
 &&& \text{in the numerator and denominator.} \\
 &= \frac{3}{4}
 \end{aligned}$$

NOTE Remember our earlier rule: We can divide *any* factor of the numerator and *any* factor of the denominator by the same nonzero number.

**CHECK YOURSELF 8***Simplify and then multiply.*

$$\frac{5}{8} \times 4\frac{4}{5} \times \frac{1}{6}$$

We encountered estimation by rounding in our earlier work with whole numbers. Estimation can also be used to check the “reasonableness” of an answer when we are working with fractions or mixed numbers.

Example 9**Estimating the Product of Two Mixed Numbers**

Estimate the product of

$$3\frac{1}{8} \times 5\frac{5}{6}$$

Round each mixed number to the nearest whole number.

$$3\frac{1}{8} \rightarrow 3$$

$$5\frac{5}{6} \rightarrow 6$$

Our estimate of the product is then

$$3 \times 6 = 18$$

Note: The actual product in this case is $18\frac{11}{48}$, which certainly seems reasonable in view of our estimate.

**CHECK YOURSELF 9***Estimate the product.*

$$2\frac{7}{8} \times 8\frac{1}{3}$$

CHECK YOURSELF ANSWERS

$$1. \text{ (a) } \frac{7}{8} \times \frac{3}{10} = \frac{7 \times 3}{8 \times 10} = \frac{21}{80}; \text{ (b) } \frac{5}{7} \times \frac{3}{4} = \frac{5 \times 3}{7 \times 4} = \frac{15}{28}$$

$$2. \frac{5}{7} \times \frac{3}{10} = \frac{5 \times 3}{7 \times 10} = \frac{15}{70} = \frac{3}{14} \quad 3. \text{ (a) } 1\frac{1}{2}; \text{ (b) } 2\frac{6}{7}$$

$$4. \frac{5}{8} \times 3\frac{1}{2} = \frac{5}{8} \times \frac{7}{2} = \frac{35}{16} = 2\frac{3}{16} \quad 5. 8\frac{1}{6} \quad 6. \frac{7}{8} \times \frac{5}{21} = \frac{\overset{1}{\cancel{7}} \times 5}{8 \times \underset{3}{\cancel{21}}} = \frac{5}{24}$$

$$7. 3\frac{1}{3} \times 2\frac{2}{5} = \frac{10}{3} \times \frac{12}{5} = \frac{\overset{2}{\cancel{10}} \times \overset{4}{\cancel{12}}}{\underset{1}{\cancel{3}} \times \underset{1}{\cancel{5}}} = \frac{8}{1} = 8 \quad 8. \frac{1}{2} \quad 9. 24$$

2.5

Exercises

Name _____

Section _____ Date _____

Multiply. Be sure to write each answer in simplest form.

1. $\frac{3}{4} \times \frac{5}{11}$

2. $\frac{2}{7} \times \frac{5}{9}$

3. $\frac{3}{4} \times \frac{7}{11}$

4. $\frac{2}{5} \times \frac{3}{7}$

5. $\frac{3}{5} \times \frac{5}{7}$

6. $\frac{6}{11} \times \frac{8}{6}$

7. $\frac{6}{13} \times \frac{4}{9}$

8. $\frac{5}{9} \times \frac{6}{11}$

9. $\frac{3}{11} \times \frac{7}{9}$

10. $\frac{7}{9} \times \frac{3}{5}$

11. $\frac{3}{10} \times \frac{5}{9}$

12. $\frac{5}{21} \times \frac{14}{25}$

13. $\frac{7}{9} \times \frac{6}{5}$

14. $\frac{8}{13} \times \frac{26}{5}$

15. $\frac{3}{4} \times \frac{6}{7}$

16. $3\frac{1}{3} \times \frac{9}{11}$

17. $\frac{2}{3} \times 2\frac{2}{5}$

18. $3\frac{1}{3} \times \frac{3}{7}$

19. $\frac{2}{5} \times 3\frac{1}{4}$

20. $2\frac{1}{3} \times 2\frac{1}{6}$

21. $2\frac{1}{3} \times 2\frac{1}{2}$

22. $\frac{3}{7} \times 14$

23. $9 \times \frac{5}{6}$

24. $15 \times \frac{5}{6}$

25. $\frac{12}{25} \times \frac{11}{18}$

26. $\frac{10}{12} \times \frac{16}{25}$

27. $\frac{14}{15} \times \frac{10}{21}$

28. $\frac{21}{25} \times \frac{30}{7}$

29. $\frac{18}{28} \times \frac{35}{22}$

30. $3\frac{2}{3} \times \frac{9}{10}$

31. $\frac{4}{9} \times 3\frac{3}{5}$

32. $5\frac{1}{3} \times \frac{7}{8}$

33. $\frac{10}{27} \times 3\frac{3}{5}$

34. $1\frac{1}{3} \times 1\frac{1}{5}$

35. $2\frac{2}{5} \times 3\frac{3}{4}$

36. $2\frac{2}{7} \times 2\frac{1}{3}$

ANSWERS

1. _____ 2. _____

3. _____ 4. _____

5. _____ 6. _____

7. _____ 8. _____

9. _____ 10. _____

11. _____ 12. _____

13. _____ 14. _____

15. _____ 16. _____

17. _____ 18. _____

19. _____ 20. _____

21. _____ 22. _____

23. _____ 24. _____

25. _____ 26. _____

27. _____ 28. _____

29. _____ 30. _____

31. _____ 32. _____

33. _____ 34. _____

35. _____ 36. _____

ANSWERS

37. _____

38. _____

39. _____

40. _____

41. _____

42. _____

43. _____

44. _____

45. _____

46. _____

47. _____

48. _____

37. $4\frac{1}{5} \times \frac{10}{21} \times \frac{9}{20}$

38. $\frac{7}{8} \times 5\frac{1}{3} \times \frac{5}{14}$

39. $3\frac{1}{3} \times \frac{4}{5} \times 1\frac{1}{8}$

40. $4\frac{1}{2} \times 5\frac{5}{6} \times \frac{8}{15}$

41. Find $\frac{2}{3}$ of $\frac{3}{7}$

42. What is $\frac{5}{6}$ of $\frac{9}{10}$?

Estimate the following products.

43. $3\frac{1}{5} \times 4\frac{2}{3}$

44. $5\frac{1}{7} \times 2\frac{2}{13}$

45. $11\frac{3}{4} \times 5\frac{1}{4}$

46. $3\frac{4}{5} \times 5\frac{6}{7}$

47. $8\frac{2}{9} \times 7\frac{11}{12}$

48. $\frac{9}{10} \times 2\frac{2}{7}$

Answers

1. $\frac{15}{44}$

3. $\frac{3}{4} \times \frac{7}{11} = \frac{21}{44}$

5. $\frac{3}{7}$

7. $\frac{8}{39}$

9. $\frac{7}{33}$

11. $\frac{3}{10} \times \frac{5}{9} = \frac{15}{90} = \frac{1}{6}$

13. $\frac{14}{15}$

15. $\frac{3}{4} \times \frac{6}{7} = \frac{3 \times 6}{4 \times 7} = \frac{18}{28} = \frac{9}{14}$

17. $1\frac{3}{5}$

19. $1\frac{3}{10}$

21. $5\frac{5}{6}$

23. $7\frac{1}{2}$

25. $\frac{22}{75}$

27. $\frac{4}{9}$

29. $1\frac{1}{44}$

31. $1\frac{3}{5}$

33. $1\frac{1}{3}$

35. 9

37. $\frac{9}{10}$

39. 3

41. $\frac{2}{7}$

43. 15

45. 60

47. 64



Using Your Calculator to Multiply Fractions



Scientific Calculator

To multiply fractions on a scientific calculator, you enter the first fraction, using the **a b/c** key, then press the multiplication sign, next enter the second fraction, then press the equals sign.

Example 1

Multiplying Two Fractions

Find the product

$$\frac{7}{15} \times \frac{5}{21}$$

The keystroke sequence is

7 **a b/c** 15 **×** 5 **a b/c** 21 **=**

The result is $\frac{1}{9}$.



CHECK YOURSELF 1

Find the product

$$\frac{24}{33} \times \frac{22}{39}$$



Graphing Calculator

When using a graphing calculator, you must choose the fraction option **1:► Frac** from the **MATH** menu before pressing **Enter**.

For the fraction problem in Example 1, $\frac{7}{15} \times \frac{5}{21}$, the keystroke sequence is

7 **÷** 15 **×** 5 **÷** 21 **1:► Frac** **Enter**

Again, the result will be $\frac{1}{9}$.

CHECK YOURSELF ANSWER

1. $\frac{16}{39}$



Calculator Exercises

Find the following products using your calculator.

1. $\frac{15}{20} \times \frac{8}{12}$

2. $\frac{7}{8} \times \frac{4}{21}$

3. $\frac{36}{55} \times \frac{33}{54}$

4. $\frac{28}{42} \times \frac{12}{35}$

5. $\frac{18}{84} \times \frac{36}{27}$

6. $\frac{6}{35} \times \frac{20}{12}$

7. $\frac{27}{26} \times \frac{13}{9}$

8. $\frac{32}{35} \times \frac{15}{16}$

9. $\frac{7}{12} \times \frac{36}{63}$

10. $\frac{8}{27} \times \frac{45}{64}$

11. $\frac{12}{45} \times \frac{27}{72}$

12. $\frac{18}{132} \times \frac{36}{63}$

13. $\frac{27}{72} \times \frac{24}{45}$

14. $\frac{81}{136} \times \frac{84}{135}$

ANSWERS

1. _____

2. _____

3. _____

4. _____

5. _____

6. _____

7. _____

8. _____

9. _____

10. _____

11. _____

12. _____

13. _____

14. _____

Answers

1. $\frac{1}{2}$

3. $\frac{2}{5}$

5. $\frac{2}{7}$

7. $\frac{3}{2}$ or $1\frac{1}{2}$

9. $\frac{1}{3}$

11. $\frac{1}{10}$

13. $\frac{1}{5}$