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Poker Hands

Get your money in with the best of it.

Before he got famous for shorting Lehman Brothers stock in 2007, David Einhorn won US$650,000 in the main event at the 2006 World Series of Poker. He told Jenny Anderson of *The New York Times*: "*Both poker and investing are games of incomplete information. You have a certain set of facts and you are looking for situations where you have an edge, whether the edge is psychological or statistical.*"[1]

Another similarity between poker and investing is that the games have layers. You can get useful insight by tackling one layer, modeling it, and solving it rigorously. One of the most important goals in poker is "*to get your chips in with the best of it.*" That means to arrange things such that when you have bet a lot of money into a large pot, you would have a positive expectation if the rest of the cards in the hand are dealt with no further betting. Of course, investors think about the same thing when putting their money to work.

Getting your money in with the best of it is by no means the only goal in poker. In fact, there are times when you bet knowing you have the worst of it. For that to make sense, you must believe that if your hand improves, you can get the other players to call large enough bets to make up for the losses when your hand does not improve. Similarly, it can make sense to fund a business project with a negative net present value, if you believe that a successful project can be expanded to make enough additional money to cover the losses in failed projects.

Nevertheless, everything is easier if you do get your money in with the best of it. You're not counting on future contingencies and skillful play, you can sit back and let the law of averages send money your way. You may lose on many hands or investments but in the long run you should do well. If you can also take advantage of future opportunities, that's just extra money.

Credit: Todd Klassy, 'A Riffle Shuffle', http://www.flickr.com/photos/latitudes/66424863/in/set-1442169/

To investigate this aspect of poker, consider a simplified version of Texas Hold'em. There are two players, each with a stack of chips worth S. The first player is called the "small blind" (SB) and has to put \$1 in the pot before the cards are dealt. The second player is the "big blind" (BB) who must put in \$2, so the pot is \$3. Now, each player is dealt two cards, which the other player cannot see.

In real no-limit Texas Hold'em, SB can fold or bet any amount of money greater than \$1 up to her full stack (which is S – \$1 at this point since she already contributed \$1 to the pot). If she folds, the hand is over and BB gets the \$3 pot. In this simplified version, if SB bets, she must bet her entire stack. If SB bets, BB can fold and let SB take the pot (which would leave SB with S + \$2 and BB with S – \$2) or call the bet, in which case BB's remaining stack goes into the pot, bringing is up to 2S. There are no more decisions in the hand, the flop is dealt, and whichever player has the better five-card poker hand takes the pot.

Before getting to the optimal play, think about which player, SB or BB, has the advantage in this game. SB has to act first but is only forced to contribute \$1 instead of \$2.

To see the solution path for this problem, consider a simpler version, in which the players each get only one card, and there is no further dealing after the betting; the higher card wins (Ace (A) is the highest, followed by King (K) down to Jack (J), then the cards in reverse numerical order from 10 (T) to 2; in a tie, the pot is split, with each player taking S). We're using a standard 52-card deck, with four cards of each of the 13 ranks.

Clearly, in this game, neither player ever folds an A. Folding gets zero. If SB bets with an A and BB folds, SB wins \$3. If BB calls with anything less than an A, SB gets S + \$1. If BB calls with an A, SB still makes \$1. All outcomes are better than folding. BB also gets zero for folding. If he calls with an A, he either makes \$2 (SB also has an A) or S + \$2 (SB has anything else).

Now, suppose SB decides to also bet with K, while BB sticks to calling only with an A. When SB has a K, 47 times out of 51 BB will not have an A and will fold, so SB makes \$3. But four times in 51, BB will have an A, so SB will lose S – \$1. $47 \times \$3 - 4 \times (S - \$1) > 0$ implies that $\$145 > 4 \times S$ or $\$36.25 > S$. So, for very large stakes, SB will bet only with an A but if the stake is less than \$36.25, SB will also bet a K. In fact, if SB's card is not an A, it doesn't matter what it is, as it always loses. So, SB will switch to betting on any card.

This is the solution as long as $S > \$33.41$. At that point, BB starts calling with a K as well as an A, and SB switches to calling with an A or K only. Table 1 shows the critical values for S. If S is between the critical values in the small blind and big blind columns, SB will bet with anything, and BB will call with that card or better. If S is larger than the critical value in the small blind column but smaller than the number in the big blind column in the row above, both players will bet or call with that card or better.

When S gets down to \$2, BB has no chips left after posting the big blind, so BB will call with anything, since he has nothing to lose. SB still has \$1 to lose, so she needs at least a 25 percent chance of winning the \$3 pot in order to bet. SB bets with a 5

Table 1: Critical values for S

	Critical value for *S*	
Card	Small blind	Big blind
A	36.25	33.41
K	17.13	15.71
Q	10.75	9.81
J	7.56	6.86
T	5.65	5.10
9	4.38	3.92
8	3.46	3.08
7	2.78	2.45
6	2.25	2.00

Table 2: Small-blind bet range, expectation range and call range

S	Small blind expectation	Small-blind bet range	Big-blind call range
1,000	0.01	AA	AA
100	0.12	KK; QQ; JJ; TT; 99; 88; 77; 66; 55; 44; AKs; AQs; 33; AJs; 22; ATs; A9s; A8s; AK; A7s; AQ	KK; QQ; JJ; TT; 99; 88; 77; 66; 55
50	0.23	A6s; AJ; A5s; KQs; AT; A4s; KJs; A9	44; AKs; AQs; 33; 22; AK; AQ
20	0.53	A3s; KTs; A8; A2s; K9s; A7; K8s; A6; K7s; A5; KQ; K6s; A4; KJ; K5s; QJs; KT; A3; K4s; QTs; A2; K9; K3s; Q9s; K8; K2s; Q8s; K7; K6; Q7s; Q6s; K5; QJ; Q5s; K4; QT	AJs; ATs; A9s; A8s; A7s; A6s; AJ; A5s; KQs; AT; A4s; A9; A3s; A8; A7; A6; A5; KQ; A4; A3
10	0.84	JTs; Q4s; K3; Q9; J9s; Q3s; K2; Q8; J8s; Q2s; Q7; J7s; Q6; J6s; Q5; JT; J5s; T9s;Q4; J4s; J9; T8s; Q3; J3s; J8; T7s; Q2; J2s; J7; T6s; J6; T5s; J5; 98s; T9	KJs; KTs; A2s; K9s; K8s; K7s; K6s; KJ; K5s; QJs; KT; K4s; QTs; A2; K9; K3s; Q9s; K8; K2s; Q8s; K7; K6; K5; QJ; K4; QT; K3; Q9; K2
6.6	1.00	T4s; J4; 97s; T8; T3s; J3; 96s; T2s; T7; J2; 95s; T6; 87s; 94s; T5; 98	Q7s; Q6s; Q5s; JTs; Q4s; J9s; Q3s; Q8; J8s; Q2s; Q7; J7s; Q6; J6s; Q5; JT; Q4; J9; Q3; Q2
4.0	1.12	86s; 93s; T4; 97; 92s; T3; 85s; 96; T2	J5s; T9s; J4s; T8s; J3s; J8; T7s; J2s; J7; T6s; J6; T5s; J5; 98s; T9; T4s; J4; 97s; T8; T3s; J3; 96s; T2s; T7; J2; T6; T5
2.8	1.09	84s; 76s; 95; 83s; 75s; 87; 94; 82s; 74s; 93; 86	95s; 87s; 94s; 98; 86s; 93s; T4; 97; 92s; T3; 85s; 96; T2; 84s; 76s; 95; 83s; 75s; 87; 94; 82s; 74s; 93; 86; 65s; 92; 73s; 85; 64s; 84; 72s; 76; 63s; 54s; 83; 75; 82; 62s; 53s; 74; 52s; 65; 43s; 73; 64; 42s; 72; 32s; 63; 54; 62; 53; 52; 43; 42; 32
2.2	1.03	65s; 92; 73s; 85; 64s; 84; 72s; 76; 63s; 54s; 83; 75; 82	
2.1	1.03	62s; 53s; 74; 52s; 65; 43s; 73; 64; 42s; 72; 32s; 63; 54; 62; 53; 52; 43	
None		42; 32	

or better and folds otherwise (with a 5, she wins if BB has any of the 12 cards below 5, ties with the three other 5s, and loses to the 36 cards higher than 5; that's 12 ´ $3 + 3 ´ $1.50 – 36 ´ $1 = $4.50; but betting with a 4 has a negative expectation). However, as BB never wins by calling with a 4 or lower, he might as well fold those hands, which keeps our pattern intact; when $S = 2$, both players bet or call with a 5 or better, and fold otherwise.

With poker, things are a bit more complicated but the same basic logic applies. With a very large *S*, both players will only play if they are dealt the strongest possible two cards, AA. Table 2 shows the results for $S = 1{,}000$. If *S* goes down to 100, SB will of course continue to play AA but will also bet with any pair, any suited hand with an A, and at least a 7 (a suited hand is indicated with an "s," such as AKs, and means that both cards are of the same suit), and AK or AQ (with no "s," these represent unsuited hands, so the cards are of different suits).

With $S = 100$, BB also expands his range but only to pairs down to 55. As *S* gets smaller, both players add more hands to their ranges but SB adds faster than BB. Things change at $S = 6.6$, when BB starts adding hands faster, and by $S = 2.8$, BB is calling with everything, while SB is still not betting with some weak hands. In fact, there are two hands – 42 and 32 unsuited – that SB will never bet on.

One interesting thing is that SB and BB do not add the hands in the same order. Of course, both of them add stronger hands before weaker but SB cares more about whether cards are suited. For example, SB adds AJs when $S = 100$, but AJ unsuited only at $S = 20$. BB adds both at $S = 20$. The reason is that BB has a smaller range than SB until *S* gets very low. That means that BB usually has the better hand. The only advantage to a suited hand is that it can make a flush, and if it does, a flush almost always wins, even if it is composed of low cards. Therefore, the flush possibility is much more valuable to the player with the weaker hand, which will be SB.

It's harder to see but you could also detect that SB cares more about whether the two cards are close together (adjacent cards like JT are called "connectors"). The closer together the cards, the greater the chance of a straight, and, like flushes, straights almost always win. For example, SB will play QJ at $S = 20$, while BB waits until $S = 10$, even for QJs.

This sort of thing is the reason why you won't find the same rankings of starting hands in any two poker books. Everyone agrees that AA is the best starting hand but which middle hands are best depends on the precise circumstances. There is even dispute about whether the worst hand is 72 or 32 (7 and 2 cannot be connected in a straight, while 32 can be part of two straights – A2345 or 23456 – but 7 is a higher card than 3).

Table 2 also has a column listing SB's expectation. BB's expectation is always \$3 – SB's. If S is very large, SB folds the 220 times out of 221 that she doesn't get dealt AA, so her expected winning is very small, and BB wins \$3 nearly all the time. But as *S* gets smaller, SB starts playing and winning more hands. Losing more hands too, but her expectation increases. At $S = 6.6$, her expectation is \$1, which means that the game is fair as she put \$1 in to start (BB put in \$2, and his expectation is \$2). So, for $S > 6.6$, BB has the advantage in this game. Unfortunately for SB, her advantage peaks at $S = 4$, when it is \$0.12. After that, it declines as *S* declines.

Figure 1: Average winning amount of various starting hands in a large sample of online poker players

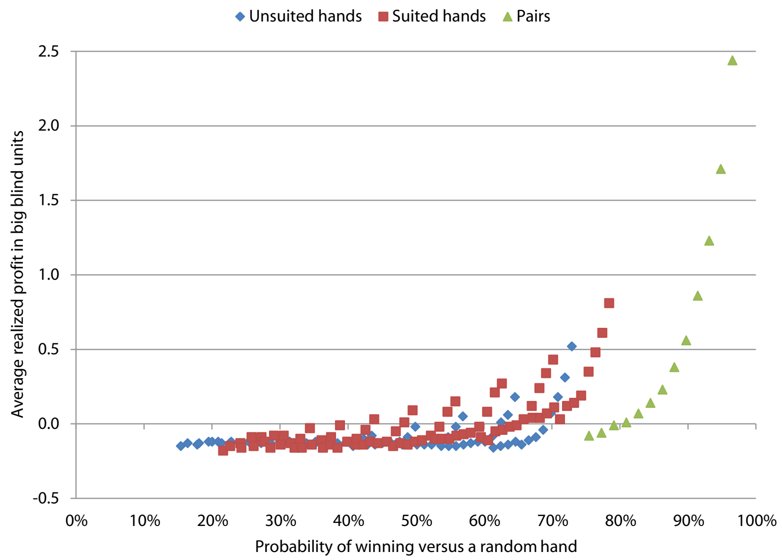

Figure 2: Connector, one-gap, two-gap, three-gap, four-gap, and unconnectable suited hands

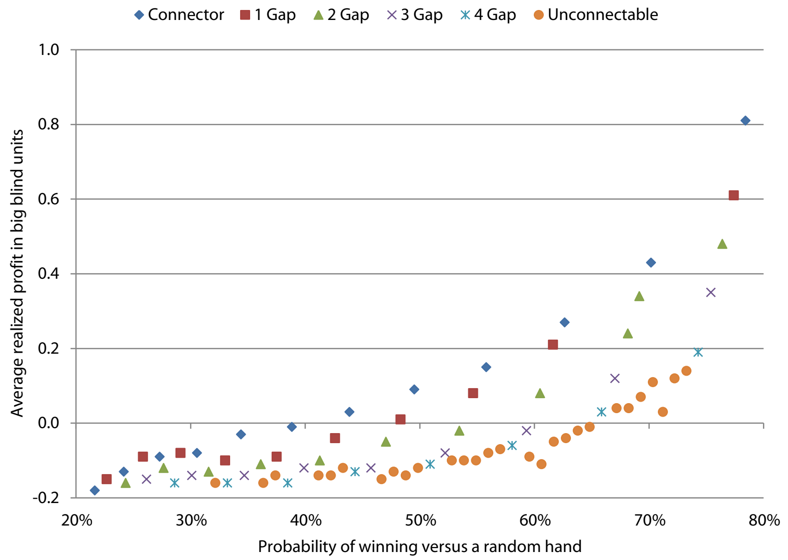

Figure 1 shows the average winning amount of various starting hands in a large sample of online poker players. The units are in big blinds. As games are played for different stakes, this helps to normalize the results. So, in a \$5/\$10 game (the SB posts \$5 and the big blind posts \$10), winning \$50 is five big blind units – the same as winning \$2,000 in a \$200/\$400 game. The horizontal axis is the probability that the hand will beat a random hand after the flop is dealt. The negative expectations for most hands reflect the rake (the poker site takes a percentage of the winnings, so the average hand loses money) and the fact that the positive expectations are concentrated in a few big winning hands. In fact, the weaker hands are nearly always folded for a small expected loss; you don't lose any more money folding 72 as folding Q9.

While realized winnings generally go up with hand strength, it is by no means monotonic. I've split the hands into unsuited hands, suited hands, and pair, and each group forms its own sequence. Even though the weakest pair (22) wins significantly more often against random hands than the strongest unsuited hand (AK), 22 loses money, on average, while AK makes more 0.5 big blinds.

Another feature of Figure 1 is that both the unsuited and suited hands seem to have branches that go above the main sequence. To understand these, In Figure 2 I've graphed only the suited hands and distinguished them by whether they are connectors (adjacent cards, like JT), one-gappers (J9, for example), and so on. The last category – unconnectable cards – have gaps greater than 4.

Realized winnings increase with hand strength but at different slopes for different gap types. There's some noise in the chart, which may be due to subtler factors, or perhaps due to sampling error.

This is a puzzling story. Hand strength does matter but in order to be a profitable poker hand, the main requirement seems to be being among the strongest hands of a type. The best unsuited hand (AK) wins 0.54 big blinds, on average, versus 0.14 big blinds for the more powerful sixth-best-suited hand (A8s). A6s wins 71 percent of the time versus a random hand, yet wins only 0.03 big blinds, on average, while T9s is much weaker, winning less than 50 percent of the time versus a random hand, but in actual play wins three times as much, 0.09 big blinds.

How does this relate to real poker? One of the key strategic decisions in a poker match is which set of starting hands to play. Of course, the actual fold, bet, or call decisions will be made in light of circumstances but it's still important to go in with a plan. In a large *S* game, it's possible to build large pots relative to the blinds; in a small *S* game, it is not. If you make a wise selection of starting hands, more often than not, you should find yourself ahead when the flop is dealt. That's no guarantee of success but it puts the wind at your back.

How about real investing? Here, a key strategic decision is what sorts of situations you will look for, ones where you think you could build an edge. In large *S* situations, potential investment amounts and returns are large relative to the cost of investigation; in small *S* situations, the reverse is true. If you choose wisely about what kind of opportunities to explore, you should find a good supply of deals in which you have that edge.

On the other hand, the complexity of even the simplified game should give you pause about using simple heuristics or models to make these decisions. After all, as the historian John Lukacs put it, "*Poker is the game closest to the Western conception of life, where life and thought are recognized as intimately combined, where free will prevails over philosophies of fate or of chance, where men are considered moral agents, and where — at least in the short run — the important thing is not what happens but what people think happens.*"

ENDNOTE

1. Anderson, J. 2006. Hedge Fund Manager Who Plays His Cards Right. *The New York Times*, August 11.