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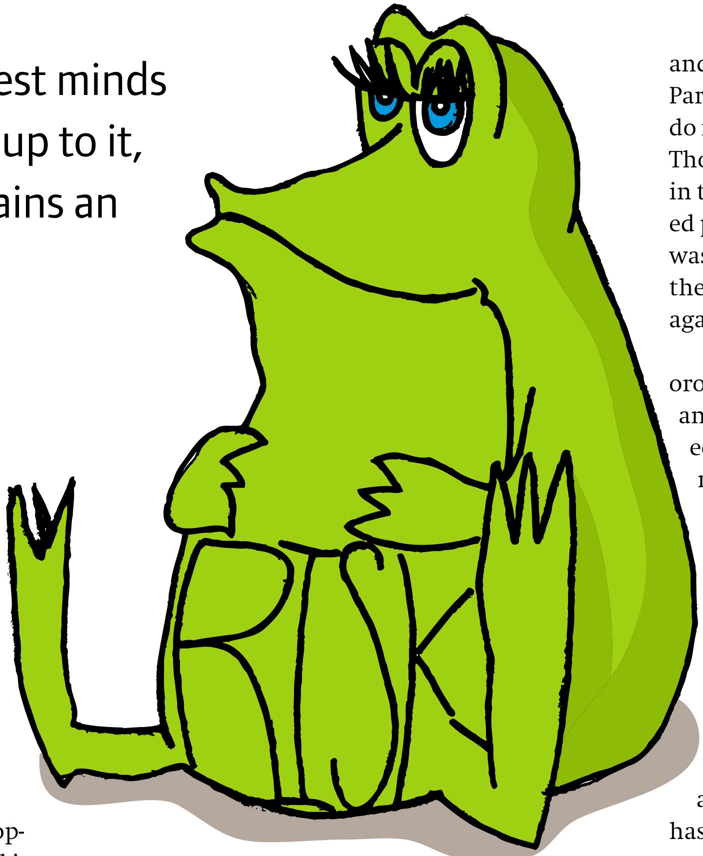
Risk: The Ugly History

Despite some of the greatest minds having mentally puckered up to it, this field of study still remains an untransformed frog

The mathematical study of risk began in 1654 with a famous exchange of letters between Pierre de Fermat and Blaise Pascal. If you like you can push the date back to Isaac Newton in 1610, Gerolamo Cardano in 1525 or Luca Pacioli in 1458, but it is still remarkably late considering that gambling is a universal human activity far older than history. Why didn't some earlier mathematician consider the problem? Why didn't some earlier gambler publish some useful inductions from experience?

The usual explanation is that philosophical and theological obstacles hindered development. But this won't convince anyone trained in finance. The more society discouraged rational approaches to gambling, the greater the rewards



After Fermat–Pascal

The mystery does not end in 1654. Fermat and

and today simple questions like the Necktie Paradox or the definition of a random number do not have fully satisfactory answers. When Ed Thorp figured out how to beat casino blackjack in the 1960s, many mathematically sophisticated people dismissed the work on the grounds it was impossible to gain an advantage by varying the bet in a game where the average odds are against you.

It's true that we've had a mathematically rigorous foundation for probability since the 1930s, and not one but four consistent and sophisticated ways to link mathematical probability to risk (by Von Neumann, Arrow-Debreu, Savage and Shannon). But this work does not correspond well with the actual risk faced by humans. In 1921, Frank Knight distinguished between "risk" and "uncertainty." With some oversimplification, he put everything modeled by probability theory in "risk" and everything people wanted to know about for practical decision-making in "uncertainty." In 1972 Daniel Kahneman and Amos Tversky began a field of study that has demonstrated the enormous gap between mathematical and behavioral concepts of risk.

I cannot think of any field of study so basic to human survival that started so late, progressed

Linguistic view

The word "risk" entered the English language in 1661. Although it comes from French and Italian, its origin and earlier history are unknown. Words that are related today had entirely different meanings. "Random" meant fast to Shakespeare, only later acquiring a connotation of careless, then haphazard, then unpredictable. The root of "danger" is the Latin "dominus" meaning "master." The word evolved to mean "under control of" then later "liable to a master." The transfer from the idea of liability or responsibility to a specific person to general possibility of harm came later. "Peril" meant to try something.

Other risk-related words had specific gambling meanings rather than uncertainty in general. "Hazard" comes from the Arabic for "dice." "Chance" meant "falling of the dice."

Of course, the fact that modern words for risk did not have their contemporary meanings does not mean there weren't words for risk in English before the 17th century. "Pleoh" is the Old English word usually translated as "danger," but it has the sense of "circumstance" like the modern "plight."

Going back farther, the most familiar passage referring to risk in the Bible is *Ecclesiastes* 9:11, "… the race is not to the swift, nor the battle to the strong … but time and chance happeneth to them all" in the King James translation. However, the Hebrew word does not imply randomness but simple circumstance. It's more "you win some, you lose some" than "wins and losses are uncaused." When the Philistines observe the path taken by an oxcart to see if their misfortunes are caused by the Hebrew God or by

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open the granary and take some of the corn," or if the owner of the house "deny that the corn was stored in his house." The penalty is the same for accidental loss, theft and fraud. The laws for physicians prescribe penalties if a patient dies without consideration of whether the physician was responsible. In certain cases, an accused is thrown into the river. The details for this are not known, but it is clear that people sometimes drowned and sometimes survived. In a more severe variant, the accused was tied up first, in which case they rarely survived. In either case, the result is not viewed as luck, because if the accused survived, he or she was assumed to be innocent and the accuser was punished.

Hammurabi is careful to excuse liability for acts of God, and the distinction lives on in modern insurance policies. The related concept force majeure was also carved in stone in 1800 BCE, Hammurabi excused carriers whose goods were seized by war enemies. So, the Code assumes someone (maybe a god or a rival king) is responsible for everything, nothing is random, there is a

the opponent cheats. One such victim, Nala, takes up service with a neighboring king. That king demonstrates his wisdom by counting the leaves and fruits on a large tree through examining a single twig. Nala offers to trade lessons in horsemanship for the secret to this feat. The king agrees, and tells Nala the secret will show him how to win at dice as well. Nala then goes home and wins back his kingdom. This appears not only to show a scientific knowledge of probability useful for dice playing, but connects that skill to reasoning from a sample. However, I know of no other evidence for either discovery.

The second exception also explains the term "premium" for insurance payments and option prices. "Bottomry" loans date back at least to the Phoenicians 3,000 years ago. They are loans secured by a ship, with the loan forgiven if the ship is lost. Bottomry lenders were granted exemptions from usury laws and allowed to charge a premium to the legal rate of interest. Legal cases are preserved in which the amount of the premium is challenged, but to my knowl-

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ping, in which A needed m heads to win and B needed n tails first.

Using modern notation, call $A(m, n)$ A's proportion of the stake. Fermat reasoned that $A(k, k) = 1/2$ for any k, because both players are in identical positions, and $A(m, n) = 1/2\ [A(m-1, n) + A(m, n-1)]$. Pascal provided the triangle (which he did not invent) to calculate these values quickly. Note, however, that the solution need not be the expected value of the outcome, in fact the coin probabilities are never used. All you need is a principle for dividing the stake when both parties are in the same situation and *reductio* (one of Fermat's signature techniques in number theory) provides the answer. This is much more similar to the binary version of the Black-Scholes argument than to the Binomial distribution in statistics. It applies more generally than expected value approach.

Pascal then made a fateful error. He confused the equity argument of Fermat with a frequentist argument, essentially that the stake should be divided by Monte Carlo simulation — completing the game many times and dividing the stake in

That would require measure theory in the 20th century, which provided the foundation but at the price of assumptions inapplicable to any real problems outside quantum physics. Bayesian statisticians attempt to avoid the dilemma by positing a subjective prior distribution, which ensures consistency but defines probability only subjectively. Nonparametric statistics also avoids the dilemma, but sacrifices a lot of power. The future of practical statistics probably lies in some combination of frequentism and non-parametric methods, something like the bootstrap, although so far no one has even figured out how to do good bootstrap regression analysis.

As a result of this confusion of theoretical approach, conventional statistics reverses the problem. It starts by introducing randomness, through the abstraction of a random variable generated by a distribution. The problem isn't that the world is predictable and we need a mechanism to introduce randomness; it's that the world is unpredictable and we need mathematics to make predictions. We have the data, and want to know the probability that a hypoth-

Despite these criticisms, probability theory works extremely well in analyzing gambling games (for which the assumption of introduced randomness is literally true) and for controlled experiments (experiments that have been converted to gambling games for the convenience of statisticians). It works for quantum physics, although a lot of people think it shouldn't. Its record in other fields is mixed. Smart, experienced, honest people, trained in probability theory and statistical practice can use mathematics to extract truth from an uncertain world. Certain tools developed for specialized problems work reasonably well most of the time. But these qualifications aside, most applications of statistics to uncontrolled situations are problematic.

The breakthrough

For 318 years, there were no statistical methods that combined rigor and practicality. The breakthrough came in 1973, with the publication of the Black-Scholes model. This led to modern derivative pricing which distinguishes between risk-neutral and simulation pricing. The first is firmly in the spirit of Fermat. It requires no assumptions about probabilities, no historical market data and no projections of future market behavior. However, it does require complete market data, which restricts its use to some simple (but important) special cases.

Monte Carlo simulation is Pascal's frequentist approach. It can price anything, but only with a probability model and the assumption that price equals risk-adjusted expected value. Many popular methods combine these two approaches, and in general they serve to verify