PSI AP Physics C – Work and Energy

(With Calculus)

Multiple Choice Questions

1. An object moves according to the function $x = t^{7/2}$ where x is the distance traveled and t is the time. Its kinetic energy is proportional to:

   (A) $t^{5/2}$ (B) $t^{7/2}$ (C) t^5 (D) $t^{3/2}$ (E) t^7

2. Which of the following best describes the relationship between force and potential energy?
   (A) Force is the anti-derivative of potential energy.
   (B) Force is the negative gradient of potential energy.
   (C) Potential energy is the negative gradient of force.
   (D) Potential energy is the derivative of force.
   (E) Force is the anti-derivative of potential energy.

3. A 5-kilogram ball moves in the x direction where x represents the ball's position. The potential energy U of the ball in Joules is given as a function: $U(x) = 4x^2 - 3x + 2$. The force on the particle at x = 4 m is:
   (A) 29 N in -x direction (B) 29N in +x direction (C) 108 N in -x direction
   (D) 45 N in -x direction (E) 108 N in +x direction

4. A student pushes a box across a rough, flat surface at a constant speed v. The box has a mass m, and the coefficient of sliding friction is represented by μ. The power supplied by the person to the box is
   (A) 0 (B) μmg/v (C) μv/mg (D) mg/μv (E) μmgv

5. The force exerted by a spring is given by: $F = \frac{kx^4}{2}$. If k is 100 N/m, find the work done by the spring on a mass from x = 0 m to x = 2 m.

   (A) 100 J (B) 320 J (C) 800 J (D) 1600 J (E) 2400 J

6. A man lifts a mass m at constant speed to a height h in time t. How much work is done by the weight lifter on the mass?

   (A) mgt (B) zero (C) mgh (D) mgh/t (E) mgt/h

7. A spring force is given by the formula F = 20x - 12x², where F is in N and x is in m. What is the change in potential energy when the spring is stretched 3 m from its equilibrium position?

(A) 18 J (B) -18 J (C) 56 J (D) -56 J (E) 64 J

8. On top of a skyscraper of height H, a ball of mass m is thrown directly downward with an initial speed v_o. What is the speed of the ball before it strikes the ground? Ignore air resistance.

(A) mgh-$\frac{1}{2}m(v_o^2-v_f^2)$ (B) mgh+$\frac{1}{2}m(v_o^2+v_f^2)$ (C) mgh+$\frac{1}{2}m(v_f^2-v_o^2)$
(D) $\sqrt{v_0^2+2gh}$ (E) $\frac{1}{2}m(v_f^2-v_o^2)$-mgh

9. A ball attached to a string rotates in a complete circle at a constant speed. The work done during each revolution is:

(A) 0 (B) U (C) U+Ke (D) Ke (E) Ke-U

10. The potential energy of two molecules is given by: U=$\frac{2}{r^7}-\frac{4}{r^5}$. If r is the distance between two molecules what is the force acting on the particles if r=1 m?

(A) 0.75 N (B) 0.67 N (C) 2 N (D) -6 N (E) 10 N

11. A force of 40 N compresses a spring with a spring constant 80 N/m. How much energy is stored in the spring?

(A) 10 J (B) 15 J (C) 20 J (D) 25 J (E) 30 J

12. What is the power delivered by gravity to a 6 kg block 4 s after it has fallen from rest?

(A) 2400 W (B) 1000 W (C) 800 W (D) 1200 W (E) 2000 W

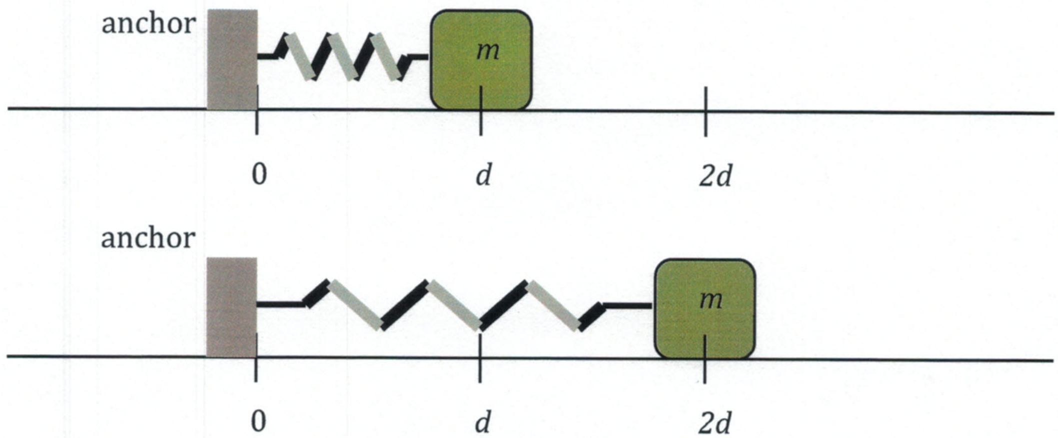
13. If F(x) = $8x^3$-$3x^2$ what is the work done from x =1 m to x =2 m?

(A) 0.5 J (B) 0.8 J (C) 2 J (D) 12 J (E) 23 J

14. A 2 kg block is pushed horizontally across a rough surface with a coefficient of kinetic friction of 0.2, at a constant speed of 4 m/s, by a force F. The work that is done by the force in 5 s is:

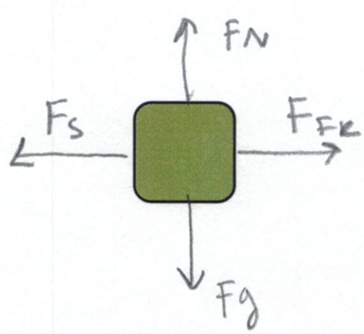
(A) 20 J (B) 40 J (C) 60 J (D) 80 J (E) 100 J

Part II. Free Response



6. A block of mass *m* rests on a rough surface, and has a light spring of spring constant *k* and unstretched length *d* attached to one side as shown, with the other end of the spring attached to an anchor. There is a static coefficient of friction μ_s between the surface and the block, and when the block is placed to the right at position 2*d*, it remains stationary on the surface. Express answers in terms of *m*, *k*, *d*, and fundamental constants.

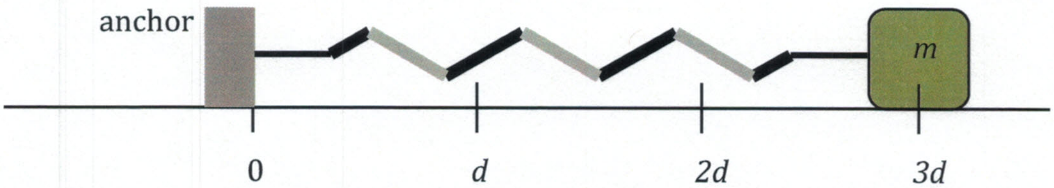
a. Draw a free-body diagram of the block at the time when it is located at position 2*d*.



b. Determine the friction force acting on the block when it is located at position 2*d*.

$F_s = F_{FR}$

$kd = F_{FR}$



c. The block is now moved to position $3d$ and released, where it remains at rest. When the block is moved slightly past this position, the block begins to slide along the surface with a kinetic coefficient of friction μ_k.

i. In terms of the variables given, what is the value of μ_s?

$$mg\mu_s = 2Kd$$

$$\mu_s = \frac{2Kd}{mg}$$

ii. How much potential energy is stored in the mass-spring system just before the block begins to move?

$$EPE = \frac{1}{2}K(2d)^2$$

$$2Kd^2$$

iii. The block slides a total distance of d before coming to a halt again. Determine the coefficient of kinetic friction μ_k.

$$KE_1 + U_1 + W_{NC} = KE_2 + U_2$$

$$0 + \frac{1}{2}K(2d)^2 - mg\mu d = 0 + \frac{1}{2}Kd^2$$

$$2Kd^2 - mg\mu d = \frac{1}{2}Kd^2$$

$$-mg\mu d = -\frac{3}{2}Kd^2$$

$$\mu_k = \frac{3Kd}{2mg}$$

iv. At what position does the block have its maximum velocity as it slides?

v where $\frac{dv}{dt}$ or acceleration $= 0$

$F_s = F_{FR}$ where $a = 0$

$mg\mu_k = Kx$

$mg\frac{3Kd}{2mg} = Kx$ $\quad x = \frac{3}{2}d$

$\frac{3}{2}d + d = \boxed{\frac{5}{2}d}$

v. What is the maximum velocity of the block as it slides?

$KE_1 + U_1 + W_{NC} = KE_2 + U_2$

$0 + \frac{1}{2}K(2d)^2 - \left[mg\frac{3}{2}\frac{Kd}{mg}\right]\frac{1}{2}d = \frac{1}{2}mv^2 + \frac{1}{2}K\left(\frac{3}{2}d\right)^2$

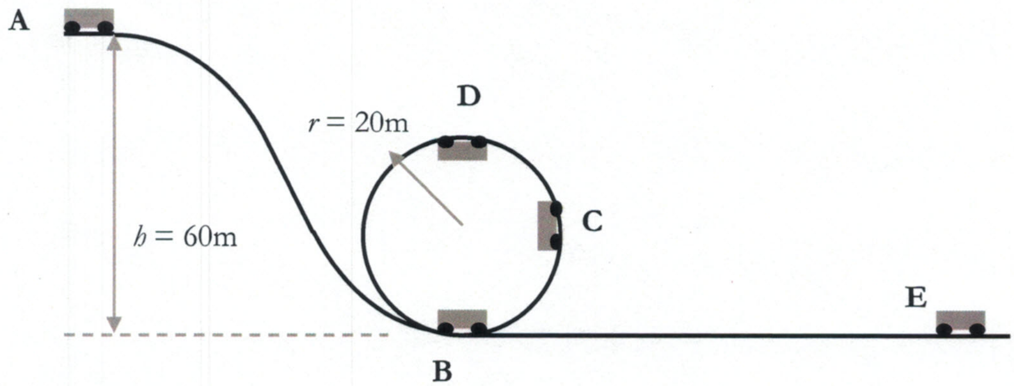
$2Kd^2 - \frac{3}{4}Kd^2 = \frac{1}{2}mv^2 + \frac{9}{8}Kd^2$

$16Kd^2 - 6Kd^2 = 4mv^2 + 9Kd^2$

$Kd^2 = 4mv^2$

$\vec{V} = \sqrt{\frac{Kd^2}{4m}}$

$\boxed{V = \frac{d}{2}\sqrt{\frac{K}{m}}}$



7. A roller coaster car of mass m = 200 kg is released from rest at the top of a 60 m high hill (position **A**), and rolls with negligible friction down the hill, through a circular loop of radius 20 m (positions **B**, **C**, and **D**), and along a horizontal track (to position **E**).

a. What is the velocity of the car at position **B**?

$U_1 = KE_2$

$(200)(9.8)60 = \frac{1}{2}(200)v^2$

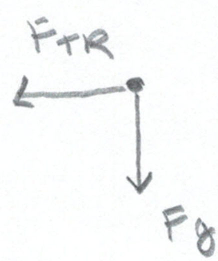
$V = 34$ m/s

b. Determine the velocity of the car at position **C**.

$200(9.8)(60) = \frac{1}{2}200v^2 + 200(9.8)(20)$

$V = 28$ m/s

c. Draw a free-body diagram of the car at position **C**.



d. Determine the velocity of the car at position **D**.

$(200)(9.8)(60) = \frac{1}{2}(200)v^2 + (200)(9.8)(40)$

$20.\ m/s$

e. Determine the force (magnitude and direction) of the track on the car at position **D**.

$F_c = \frac{(200)(20)^2}{20} = 4000N$

$F_g = (200)(9.8) = 1960$

$F_{TR} = 4000 - 1960$ $2040N$

f. After completing the loop, the rollercoaster car is travelling horizontally at velocity v_0 and subjected to a braking force $F_{braking} = -kv$, where k is a constant, v is the instantaneous velocity of the car, and time t is the amount of time that the braking force has been applied.

i. Develop a definite integral—expressed in terms of initial velocity v_0, k, m, and time t—that could be used to evaluate the velocity of the car along the horizontal track.

$ma = -KV$

$m\frac{dv}{dt} = -KV$

$\int_{V_0}^{V} \frac{dV}{V} = \int_0^t \frac{-K\,dt}{m}$

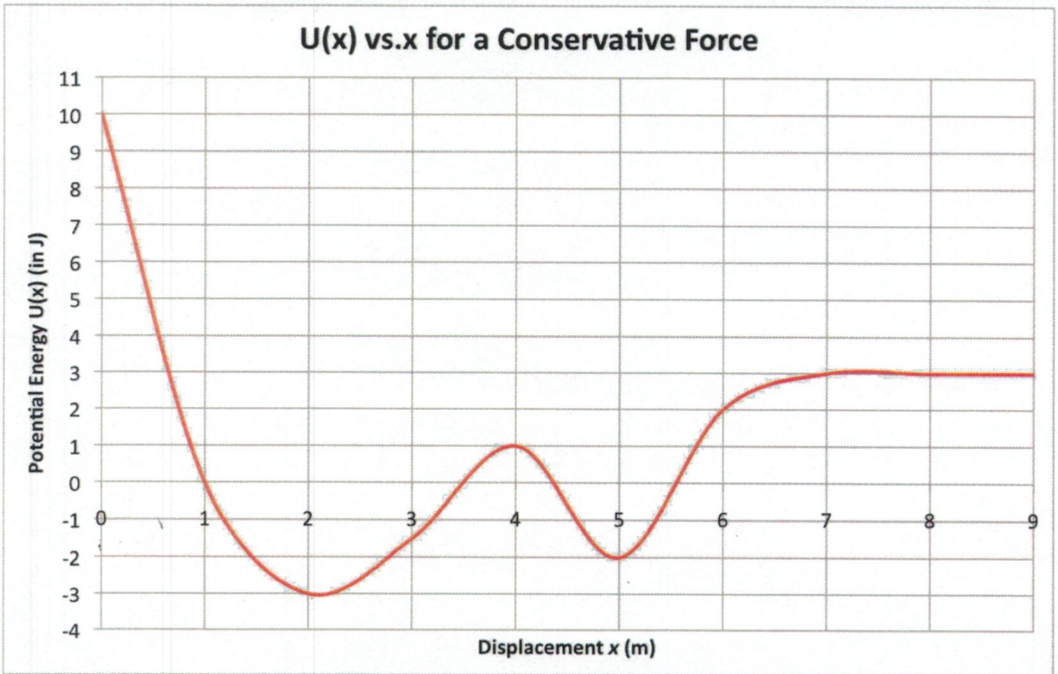
ii. Solve the integral to determine an equation that could be used to calculate the horizontal velocity of the car as a function of initial velocity v_0, k, m, and time t.

$\left[\ln V\right]_{V_0}^{V} = \left[\frac{-K}{m}t\right]_0^t$

$\ln V - \ln V_0 = \frac{-Kt}{m}$

$\ln\left(\frac{V}{V_0}\right) = \frac{-Kt}{m}$

$V = V_0 e^{\frac{-Kt}{m}}$



8. A conservative force acts in the x-direction on a particle of mass m = 2.0 kg to produce a potential energy curve as shown above.

a. A particle is released from rest at the 0.5 meter position.

i. What is the potential energy of the particle at this position?

$U_{.5} = 5\text{ J}$

ii. What is the velocity of this particle at x = 2 meters?

$U_1 = U_2 + KE_2$

$5 = -3 + \frac{1}{2}(2.0)v^2$

$v = 2.8\text{ m/s}$

iii. Describe the point on the U curve at x = 4m briefly, and what happens when the released particle reaches this position.

$U_{(4)} = 1\text{ J}$ and has a net force equal to zero since the slope is equal to zero. The mass will keep moving since it has 4 J of KE.

iv. Does the released particle reach x = 7 m? If not, explain why not. If so, describe the particle's behavior. IT CAN REACH THIS POINT SINCE $U_7 = 3\text{J}$ IS LESS THAN $U_{.5} = 5\text{J}$. IT WILL NOW MOVE AT CONSTANT VELOCITY.

b. A second particle, also of mass m, is released from rest at $x = 8$ m. Briefly describe the behavior of this particle.

This particle will have no acceleration And will Remain at Rest since The Force = 0 N

c. Sketch a graph of the conservative force that produces this potential energy curve.