

# An introduction to Matai - The Treasury's new macro-economic forecasting model

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An introduction to Matai - The Treasury's new macro-economic forecasting model

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## Executive summary

The Treasury uses a macro-economic forecasting model of the New Zealand economy to aid in the production of its economic and tax forecasts. During the Budget 2018 forecast round we have introduced a new model called Matai which attempts to provide a more accessible and up to date tool for the forecasters. It was designed with the following objectives in mind:

- a) To use standard definitions of the model variables that are widely used and understood by the broader economic community
- b) A special focus on forecasting nominal GDP (a key determinant of the tax base)
- c) A very simple/intuitive economic structure but calibrated to reflect as closely as possible the key results from in depth off model economic research
- d) Adopt a software package called EViews, which is widely used in the economic community

In Matai, all variables are split into trend and cycle. The (smooth) trend attempts to capture the elements of the data series that are expected to persist longer than the business cycle (structural elements). All short term and business cycle elements are captured in the gap between actual and trend. Potential output is forecast using a single production function that weights together trend estimates of inputs - the capital stock, labour, and multifactor productivity. The Output Gap is modelled as a sum of the individual expenditure gaps such as the Private Consumption Gap, Residential Investment Gap etc. Each expenditure component gap equation is a weighted sum of other gaps. For example the Consumption Gap is a function of the Real Wage Gap, House Price Gap, Interest rate Gap and the Personal tax rate Gap.

Matai focuses on forecasting Nominal GDP which is the product of four components – Trend real GDP (potential output), the Output Gap (real), the GDP deflator relative to the CPI, and the CPI.

A flexible structure allows the model to capture many types of important off model information. Model flexibility is illustrated by using a Fiscal shock to generate either Ricardian or Keynesian type results.

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## 1 Introduction

For over 10 years the Treasury has used a macro-economic model of the New Zealand economy to aid in the production of the economic and tax forecasts. The Budget 2018 forecast round saw the introduction of a new macro-economic forecasting model called Matai. This paper provides a non-technical description of Matai. The relative complexity of the previous model (NZTM) resulted in a number of disadvantages such as creating significant key person risk<sup>1</sup>.

Matai attempts to provide a more accessible and up to date tool for the forecasters. The key objective was to design a simpler, more transparent approach so that a much wider group of users were able to understand, operate and scrutinise the model forecasts.

## 2 Design strategy

We adopted a number of design choices to simplify the system namely:

- Use standard definitions of the model variables that are widely used and understood by the broader economic community.
- Focus the model on forecasting nominal GDP. The size of the model can be reduced if it is well targeted since more variables will only be added if it can be shown these have a material impact on forecasts of the key variable/s. In general the more key variables targeted, the larger the model. A smaller model, all else unchanged, is a simpler model.
- Adopt a very simple/intuitive economic structure but calibrate it to reflect off-model research. Matai employs a trend/cycle decomposition which is considered to be the simplest structure. A model becomes easier to understand if it adopts a common/consistent structure. Most macro forecast models have a trend/cycle decomposition in one or more areas but often mix this with a different approach in other parts of the model. For example an Output Gap (cycle) or Unemployment Gap is often used to explain domestic inflation. Matai applies the decomposition consistently to all variables and consistent treatment aids transparency. Apart from the consistency argument, a trend/cycle approach usually takes a very simple agnostic view of the trends, modelling them using a statistical filter. This decomposition has been adopted for many years in Real Business cycle and Dynamic Stochastic General Equilibrium (DSGE) models as a simplification device<sup>2</sup>. Matai's flexible semi-structural approach makes it relatively simple to include key results from DSGE, VAR models and any other relevant sources.

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<sup>1</sup> The degree of complexity severely limited the ability to pass on the skills required to understand and operate the model. Nevertheless it should be noted that the development, maintenance and use of NZTM played a successful and significant role in the production of economic forecasts for a number of years. The shift to the new model therefore reflects the evolution of the forecast process now that the use of an economic model is successfully embedded in the forecasting approach.

<sup>2</sup> DSGE models of the cycle are typically much more complicated than Matai as they include forward looking variables and micro-founded equations.

- Calibrate the model to reflect robust historical reduced form relationships. The aim is to minimise the need to re-calibration the model. Re-calibration creates additional complications when reconciling the new forecast with the old. In terms of equation contributions one not only needs to account for changes in the historical data but also changes in parameters<sup>3</sup>.
- Adopt a commonly used economic software package called EViews. The Previous model NZTM, was written in TROLL which is not nearly as widely used and understood.
- Create a suite of interactive diagnostic tools. If the model produces results that surprise the operator, it is essential to have the ability to quickly understand why. If the results are too difficult to understand, they are likely to be overwritten, negating the potentially useful insights produced by the model. Matai has a suite of add-ins that enable the user to quickly and easily understand the model results. Tools include graphical comparison functions along with equation and shock decomposition functions.

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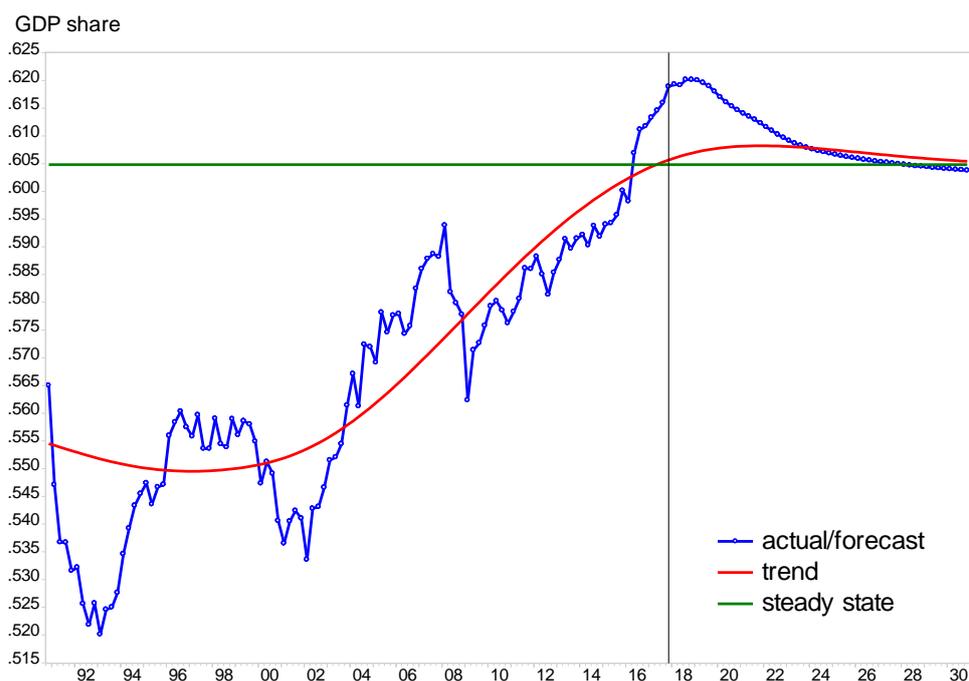
<sup>3</sup> Many national accounts revisions impact the level rather than the pattern in short term growth rates. Very often a statistical calculation of the trends automatically adjust to the revised level leaving the gap component much the same. In this instance there is little need to re-calibrate the equation parameters.

### 3 Model structure

#### 3.1 Trend/Cycle decomposition

All Matai variables are split into a trend and cycle(gap) component. The trends in the forecast period converge to an assumed steady-state (long run) position (see the Private Consumption example in figure 1 below).

Figure 1. Private Consumption share<sup>4</sup> of Potential GDP



The trend (red line) attempts to capture the elements of the data series that are expected to persist longer than the business cycle (structural elements). All short term and business cycle elements are captured in the gap between actual and trend (blue less red line). In the long-run the trend converges to the steady-state value (green line). By definition cycle components are temporary i.e all gaps are forecast to eventually close (blue line converges to red).

While most trends are just statistically smoothed (Hodrick-Prescott filtered) versions of the actual data and steady-state assumptions, key trends such as potential output (discussed below) are modelled in more detail.

Theory is used to create an economically coherent set of steady values (see table 1). The steady-state flow of Net Exports is set to be consistent with a stable Foreign Debt<sup>5</sup>

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<sup>4</sup> We forecast the share using Potential GDP as a scale factor i.e. for a given share, if the economy is twice as big, Private Consumption will be twice as big. Some approaches use population as this scale factor and model per capita consumption rather than modelling the share of Potential GDP.

<sup>5</sup> See Appendix 2 for more detail

(Net International Investment position or NIIP) to nominal GDP ratio around its historical average. Similarly, the Investment share is set consistent with producing a constant capital/output ratio. Public Consumption is simply assumed exogenously leaving Private Consumption as a residual that ensures all shares sum to unity. A higher assumption for Public Consumption would, all else unchanged, result in an exact offset in Private Consumption. If the terms of trade (prices) were higher, Net Exports (volumes) would be lower in order to produce the same value for the trade balance (the balance consistent with stable foreign debt to GDP). So a higher terms of trade results in a lower Net Exports share and higher Private Consumption share.

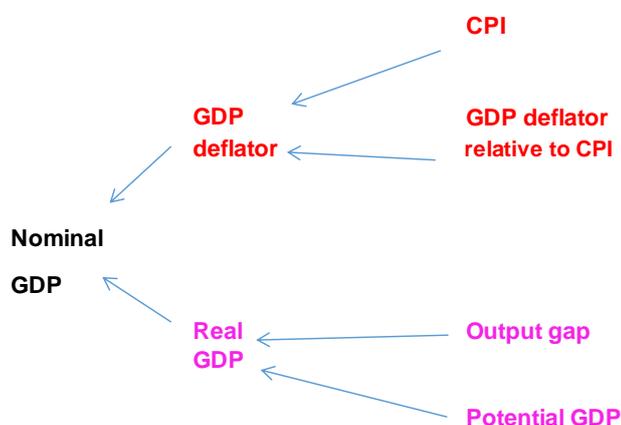
Table 1. Steady-state real expenditure shares of GDP

Expenditure component	Steady-state property	Value
Private Consumption	Residual	.605
Public Consumption	Exogenous	.18
Investment	Constant capital/output ratio	.275
Net Exports	Constant NIIP/GDP ratio	-.06

### 3.2 Nominal GDP forecast

Another design feature of Matai is the focus on forecasting nominal GDP which is modelled as four components – Trend real GDP (potential output), the Output Gap (real), the GDP deflator relative to the CPI, and the CPI. The product of the first two produces real GDP while the product of the last two creates the GDP deflator (figure 2).

Figure 2. Decomposition of Nominal GDP

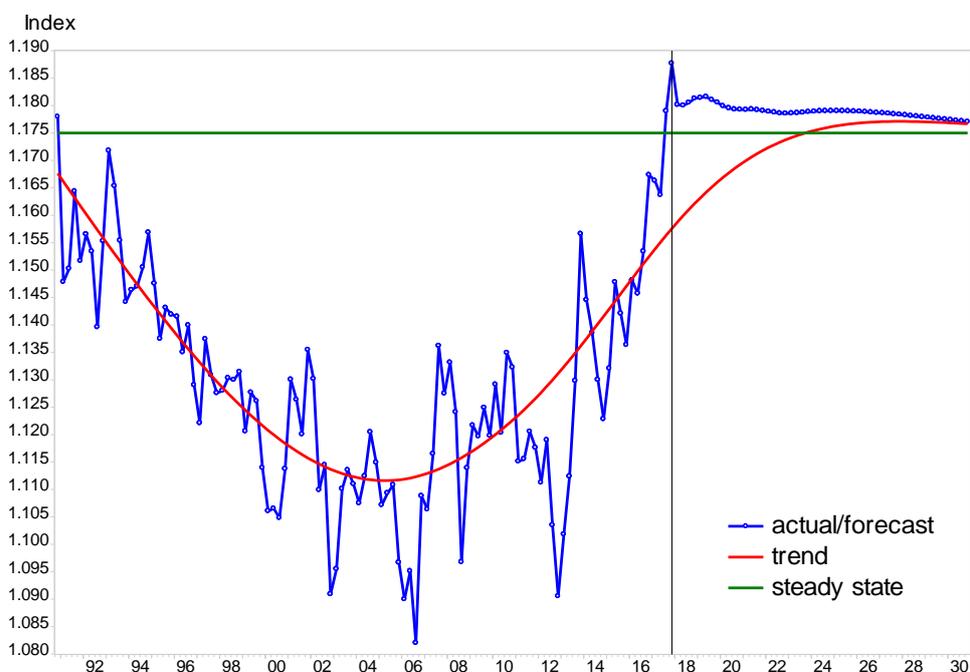


Potential output is forecast using a Cobb-Douglas production function that weights together trend estimates of the capital stock, labour, and total factor productivity<sup>6</sup>.

The Output Gap is modelled as a sum of the individual expenditure gaps such as the Private Consumption Gap, Residential Investment Gap etc. Each expenditure component gap equation is a weighted sum of other gaps<sup>7</sup>. For example the Private Consumption Gap is a function of the after tax Real Wage Income Gap, House Price Gap (deviation from fair value), and Interest rate Gap (deviation from assumed neutral interest rate).

The GDP deflator relative to CPI (PY/CPI) trend converges to an assumed steady-state (Figure 3). The cycles are modelled as the weighted sum of expenditure deflator gaps. Terms of trade cycles explain a significant proportion of the PY/CPI Gap.

Figure 3. GDP deflator relative to CPI



CPI inflation is modelled as the weighted sum of Non-tradable and Tradable inflation. Non-tradable CPI inflation is modelled using backward looking inflation expectations and the Output Gap while Tradable inflation is a function of inflation expectations and the Import Price Gap<sup>8</sup>.

The interest rate response in Matai is based on a Taylor rule. The Monetary policy response has quite important implications for the nominal GDP forecasts, since the

<sup>6</sup> See Appendix 2 for more detail

<sup>7</sup> See Appendix 3 for more detail

<sup>8</sup> See Appendix 4 for more detail

speed of interest rate responses to shocks impact the final level of the CPI. For example a slower interest rate response to a positive inflation shock will result in a permanently higher CPI level and hence a higher GDP deflator.

### 3.3 Key equations

Matai can be characterised as semi-structural or put another way – unlike DSGE models, equations are not rigorously derived from micro economic theory. Equations have been calibrated to reflect institutional knowledge gained by the practical use of many forecasting models in a live policy environment. As such they reflect experience with a broad range of modelling approaches each with differing degrees of theoretical and empirical rigor. The choice of right hand side variables is influenced by factors often referred to when monetary and fiscal authorities explain their forecasts. Equation coefficients are calibrated based on historical data but the choices also reflect as desire to produce plausible impulse responses. There are only a few (semi) structural cross equation coefficient restrictions reflecting the fact that the data doesn't perfectly measure many of the theoretical concepts employed in the model<sup>9</sup>.

#### Private Consumption Gap

$$cpg = 0.4 * wy\_pg - 0.1 * trwyg + 0.1 * phg - 0.1 * rg(-2) + shock$$

Economic theory suggests consumption should be related to interest rates, after tax real incomes and perceived financial wealth (house prices are used as a proxy). In the above equation we calibrate coefficients on the gaps in real compensation of employees (wy\_pg), personal tax rates (trwyg), house prices (phg) and interest rates (rg).

#### Residential investment

$$lhg = 38.1 * dlpopg + 0.28 * phg + 3.3 * (yg) - rg(-2) + shock$$

Population growth cycles (dlpopg) are the most important exogenous influences on housing investment. Existing house prices (phg) are included to pick up a Tobin's Q effect. The output gap (yg) has been added to capture confidence effects. Interest rates are included for theoretical reasons but are calibrated to have a small direct impact. Most of the interest rate impact on Residential Investment comes indirectly via house prices.

#### Business investment

$$lbg = 2.8 * yg(-1) + 0.43 * pxgf\_pfg(-1) - 0.3 * \underline{annual\_average(pmg)} - 0.5 * rg + shock$$

---

<sup>9</sup>For example CPI non-tradable inflation is a collection of regimen items considered to be mostly non-tradable such as construction costs. However the construction sector still uses a significant amount of imported (tradable) goods.

The output gap (yg) explains the bulk of business investment cycles. All else unchanged, a higher level of output increases the marginal product of capital which provides the motivation to increase investment. Commodity prices (pxg\_pfg) are included to reflect confidence effects. Interest rates (rg) and import prices (pmg) have minor effects but are included to reflect the cost of capital.

## CPI

$$dp = \lambda * dpn + (1 - \lambda) * dpt + \text{shock}$$

$$dpn = 0.1 * yg + dpe + \Theta + \text{shock}$$

$$dpt = dpe - \Theta \lambda / (1 - \lambda) + 0.03 * pmg + \text{shock}$$

$$\lambda = 0.55$$

$$\Theta = 0.2$$

Quarterly CPI inflation (dp) is modelled as the weighted sum of tradeable (dpt) and non-tradable (dpn) inflation rates. Data is seasonally adjusted so the shock term in dp captures statistical residuals (seasonal, weighting and rounding). Non-tradable CPI inflation is explained by the Output Gap (yg) and inflation expectations (dpe) while Tradable inflation is explained by the Import Price Gap (pmg) and expectations. Both equations have constants which reflect the average difference between Headline CPI inflation and the respective component. These constants are added to the aggregate CPI inflation expectation measure to create a component specific measure. E.g. aggregate expectations (dpe) plus the non-tradable constant ( $\Theta$ ) is a measure of non-tradable inflation expectations (dpe +  $\Theta$ ).

## CPI expectations

$$dpe = dpe(-1) * \sigma + dp * (1 - \sigma)$$

$$\sigma = 0.9$$

Expectations are modelled as an adaptive process with a weight of 0.9 on lagged expectations and 0.1 on current quarterly CPI inflation. The resulting time series broadly follows the RBNZ 1 year ahead survey measure post 2000.

## Exchange rate

$$vg = (1 + (1.4 * ((r - rt) - (rf - rft)) + 0.5 * (pxgf\_pfg)) / 100) + \text{shock}$$

The Real Exchange rate Gap (vg) is modelled using 90 day foreign (rf-rft) versus domestic (r-rt) interest rate differentials. World export prices (pxgf\_pfg) are also included to reflect terms of trade cycles.

## Domestic 90 day interest rate

$$rg = \psi * rg(-1) + (1 - \psi) * (yg + 10 * (dpe - dptar) + 0.5 * (d4p/4 - dptar)) + \text{shock}$$

$$\psi = 0.6$$

The monetary policy reaction function in Matai is a Taylor rule with a relatively high weight on the deviation of inflation expectations (dpe) from target (dptar). Annual inflation deviations from target (d4p/4-dptar) have a weight of 0.5. A 0.6 weight on the lag of nominal interest rates acts as an interest rate smoother.

#### Wage inflation

$$Q\%(wn)=\text{Annual\_average}(dptar/2+dpe/2)+ Q\%(yt/ht)+.1*yg(-3)+5*\log(wy\_p\_yt/(wy/p/yt*1000)) \text{ +shock}$$

Quarterly wage inflation is modelled with an inflation expectations term (dpe/2+dptar/2) plus trend labour productivity(yt/ht), the Output Gap (yg) and a level correction term which moves the labour share (wp/p/yt) towards its trend (closes the labour share gap). Expectations are a 50:50 weighting of CPI expectations (backward looking) and the CPI inflation target (which is forward looking in the sense that the model will eventually deliver targeted inflation). The approach reflects an assumption that wages are stickier than prices hence expectations are more forward looking than those used in the CPI inflation equations. The combination of the inflation expectations term and trend labour productivity growth term cause the labour share of income to eventually stabilise at a constant.

#### House price inflation

$$Q\%(ph/pht) = 8.8*( Q\%(lpop(-1)/lpopt(-1))) -24*rg +3.6*\log(pht(-1)/ph(-1)) \text{ +shock}$$

Quarterly house price inflation deviation from trend (ph/pht) is explained mostly by the population growth gap (lpop/lpopt) and the interest rate gap (rg). A levels correction term (pht/ph) is included to close the House Price Gap which is a key element in the private consumption equation.

## 4 Example scenarios

### 4.1 Two Public Consumption scenarios

This section illustrates the use of Matai by constructing two scenarios where each has a stronger path for Public Consumption. Internationally there is great deal of uncertainty around empirical estimates of fiscal multipliers as evident in table 2 below. The VAR estimate for New Zealand by Hamer-Adams and Wong at 0.87 is close to the maximum Matai value of 1.0. The two fiscal scenarios below explain how multipliers in Matai can range between 0 and 1.0.

Table 2. Cumulative fiscal multipliers

(4 quarter sum of change in output over 4 quarter sum of change in Real Public Consumption)

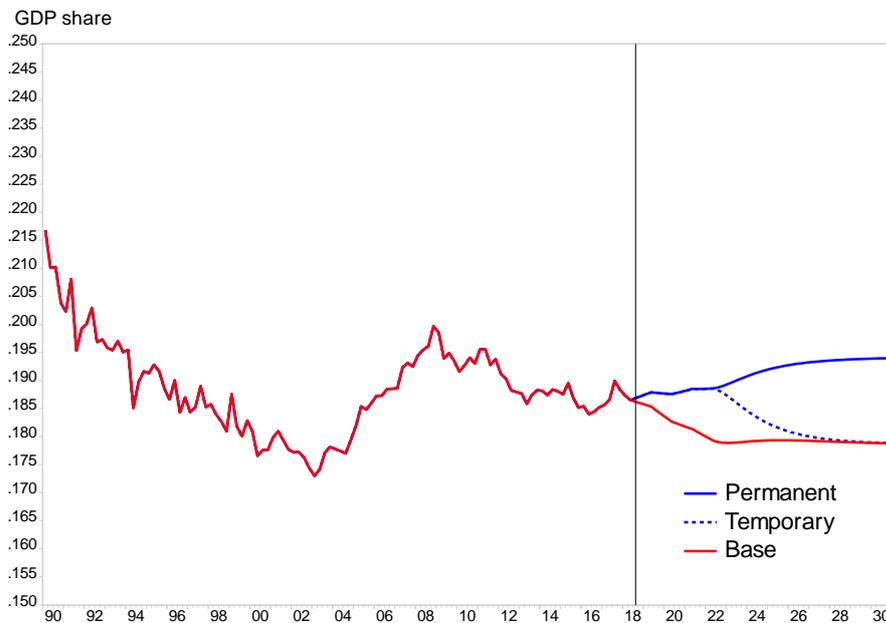
	Mean (range)	Source
International	0.85 (-2.0 to 3.6)	Gechert & Rannenburg (2014)
New Zealand	0.87	Hamer-Adams & Wong (2017)
	0.5 (0 to 1.0)	Matai

In Matai the Real Public Consumption shock can be implemented as either permanent (changed steady-state/trend) or temporary (just the cycle component changes)

- An assumed permanent shock can produce Ricardian equivalence – i.e. no change in output, interest rates and nominal GDP
- An assumed temporary shock delivers a Keynesian-type result - a strong cyclical impact on output, interest rates and nominal GDP

The two scenarios assume stronger paths for Public Consumption as shown in Figure 4 below. While the paths are identical up until 2022q2 they produce quite different effects over this horizon.

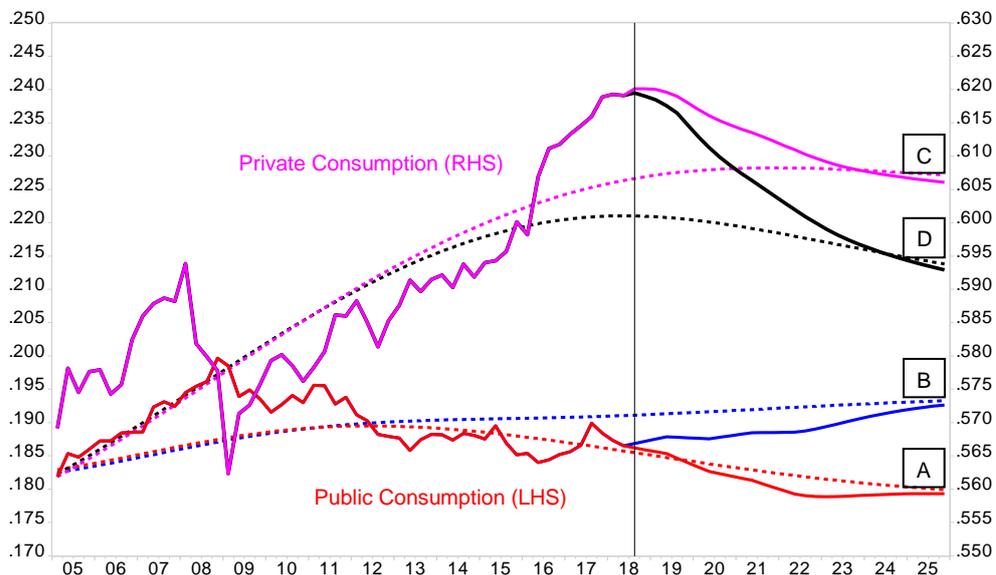
Figure 4. Two scenarios for the Public Consumption share of Potential GDP



## 4.2 Permanent Fiscal shock

In Figure 5 below the steady-state share of Public Consumption moves from A up to B. The trend is revised up and converges to this higher share. Since all steady-state shares must sum to unity and Private Consumption is the residual, the increase in Public Consumption is exactly offset by a decrease in Private Consumption which moves from C to D. The lower end point for Private Consumption causes a downward revision in the path of the trend. The forecasts for Private Consumption converge to this lower trend. Lower Private Consumption more or less offsets higher Public Consumption. While the composition of the Output Gap changes, the aggregate gap is mostly unchanged hence inflation and interest rates are unaffected.

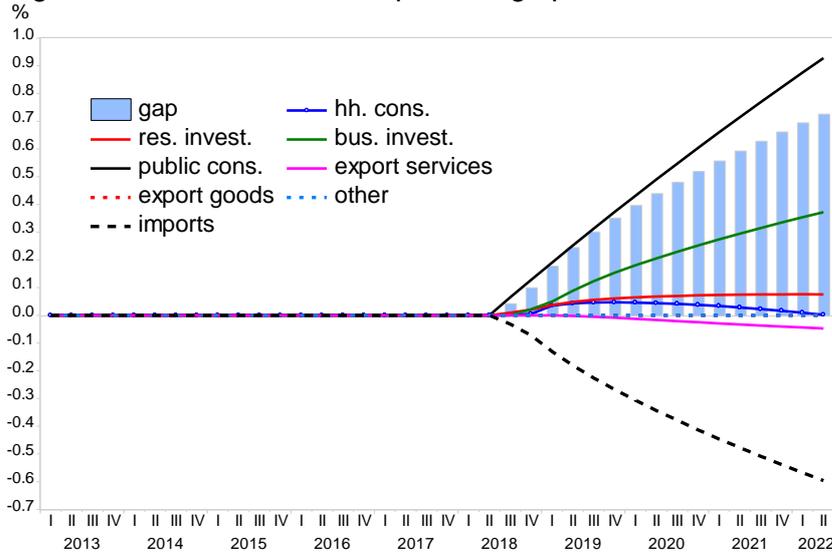
Figure 5. Private and Public consumption shares of Potential GDP (dotted lines are trends)



## 4.3 Temporary Fiscal shock

In the temporary shock scenario, the trend is unchanged, hence all of the higher Public Consumption appears in the gap (cycle component). Since Public Consumption is a component of the output gap, it directly boosts the Output Gap (the solid black line in Figure 6). The impact on the Output Gap is moderated by higher imports but boosted by an Output Gap accelerator effect on Business Investment. There are also small positive contributions from Private Consumption and Residential Investment. One can also see from figure 6 that the sum of Output Gaps over the first year is the same as the sum of the direct contributions from Public Consumption (i.e. a cumulative multiplier of 1).

Figure 6. Contributions to the percentage point difference in the Output Gap



The higher Output Gap (Figure 7) produces higher inflation (Figure 8) which the central bank partly offsets with higher interest rates (Figure 9). Nominal GDP (Figure 10) is around 1.1% points higher by 2022q2, with two thirds of this difference explained by a higher Output Gap and the rest by a higher CPI level (Figure 11).

Figure 7. Output Gap

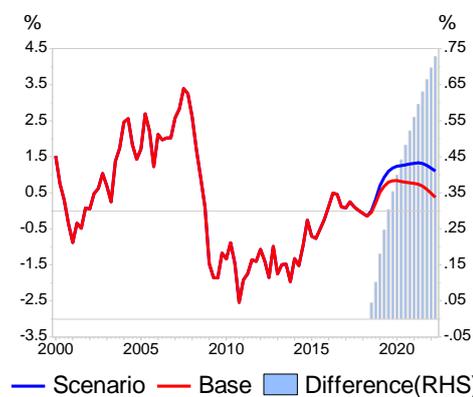


Figure 8. Annual CPI inflation

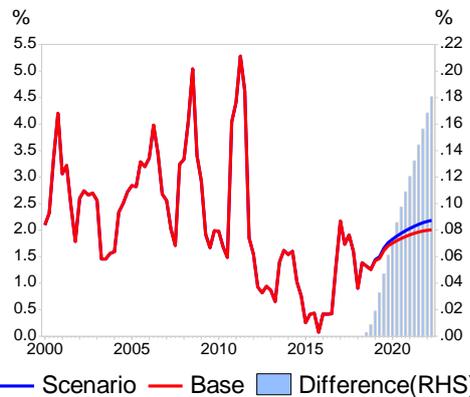


Figure 9. 90 day interest rate

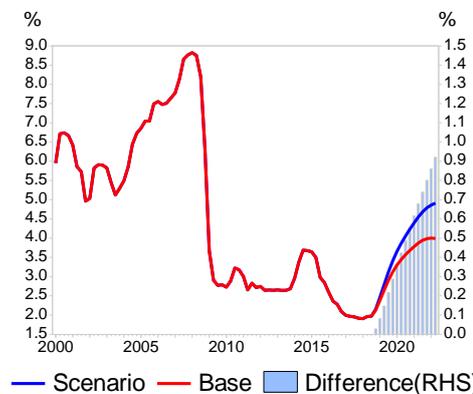


Figure 10. Nominal GDP

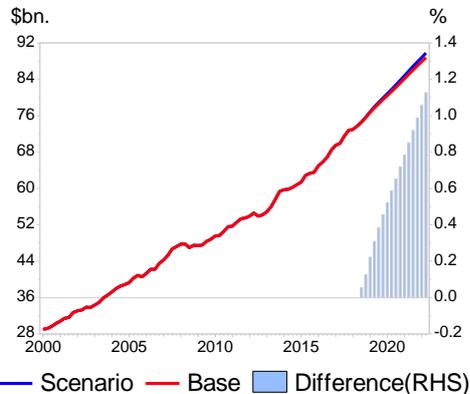
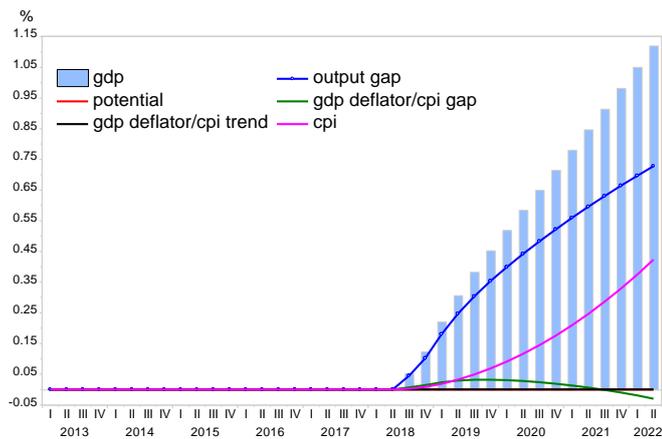


Figure 11. Contributions to the percentage point difference in nominal GDP



## 5 Next steps

We are planning to formally estimate the model and conduct sensitivity analysis. The plan is to include these results in a paper that provides a more detailed technical description of Matai.

We also plan to develop the fiscal block (which forecasts the operating balance) which is currently simply a place holder.

## References

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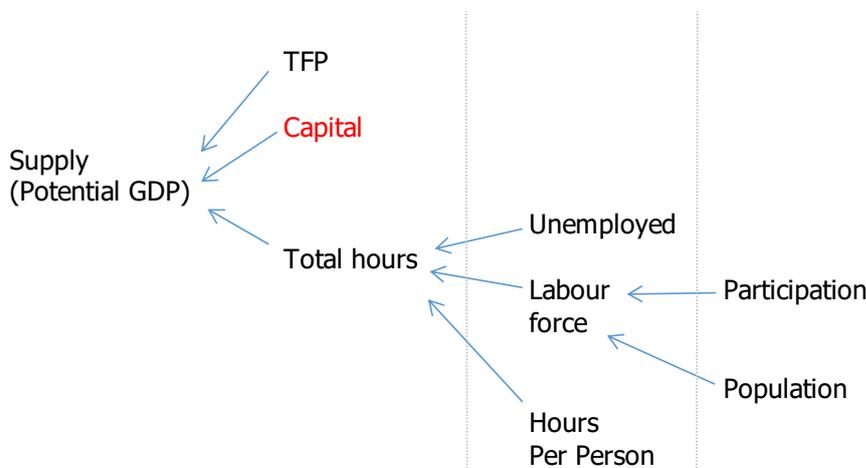
## Appendix 1: Potential Output

Potential GDP is forecast using a nested Cobb-Douglas production function,

$$Y_t = TFP_t K_t^\alpha H_t^{1-\alpha}, \quad (1)$$

in which  $Y_t$  is potential GDP,  $TFP_t$  is trend total factor productivity,  $K_t$  the trend capital stock, and  $H_t$  the trend total hours worked by the labour force. The output elasticities of capital ( $\alpha$ ) and labour are set to 0.4 and 0.6 respectively. Total hours is further broken down into trends for the employment rate (1-unemployment rate), hours worked per person, participation rate and working age population (Figure 10).

Figure 10



The trends for each component are forecast with an autoregressive process which has one of three forms:

Where  $0 < \mu < 1$

AR1 in growth rates (used for population<sup>10</sup> and TFP)

$$\Delta X(t) = \mu \Delta X(t-1) + (1 - \mu) \Delta X_{\text{steady\_state}} \dots \dots \dots (1)$$

Levels error correction model (used for participation, unemployment and hours per person)

$$\Delta X(t) = \mu \Delta X(t-1) + \psi (X_{\text{steady\_state}} - X(t-1)) \dots \dots \dots (2)$$

In the forecasting period, the growth rate of capital stock (sum of residential and business capital stock) follows the following process:

$$\Delta X(t) = \mu \Delta X(t-1) + (1 - \mu) (\text{Potential\_growth}) \dots \dots \dots (3)$$

---

<sup>10</sup> Also includes a level correction term to converge trend population to forecasts of actual population

Capital growth rate is endogenous and will slowly converge to the potential GDP growth rate implying a constant capital/output ratio in the steady-state.

The assumed steady-state values at the time of writing are contained in Table 3

Table 3 Steady-state assumptions

Variables	Steady-state value
Capital trend (kt)	Potential GDP growth rate
Population trend (lpopt)	0.9% per annum
Participation rate trend (lppt)	69.5%
Unemployment rate trend (ut)	4.25%
Hours per person trend (hppt)	33.5 hours
TFP trend (tfpt)	0.8% per annum

## Appendix 2: **Steady-state Foreign Debt ratio**

This appendix explains the derivation of the net export steady-state share of potential output assumed in Matai.

First we derive a steady-state value for the current account balance as a share of nominal GDP. The current account balance as a percentage of GDP can be stated as the change in debt as a percentage of GDP (i.e. assume no revaluations or statistical residuals in the steady-state). This can be expressed as follows:

$$CA/YN = \Delta B/YN$$

Where CA is the current account balance YN is nominal GDP and  $\Delta B$  is the change in NIIP.

$$CA/YN = B/YN - B_{-1}/Y_{N-1}$$

Multiplying the final term by  $Y_{N-1}/Y_{N-1}$  simplifies to

$$CA/YN = B/YN - B_{-1}/Y_{N-1} * (Y_{N-1}/Y_N)$$

Using  $b$  (lower case) to represent the NIIP ratio,  $B/YN$ , we can express as follows

$$CA/YN = b - b_{-1} * (Y_{N-1}/Y_N)$$

The Matai definition of the steady-state is that the NIIP ratio must be constant i.e.  $b = b_{-1}$  so we can write

$$CA/YN = b * (1 - Y_{N-1}/Y_N)$$

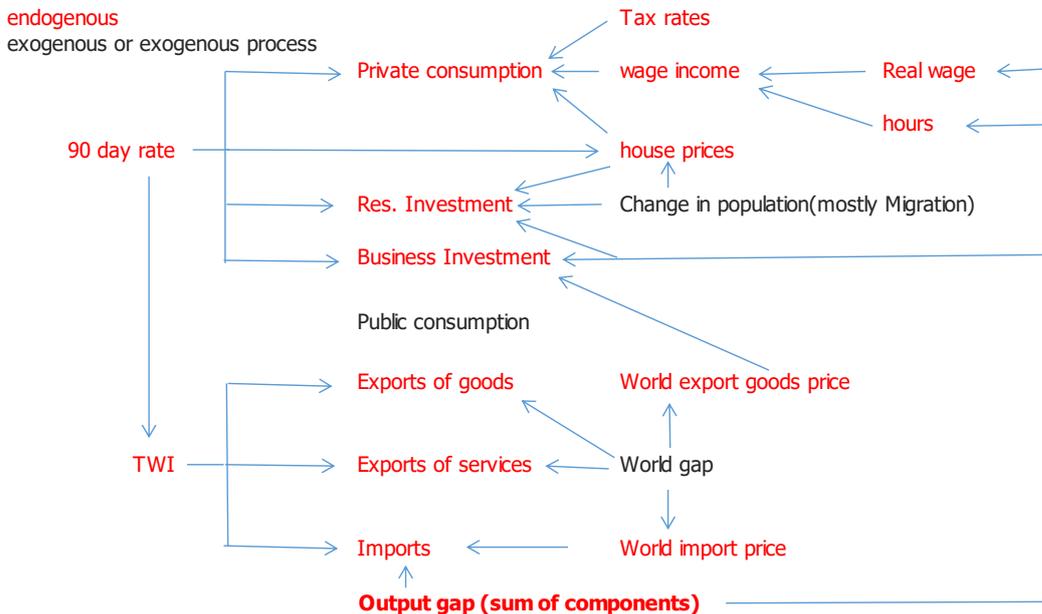
Where  $g$  is nominal GDP growth  $(Y_N/Y_{N-1} - 1)$  we can write

$$CA/YN = b * g / (1 + g)$$

Calculating the current account balance as a percentage of GDP consistent with a constant NIIP ratio is then simply a matter of making some assumptions about the steady-states for nominal GDP growth and the NIIP ratio. Historically, the NIIP ratio (net NIIP to GDP) has moved around -70%. Trend potential GDP growth is assumed to be 4.5% (2.5% real 2% inflation). If these assumptions are used then the current account ratio would be -3.0%. We then assume a net transfers to GDP balance of -3.8% to derive a trade balance share of 0.8%. Finally we assume a steady-state level for the terms of trade which allows us to derive the net export share of -0.6%.

## Appendix 3: The Output Gap

Figure 11



Key exogenous influences on the Output Gap are

- The world Output Gap which influences the terms of trade and net exports
- Personal income tax rates
- Migration

Key endogenous influences (feedback loops) from the Output Gap and back again are

- Imports
- Business Investment
- Hours worked and with a lag, wages. Both these combine to produce higher real wage incomes and stronger consumption

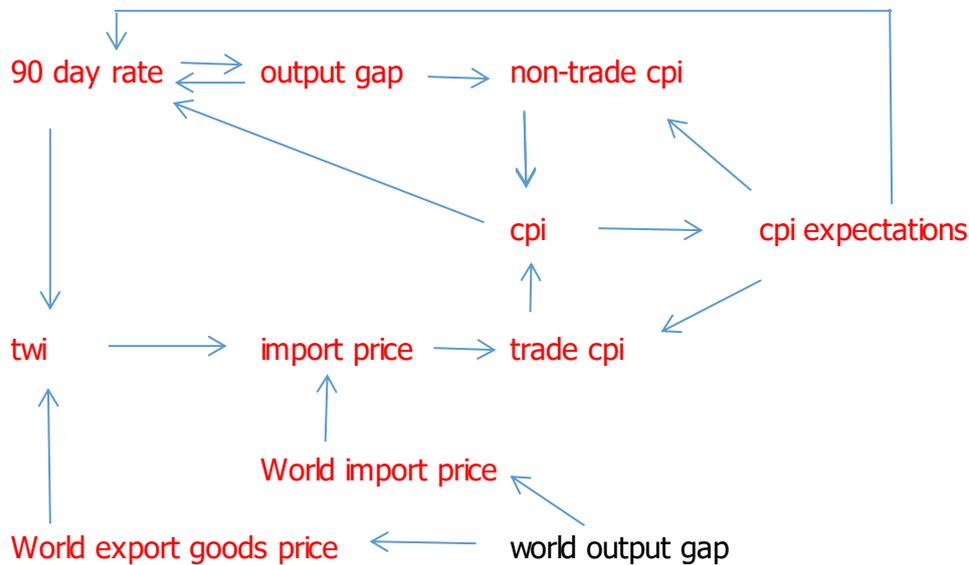
In the absence of ongoing shocks, endogenous monetary policy eventually delivers an Output Gap consistent with targeted inflation. For example, if inflation expectations are above target, interest rates will adjust to achieve the inflation target by creating an offsetting negative Output Gap. When inflation is held at the target, inflation expectations will gradually converge back to target and the Output Gap will slowly close.

Monetary policy is transmitted to the Output Gap through a number of channels. Interest rates impact on Private Consumption directly and also indirectly through their impact on house prices. Interest rates also directly affect residential and business investment. On the real side the interest rates impact on the exchange rate, causes a shift in relative prices and therefore net export volumes.

## Appendix 4: CPI inflation

Figure 20

### CPI gaps model



CPI inflation is controlled by a monetary policy reaction function which is modelled as a Taylor rule with a large weight on expected CPI inflation. The transmission of monetary policy to inflation occurs through interest rates and their impact on the Output Gap (non-tradeable inflation) and the exchange rate (domestic import prices and tradable inflation).

CPI inflation feeds in to CPI expected inflation which is modelled as a weighted average of past actual CPI inflation (using geometrically declining weights). CPI expectations for tradables and non-tradables are the aggregate CPI inflation expectation plus a constant reflecting the ratio of tradables to non-tradables in the CPI regimen.

Terms of trade cycles are a function of the world Output Gap and these impact on inflation directly through changes in world import prices and indirectly through the impact of world export prices on the exchange rate. World exports of goods price cycles move by a factor of 3 with the world Output Gap while import prices move by a factor of 1.5 making the terms of trade pro-cyclical to world activity.

Non-tradeable inflation is a function of the Output Gap and non-tradeable inflation expectations.

Tradable inflation is the function of the Import Price Gap and tradable inflation expectations. Note that tradable inflation is often modelled in change form i.e. tradable inflation reflects import price inflation not the Import Price level. In Matai tradable

inflation reflects the Import Price level (gap). If the Import Price Gap is positive, then tradable inflation will be above its mean (assuming expectations are on target).

## Appendix 5: Model variable index

Variable types can be exogenous, endogenous, identity, exogenous parameter and exogenous process (i.e. just a function of its own lags). Endogenous and exogenous processes have equation add factors (also referred to as equation residuals or shocks).

Block	Name	Description	Type
supply	lpop	Working age population	exog proc
supply	dlpop	log change in Working age population *100	identity
supply	lpr	Labour force participation rate	endog
supply	u	Unemployment rate	endog
supply	hpp	Hours worked per worker	endog
supply	h	Total hours	identity
supply	lpopt	Working age population trend	endog
supply	dlpopt	log change in Working age population trend *100	identity
supply	lprt	Labour force participation rate trend	exog proc
supply	ut	Unemployment rate trend	exog proc
supply	hppt	Hours worked per worker trend	exog proc
supply	ht	Total hours trend	identity
supply	tfpt	Total factor productivity trend	exog proc
supply	kt	Capital stock trend	endog
supply	yt	Potential Prodn. Real GDP	identity
supply	dyt	Q% Potential Prodn. Real GDP	identity
supply	dlpopg	log change in Working age population gap *100	identity
supply	lprg	Labour force participation rate gap	identity
supply	ug	Unemployment rate gap	identity
supply	hppg	Hours worked per worker gap	identity
supply	dlpop_ss	A% steady-state Population growth	exog param
supply	lpr_ss	Steady-state participation rate	exog param
supply	u_ss	Steady-state unemployment rate	exog param
supply	hpp_ss	Steady-state hours per person	exog param
supply	dtfp_ss	A% steady-state TFP growth	exog param
demand	y	Prodn. Real GDP	identity
demand	dy	Q% Prodn. Real GDP	identity
demand	d4y	A% Prodn. Real GDP	identity
demand	cp	Real private consumption	endog
demand	ih	Real housing investment	endog
demand	ib	Real non-housing investment	endog
demand	cg	Real Public consumption	endog
demand	xs	Real export services	endog
demand	xg	Real exports of goods	endog
demand	m	Real total imports	endog
demand	yo	Real gdp balancing item	endog

demand	cp_yt	Real private consumption trend share of potential	identity
demand	ih_yt	Real housing investment trend share of potential	exog
demand	ib_yt	Real investment (non-housing) trend share of potential	exog
demand	cg_yt	Real Public consumption trend share of potential	exog
demand	xs_yt	Real export services trend share of potential	exog
demand	xg_yt	Real exports of goods trend share of potential	exog
demand	m_yt	Real total imports trend share of potential	exog
demand	yo_yt	Real gdp balancing item trend share of potential	exog
demand	yg	Output Gap	identity
demand	cpg	Real private consumption gap	identity
demand	ihg	Real housing investment gap	identity
demand	ibg	Real non-housing investment gap	identity
demand	cgg	Real Public consumption gap	identity
demand	xsg	Real export services gap	identity
demand	xgg	Real exports of goods gap	identity
demand	mg	Real total imports gap	identity
demand	yog	Real gdp balancing item gap	identity
prices	py	GDP production deflator	identity
prices	dpy	Q% GDP production deflator	identity
prices	pcp	Private consumption deflator	endog
prices	pih	Residential investment deflator	endog
prices	pib	Real non-housing investment deflator	endog
prices	pcg	Govt. consumption deflator	endog
prices	pxs	Export of services deflator	endog
prices	pxgf	Export goods prices (in foreign currency)	identity
prices	pxg	Export of goods deflator	endog
prices	pmf	Import prices (in foreign currency)	identity
prices	pm	Imports of deflator	endog
prices	tot	G+S terms of trade	identity
prices	py_pt	GDP production deflator to cpi trend	exog
prices	pcp_pyt	Private consumption deflator to py trend	exog
prices	pih_pyt	Residential investment deflator to py trend	exog
prices	pib_pyt	Real non-housing investment deflator to py trend	exog
prices	pcg_pyt	Govt. consumption deflator to py trend	exog
prices	pxs_pyt	Exports of servicest deflator to py trend	exog
prices	pxgf_pft	Real export goods prices (in foreign currency) trend	exog
prices	pxg_pyt	Exports of goods deflator to py trend	exog
prices	pmf_pft	Real import prices (in foreign currency) trend	exog
prices	pm_pyt	Imports deflator to py trend	exog
prices	py_pg	GDP production deflator to cpi gap	identity
prices	pcpg	Private consumption deflator gap	identity
prices	pihg	Residential investment deflator gap	identity

prices	pihg	Real non-housing investment deflator gap	identity
prices	pcgg	Govt. consumption deflator gap	identity
prices	pxsg	Export of services deflator gap	identity
prices	pxgf_pfg	Real export goods prices (in foreign currency) gap	identity
prices	pxgg	Export of goods deflator gap	identity
prices	pmf_pfg	Real import prices (in foreign currency) gap	identity
prices	pmg	Imports deflator gap	identity
prices	p	CPI splice ex interest s.a	identity
prices	dp	Q% CPI splice ex interest s.a	endog
prices	d4p	A% CPI splice ex interest s.a	identity
prices	dpn	Q% inflation rate of CPI non-tradables (sa)	endog
prices	dpt	Q% inflation rate of CPI tradables (sa)	endog
prices	dptar	Q% inflation target	endog
prices	dpe	Q% inflation expectations	endog
prices	ph	House price index (sa)	endog
prices	wn	QES hourly wage rate	endog
prices	pht	House price index (sa) trend	endog
prices	phg	House price index (sa) gap	identity
nominal	yn	Nominal prodn. GDP	identity
nominal	dyn	Q% Nominal prodn. GDP	identity
nominal	cpn	Nominal private consumption	identity
nominal	ihn	Nominal housing investment	identity
nominal	ibn	Nominal non-housing investment	identity
nominal	cgn	Nominal Public consumption	identity
nominal	xsn	Nominal export services	identity
nominal	xgn	Nominal exports of goods	identity
nominal	mn	Nominal total imports	identity
nominal	yon	Nominal gdp balancing item	endog
nominal	wy	Compensation of employees	endog
nominal	yon_ynt	Nominal gdp balancing item to GDP trend	exog
nominal	wy_pt	Compensation of employees to cpi trend	identity
nominal	wy_p_yt	Real Compensation of employees to potential GDP trend	exog
nominal	wy_pg	Compensation of employees to cpi gap	identity
financial	r	90-day bank bills	endog
financial	s	Nominal twi exchange rate	endog
financial	v	Real exchange rate index	identity
financial	rt	90-day bank bills trend	exog
financial	st	Nominal twi exchange rate trend	identity
financial	vt	Real exchange rate index trend	exog
financial	pf_pt	Foreign to domestic cpi - trend	endog
financial	vg	Real exchange rate index gap	identity
financial	rg	90-day bank bills gap	identity

foreign	pf	Foreign cpi	endog
foreign	rf	Foreign 90 day interest rate	exog proc
foreign	yf	Foreign GDP	endog
foreign	rft	Foreign 90 day interest rate trend	exog
foreign	yft	Foreign potential GDP	exog proc
foreign	yfg	Foreign Output Gap	identity
fiscal	twy	Tax on wage income	identity
fiscal	tgst	GST	identity
fiscal	tb	Tax on other personal and company (business)	identity
fiscal	go	Other net Public spending	identity
fiscal	trwy	Effective tax rate on wage income	exog proc
fiscal	trgst	GST rate	exog proc
fiscal	trb	Tax rate on other personal and company (business)	exog proc
fiscal	trwyt	Effective tax rate on wage income trend	endog
fiscal	trwyg	Effective tax rate on wage income gap	identity
fiscal	ob	OBEGAL	identity
fiscal	ob_yn	OBEGAL as % of nominal GDP	identity
fiscal	d4ob_yn	Annual change in OBEGAL as % of nominal GDP	identity
Fiscal	go_yn_ss	Other net Public spending steady-state GDP share	exog param

## Appendix 6: Model equations

Shocks are in red and have an \_a suffix

### Supply 100

$$\begin{aligned} d\log(lpopt) &= d\log(lpopt(-1)) * \omega + (1 - \omega) * d\log(lpopt\_ss/400) + lpopt\_a & \omega &= 0.85 \\ d\log(lpopt) &= d\log(lpopt) * 100 \\ lpr &= lpr * (1 + 0.15 * yg/100) + lpr\_a \\ u &= ut - yg/1.5 + u\_a \\ hpp &= hpp * (1 + 0.15 * yg/100) + hpp\_a \\ h &= lpopt * lpr / 100 * (1 - u/100) * hpp \\ d\log(lpopt) &= \pi * d\log(lpopt(-1)) + (1 - \pi) * d\log(lpopt\_ss/400) - .005 * \log(lpopt/lpop) + lpopt\_a & \pi &= 0.9 \\ d\log(lpopt) &= d\log(lpopt) * 100 \\ d(lprt) &= .9 * d(lprt(-1)) + .001 * (lpr\_ss - lprt(-1)) + lprt\_a \\ d(ut) &= .9 * d(ut(-1)) + .01 * (u\_ss - ut(-1)) + ut\_a \\ d(hppt) &= .9 * d(hppt(-1)) + .001 * (hpp\_ss - hppt(-1)) + hppt\_a \\ ht &= lpopt * lprt / 100 * (1 - ut/100) * hppt \\ d\log(tfpt) &= \beta * d\log(tfpt(-1)) + (1 - \beta) * \log(1 + dtfp\_ss/400) + tfpt\_a & \beta &= 0.95 \\ d\log(kt) &= \mu * d\log(kt(-1)) + (1 - \mu) * dyt/100 + kt\_a & \mu &= 0.95 \\ \log(yt) &= (1 - \alpha) * \log(ht) + \alpha * \log(kt) + \log(tfpt) & \alpha &= 0.4 \\ dyt &= @pc(yt) \\ d\logpop &= d\logpop - d\logpop \\ lprg &= lpr - lprt \\ ug &= u - ut \\ hppg &= hpp - hppt \end{aligned}$$

### Demand 200

$$\begin{aligned} y &= cp + ih + ib + cg + xs + xg - m + yo \\ dy &= @pc(y) \\ d4y &= @pcy(y) \\ cp &= cp\_yt * yt * (1 + (0.4 * wy\_pg - 0.1 * trwyg + 0.1 * phg - 0.1 * rg(-2)) / 100) + cp\_a \\ ih &= ih\_yt * yt * (1 + (3.3 * (yg) + 38.12 * d\logpopg + 0.28 * phg - rg(-2)) / 100) + ih\_a \\ ib &= ib\_yt * yt * (1 + (0.43 * pxgf\_pfg(-1) + 2.8 * yg(-1) - .3 * @movav(pmg, 4) - .5 * rg) / 100) + ib\_a \\ cg &= cg\_yt * yt * (1 + (.8 * cgg(-1)) / 100) + cg\_a \\ xs &= xs\_yt * yt * (1 + (-(.3 * vg(-1) + .4 * vg(-2) + .3 * vg(-3)) * .5 + yfg(-2)) / 100) + xs\_a \\ xg &= xg\_yt * yt * (1 + (0.5 * yfg(-1) + .5 * xgg(-1)) / 100) + xg\_a \\ m &= m\_yt * yt * (1 + (1.3 * (yg + .3 * mg) - 0.1 * @movav(pmg, 4)) / 100) + m\_a \\ yo &= yo\_yt * yt * (1 + (0.0 * yog(-1)) / 100) + yo\_a \\ cp\_yt &= 1 - ih\_yt - ib\_yt - cg\_yt - xs\_yt - xg\_yt + m\_yt - yo\_yt \\ yg &= \log(yt) * 100 \\ cpg &= (cp / (cp\_yt * yt) - 1) * 100 \\ ihg &= (ih / (ih\_yt * yt) - 1) * 100 \\ ibg &= (ib / (ib\_yt * yt) - 1) * 100 \\ cgg &= (cg / (cg\_yt * yt) - 1) * 100 \\ xsg &= (xs / (xs\_yt * yt) - 1) * 100 \end{aligned}$$

$xgg=(xg/(xg\_yt*yt)-1)*100$   
 $mg=(m/(m\_yt*yt)-1)*100$   
 $yog=(yo/yt-yo\_yt)*100$

### Prices 300

$py= yn/y$   
 $dpy=@pc(py)$   
 $pcp=p*pcp\_pyt*py\_pt*(1+( +0.8*pcpg(-1)+0.016*pmg )/100 ) +pcp\_a$   
 $pih=p*pih\_pyt*py\_pt*(1+( pihg(-1)*0.88+0.048*phg )/100) +pih\_a$   
 $pib=p*pib\_pyt*py\_pt*(1+( pibg(-1)*.54+0.13*pmg )/100) +pib\_a$   
 $pcg=p*pcg\_pyt*py\_pt*(1+( pcgg(-1)*.61+.036*pmg )/100) +pcg\_a$   
 $pxs=p*pxs\_pyt*py\_pt*(1+( pxsg(-1)*.63-.057*vg )/100) +pxs\_a$   
 $pxgf=pxgf\_pft*(1+yfg/100*3)*pf +pxgf\_a$   
 $pxg=p*pxg\_pyt*py\_pt*(pxgf\_pfg/100+1)/(vg/100+1)$   
 $pmf=pmf\_pft*(1+yfg/100*1.5)*pf +pmf\_a$   
 $pm=p*pm\_pyt*py\_pt*(pmf\_pfg/100+1)/(vg/100+1)$   
 $tot=(pxs*xs+pxg*xg)/(xs+xg)/pm$   
 $py\_pg=(py/p/py\_pt-1)*100$   
 $pcpg=(pcp/p/(pcp\_pyt*py\_pt)-1)*100$   
 $pihg=(pih/p/(pih\_pyt*py\_pt)-1)*100$   
 $pibg=(pib/p/(pib\_pyt*py\_pt)-1)*100$   
 $pcgg=(pcg/p/(pcg\_pyt*py\_pt)-1)*100$   
 $pxsg=(pxs/p/(pxs\_pyt*py\_pt)-1)*100$   
 $pxgf\_pfg=(pxgf/p/(pxg\_pyt*py\_pt*st)-1)*100$   
 $pxgg=(pxg/p/(pxg\_pyt*py\_pt)-1)*100$   
 $pmf\_pfg=(pmf/p/(pm\_pyt*py\_pt*st)-1)*100$   
 $pmg=(pm/p/(pm\_pyt*py\_pt)-1)*100$   
 $p=p(-1)*(1+dp/100)$   
 $dp= \lambda *dpn+(1- \lambda)*dpt +dp\_a$   $\lambda=0.55$   
 $d4p=@pcy(p)$   
 $dpn=0.1*yg+dpe+.2 +dpn\_a$   
 $dpt=dpe-.2 \lambda /(1- \lambda)+.03*pmg +dpt\_a$   $\lambda=0.55$   
 $dptar=dptar(-1) +dptar\_a$   
 $dpe=dpe(-1)* \sigma +dp*(1- \sigma)$   $\sigma=0.9$   
 $@pc(ph)=@pc(pht)+8.8*( @pc(lpop(-1))- @pc(lpopt(-1)))-.24*rg+2*1.8*log(pht(-1)/ph(-1))+.0*pxgf\_pfg+ph\_a$   
 $@pc(wn)=@movav(dptar/2+dpe/2,4)+@pc(yt/ht)+.1*yg(-3)+5*log(wy\_p\_yt/(wy/p/yt*1000)) +w\_a$   
 $dlog(pht)=dlog(pht(-1))*\phi+dlog(yt*py\_pt*p)*(1- \phi ) +pht\_a$   $\phi=0.9$   
 $phg=(ph/pht-1)*100$

### Nominal 400

$yn=cpn+ihn+ibn+cgn+yon+xsn+xgn-mn$   
 $dyn=@pc(yn)$   
 $cpn=pcp*cp$   
 $ihn=pih*ih$

$ibn = pib * ib$   
 $cgn = pcg * cg$   
 $xsn = pxs * xs$   
 $xgn = pxg * xg$   
 $mn = pm * m$   
 $yon = yon\_ynt * yn + yon\_a$   
 $dlog(wy) = dlog(wn) + dlog(h) + wy\_a$   
 $wy\_pt = wy\_p\_yt * yt$   
 $wy\_pg = (wy / (p / 1000) / wy\_pt - 1) * 100$

### Financial 500

$r = rt + \psi * (rg(-1)) + (1 - \psi) * (1 * (yg) + 10 * (dpe - dptar) + 0.5 * (d4p / 4 - dptar)) + r\_a$   $\psi = 0.6$   
 $s = st * (1 + (1.4 * ((r - rt) - (rf - rft)) + 0.5 * (pxgf\_pfg)) / 100) + s\_a$   
 $v = s * p / pf / 70$   
 $st = vt * pf\_pt * 70$   
 $pf\_pt = pf / p + pf\_pt\_a$   
 $vg = (v / vt - 1) * 100$   
 $rg = r - rt$

### Foriegn 600

$dlog(pf) = \epsilon * dlog(pf(-1)) + (1 - \epsilon) * .005 + pf\_a$   $\epsilon = 0.9$   
 $rf = rf(-1) * \varepsilon + rft * (1 - \varepsilon) + rf\_a$   $\varepsilon = 0.5$   
 $yf = yft * (1 + yfg(-1) / 100 * .8) + yf\_a$   
 $dlog(yft) = dlog(yft(-1)) * .95 + (1 - \epsilon) * 0.0055 + yft\_a$   $\epsilon = 0.95$   
 $yfg = (yf / yft - 1) * 100$

### Fiscal 700

$twy = trwy * wy$   
 $tgst = trgst * (pcp * cp + pih * ih + pxs * xs * .8)$   
 $tb = trb * (yn - .14 * py\_pt * p * yt - tgst - wy)$   
 $go = py\_pt * p * yt * (@movav(go(-1)) / (py\_pt(-1) * p(-1) * yt(-1)), 8) * \Gamma + (1 - \Gamma) * go\_yn\_ss / 100 + go\_a$   $\Gamma = 0.5$   
 $trwy = trwy(-1) + trwy\_a$   
 $trgst = trgst(-1) + trgst\_a$   
 $trb = trb(-1) + trb\_a$   
 $trwyt = \Phi * trwyt(-1) + (1 - \Phi) * trwy + trwyt\_a$   $\Phi = 0.9$   
 $trwyg = (trwy / trwyt - 1) * 100$   
 $ob = twy + tgst + tb + go - pcg * cg$   
 $ob\_yn = ob / yn * 100$   
 $d4ob\_yn = (ob / yn - ob(-4) / yn(-4)) * 100$