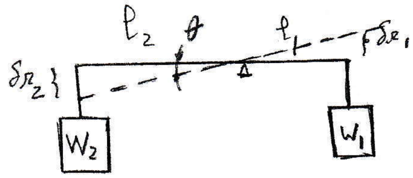
2.1



$\sum \vec{F} \cdot \delta \vec{r} = 0$ $\quad \delta r_1 = l_1 \operatorname{tg} \theta$ $\quad w_1 l_1 \operatorname{tg}\theta - w_2 l_2 \operatorname{tg}\theta = 0$

$\delta r_2 = -l_2 \operatorname{tg} \theta$ $\quad \boxed{w_1 l_1 = w_2 l_2}$

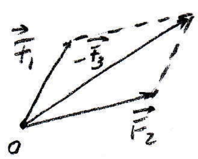
2.3

$\sum_i \vec{F}_i \cdot \delta\vec{r}_1 = 0$

a) $F_1 \cdot \delta r_1 = 0$, e como δr_1 é qualquer, então $F_1 = 0$

b) $\vec{F}_1 \cdot \delta\vec{r}_1 + \vec{F}_2 \cdot \delta\vec{r}_2 = 0$ mas $\delta\vec{r}_1 = \delta\vec{r}_2 = \delta\vec{r}$, pelo que $(\vec{F}_1 + \vec{F}_2)\delta\vec{r} = 0 \Rightarrow \vec{F}_1 = -\vec{F}_2$ isto é, as forças são iguais em grandeza, colineares e opostas em sentido

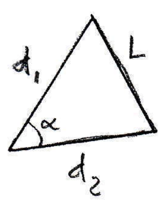
c) $(\vec{F}_1 + \vec{F}_2 + \vec{F}_3)\cdot\delta\vec{r} = 0 \Rightarrow \vec{F}_3 = -(\vec{F}_1 + \vec{F}_2)$ e a soma de 2 vectores dá um vector coplanar com eles. Logo $\vec{F}_3$ é coplanar com $\vec{F}_1$ e $\vec{F}_2$ e, além disso, as linhas de acção das 3 forças intersectam-se no mesmo ponto O.



d) $\sum_i \vec{F}_i \cdot \delta\vec{r}_i = 0$; $\sum F_i \, \delta r_i \cos \Delta_i = 0$; $(\sum F_i \cos \Delta_i)\, \delta r_i = 0$

e como δr_i é qualquer, vem: $\sum_i F_i \cos \Delta_i = 0$

2.4



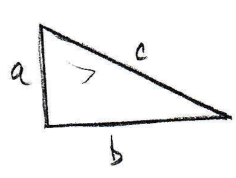
a) $L^2 = d_1^2 + d_2^2 - 2 d_1 d_2 \cos\alpha$ e se α varia de $\Delta\alpha$, vem:

$(L + \Delta L)^2 = d_1^2 + d_2^2 - 2 d_1 d_2 \cos(\alpha + \Delta\alpha)$

$L^2 + 2L\Delta L + (\Delta L)^2 = d_1^2 + d_2^2 - 2 d_1 d_2 (\cos\alpha \cos\Delta\alpha - \operatorname{sen}\alpha \operatorname{sen}\Delta\alpha)$ e se $\Delta\alpha$ pequeno

$\cos\Delta\alpha \simeq 1$ e $\operatorname{sen}\Delta\alpha \simeq \Delta\alpha$ e vem:

$L^2 + 2L\Delta L + (\Delta L)^2 = \underbrace{d_1^2 + d_2^2 - 2 d_1 d_2 \cos\alpha}_{L^2} + 2 d_1 d_2 \operatorname{sen}\alpha\, \Delta\alpha$ e, desprezando $(\Delta L)^2$ vem: $2L\Delta L = 2 d_1 d_2 \operatorname{sen}\alpha\,\Delta\alpha$ ou $\boxed{\Delta L = \dfrac{d_1 d_2}{L} \operatorname{sen}\alpha\, \Delta\alpha}$



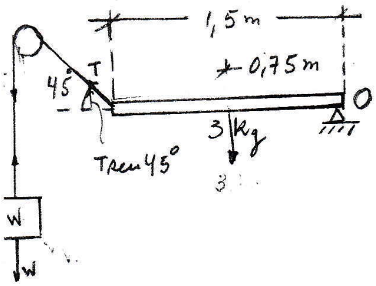
b) $c^2 = a^2 + b^2$

$(c + \Delta c)^2 = (a + \Delta a)^2 + (b + \Delta b)^2$; $c^2 + 2c\Delta c + (\Delta c)^2 = a^2 + 2a\Delta a + (\Delta a)^2 + b^2 + 2a\Delta a + (\Delta b)^2$

e desprezando $(\Delta c)^2, (\Delta a)^2, (\Delta b)^2$ e simplificando dá: $\boxed{c\Delta c = a\Delta a + b\Delta b}$

2.5

2.5

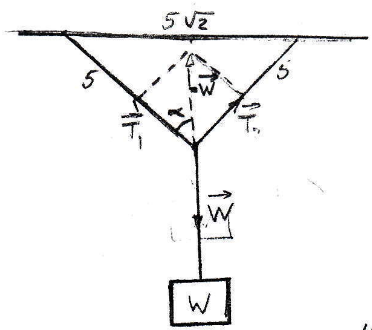


No equilíbrio estático temos: $T = W$

$$T \operatorname{sen} 45 \cdot 1{,}5 - 3\,\frac{1{,}5}{2} = 0$$ $\sum_i$ momentos em relação a O.

$$T = \frac{3 \cdot \frac{1{,}5}{2}}{\frac{\sqrt{2}}{2}\,1{,}5} = \frac{\frac{3}{2}}{\frac{\sqrt{2}}{2}} = \frac{3}{\sqrt{2}}\ \text{Kgf} = W$$

2.9



$$\operatorname{sen}\alpha = \frac{5\sqrt{2}}{2}\,\frac{1}{5} = \frac{1}{\sqrt{2}} = \cos\alpha$$

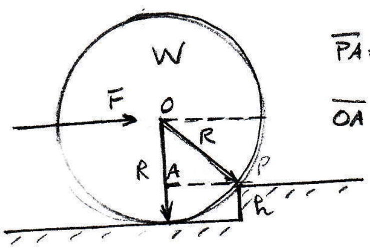
$$T_1 = W\cos\alpha = \frac{1}{\sqrt{2}}\,50 = \frac{50}{\sqrt{2}}$$

$$\vec{T}_1 + \vec{T}_2 + \vec{W} = 0 \qquad |\vec{T}_1| = |\vec{T}_2| = T$$

$$W = T_1\cos\alpha + T_2\cos\alpha = 2T\cos\alpha$$

$$T = \frac{1}{2\cos\alpha}W = \frac{W}{2\frac{1}{\sqrt{2}}} = \frac{1}{\sqrt{2}}W$$

2.11



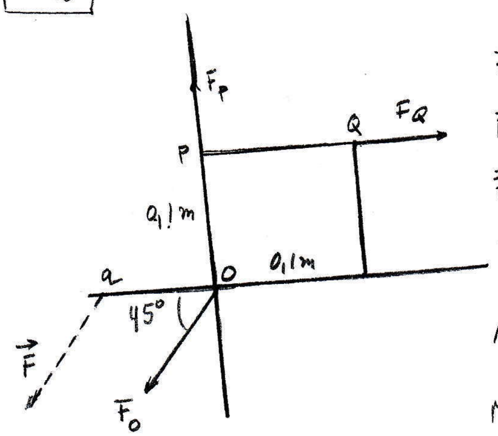
$$\overline{PA} = \sqrt{R^2 - (R-h)^2}$$

$$\overline{OA} = R - h$$

$$\sum \text{momentos em relação a } P = W\sqrt{R^2-(R-h)^2} = F(R-h)$$

$$F = \frac{\sqrt{R^2 - R^2 + 2Rh - h^2}}{R-h}\,W = \frac{\sqrt{h(2R-h)}}{R-h}\,W$$

2.13



$$\vec{F}_p = 50\cdot\hat{j}$$

$$\vec{F}_Q = 50\cdot\hat{i}$$

$$\vec{F}_O = -50\cos 45\,\hat{i} - 50\operatorname{sen}45\,\hat{j} = -\frac{50}{\sqrt{2}}\,\hat{i} - \frac{50}{\sqrt{2}}\,\hat{j}$$

$$\vec{F}_p + \vec{F}_Q + \vec{F}_O = 50\left(1-\frac{1}{\sqrt{2}}\right)\hat{i} + 50\left(1-\frac{1}{\sqrt{2}}\right)\hat{j} = 50\left(1-\frac{1}{\sqrt{2}}\right)(\hat{i}+\hat{j}) = \vec{R} = -\vec{F}$$

$M_{F_p/O}$ = momento da força F_p em relação a $O = 0$

$M_{F_Q/O}$ = " F_Q " $= 50\cdot 0{,}1\cdot(-\hat{k})$

$M_{F_O/O}$ = " F_O " $= 0$

$$M_{\vec{R}/O} = \vec{r}\wedge(-\vec{F}) = -\left[a\,\hat{i}\wedge 50\left(1-\frac{1}{\sqrt{2}}\right)(\hat{i}+\hat{j})\right] = \begin{vmatrix} i & j & k \\ a & 0 & 0 \\ 1 & 1 & 0 \end{vmatrix}\,50\left(1-\frac{1}{\sqrt{2}}\right) =$$

$$= -a\cdot 50\left(1-\frac{1}{\sqrt{2}}\right)\hat{k}$$

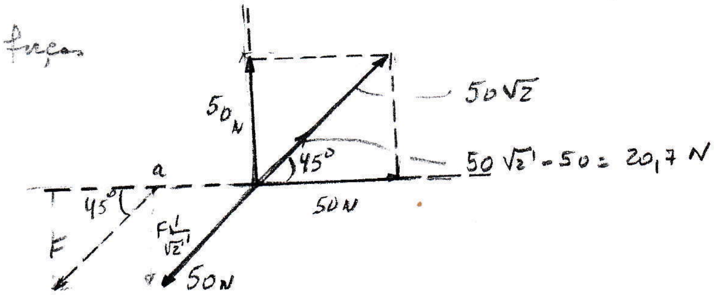
Mas $\sum$ Momentos $= 0$ e dá $\left[0 - 50\cdot 0{,}1 - a\,50\left(1-\frac{1}{\sqrt{2}}\right)\right]\hat{k} = 0$ ou $-a\,50\left(1-\frac{1}{\sqrt{2}}\right) = 50\cdot 0{,}1$

$$a = \frac{-0{,}1}{1-\frac{1}{\sqrt{2}}} = \underline{-0{,}34\text{ m}}$$

com $\vec{F} = -50\left(1-\frac{1}{\sqrt{2}}\right)(\hat{i}+\hat{j})$; $|\vec{F}| = 50\left(1-\frac{1}{\sqrt{2}}\right)\sqrt{1^2+1^2} = \underline{20{,}7\,N}$

2.13 Contin.   Contin 2.13

A geometria da figura possibilita uma resolução mais directa. Calculemos a resultante das 3 forças



O ponto de aplicação é tal que: $F_Q \cdot 0{,}1 + |\vec{F}|\frac{1}{\sqrt{2}} \cdot a = 0$ $a = -0{,}3416$ m

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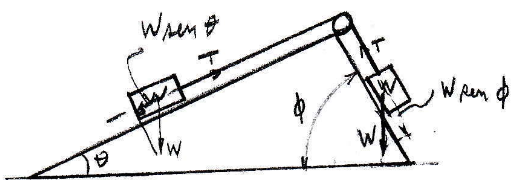
2.15   2.15



$$W \operatorname{sen}\theta + \frac{W}{g}\ddot{l}_\theta = T \qquad \text{mas } \ddot{l}_\theta = \ddot{l}_\phi$$

$$W \operatorname{sen}\phi - \frac{W}{g}\ddot{l}_\phi = T$$

$$\frac{W}{g}\ddot{l}_\theta + W\operatorname{sen}\theta = W\operatorname{sen}\phi - \frac{W}{g}\ddot{l}_\theta \ ; \ 2\frac{W}{g}\ddot{l}_\theta = W(\operatorname{sen}\phi - \operatorname{sen}\theta); \ \ddot{l}_\theta = \frac{g}{2W}W(\operatorname{sen}\phi - \operatorname{sen}\theta)$$

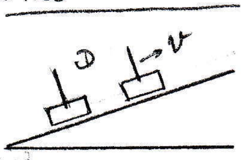
$\ddot{l}_\theta = \frac{1}{2}g(\operatorname{sen}\phi - \operatorname{sen}\theta)$ $\dot{l}_\theta = \frac{1}{2}g(\operatorname{sen}\phi - \operatorname{sen}\theta)t + C$ e como $\dot{l}_\theta(t=0) = 0 \Rightarrow C = 0$

$l_\theta = \frac{1}{4}g(\operatorname{sen}\phi - \operatorname{sen}\theta)t^2 + C$ mas $l_\phi(t=0) = 0 \Rightarrow C = 0$ donde $t = \sqrt{\frac{4l_\theta}{g(\operatorname{sen}\phi - \operatorname{sen}\theta)}}$

que substituindo na equação da velocidade $\dot{l}_\theta = \frac{1}{2}g(\operatorname{sen}\phi - \operatorname{sen}\theta)\cdot\sqrt{\frac{4l_\theta}{g(\operatorname{sen}\phi - \operatorname{sen}\theta)}} =$

$= \sqrt{l_\theta g(\operatorname{sen}\phi - \operatorname{sen}\theta)}$ donde $v = \sqrt{Dg(\operatorname{sen}\phi - \operatorname{sen}\theta)}$

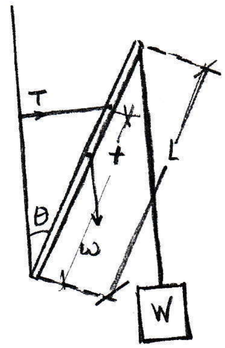
OUTRO MÉTODO: Pela conservação da energia podemos escrever:



$$\frac{1}{2}\frac{W}{g}v^2 + \frac{W}{g}gD\operatorname{sen}\theta + \frac{1}{2}\frac{W}{g}v^2 - \frac{W}{g}gD\operatorname{sen}\phi = 0$$

$2\cdot\frac{1}{2}v^2 = gD(\operatorname{sen}\phi - \operatorname{sen}\theta)$ donde $v = \sqrt{gD(\operatorname{sen}\phi - \operatorname{sen}\theta)}$

2.17 2.17



$$\Sigma \vec{M} = 0$$

$$\frac{L}{2} \operatorname{sen}\theta \cdot \omega + L \operatorname{sen}\theta\, W - T \times \cos\theta = 0$$

$$T = \frac{L}{x}\left(\frac{\omega}{2} + W\right) \operatorname{tg}\theta$$

2.19 2.19

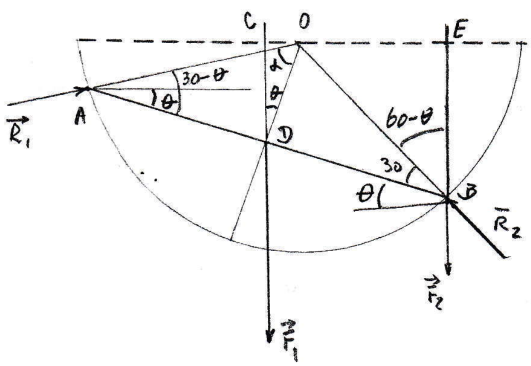
O sistema está em equilíbrio estático. Então as reacções em A e B não têm componente tangencial, pois se tivessem e como a barra está em equilíbrio estático e não há atrito entre as extremidades da barra e a superfície em que se apoia, a barra não estaria em equilíbrio estático, como, por hipótese está.

Calculemos a soma dos momentos em relação ao ponto O.

Os momentos das forças $\vec{R}_1$ e $\vec{R}_2$ em relação a O são nulos pois a linha de acção dessas forças passam por O, porque não há componente tangencial.

$\overline{AB} = \ell = \sqrt{3}\,R$

$R \operatorname{sen}\alpha = \frac{\ell}{2} = \frac{\sqrt{3}}{2} R$; $\operatorname{sen}\alpha = \frac{\sqrt{3}}{2}$; $\alpha = 60°$



$\overline{OD} = R \cdot \operatorname{sen} 30° = \frac{R}{2}$

$\overline{OC} = \frac{R}{2} \operatorname{sen}\theta$

$\overline{OE} = R \operatorname{sen}(60-\theta)$

Momento de F_1 em relação a $O = F_1 \cdot \overline{OC} = W \frac{R}{2} \operatorname{sen}\theta$

" " F_2 " " " " $= F_2 \cdot \overline{OE} = \frac{W}{2} R \operatorname{sen}(60-\theta)$

e a sua soma deve ser nula, e vem:

$$W \frac{R}{2} \operatorname{sen}\theta - \frac{W}{2} R \operatorname{sen}(60-\theta) = 0$$

$$\operatorname{sen}\theta - \operatorname{sen}(60° - \theta) = 0$$

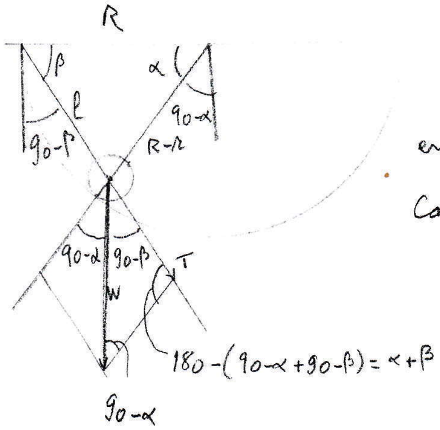
$$\operatorname{sen}\theta - \operatorname{sen}60° \cos\theta + \operatorname{sen}\theta \cos 60° = 0 \; ; \; \operatorname{sen}\theta\left(1 + \frac{1}{2}\right) = \frac{\sqrt{3}}{2}\cos\theta$$

$$\operatorname{tg}\theta = \frac{\frac{\sqrt{3}}{2}}{1 + \frac{1}{2}} = \frac{\sqrt{3}}{3} \quad \text{pelo que } \theta = 30°$$

2.21 | 2.21

$R = 49$ cm

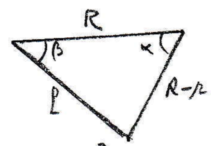
$r = 4,5$ cm



$$\frac{W}{\operatorname{sen}(\alpha+\beta)} = \frac{T}{\operatorname{sen}(90-\alpha)} \qquad T = \frac{\operatorname{sen}(90-\alpha)}{\operatorname{sen}(\alpha+\beta)} W = \frac{\cos\alpha}{\operatorname{sen}(\alpha+\beta)} W$$

em que T é a tensão no fio para $l = 40$ cm

Calculo de α e de β:



$$l^2 = R^2 + (R-r)^2 - 2R(R-r)\cos\alpha \;; \quad \cos\alpha = \frac{R^2 + (R-r)^2 - l^2}{2R(R-r)}$$

$$\cos\alpha = \frac{49^2 + (49-4,5)^2 - 40^2}{2 \cdot 49 \cdot (49-4,5)} = 0,6377 \quad \text{donde} \quad \alpha = 50,37^\circ$$

Por outro lado $(R-r)^2 = l^2 + R^2 - 2lR\cos\beta$; $\cos\beta = \dfrac{l^2 + R^2 - (R-r)^2}{2lR} = \dfrac{40^2 + 49^2 - (49-4,5)^2}{2 \cdot 40 \cdot 49} = 0,515$

donde $\beta = 58,96^\circ$ e então vem:

$$T = \frac{\cos 50,37}{\operatorname{sen}(50,37 + 58,96)} W = 0,67\, W$$

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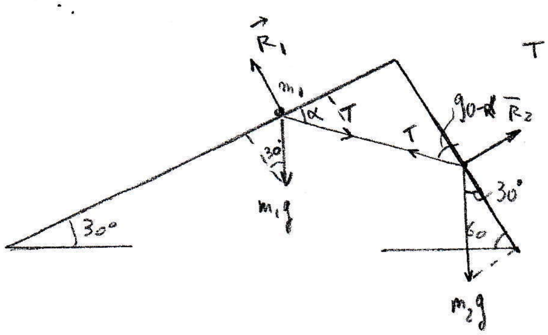
2.23 | 2.23



$$T\cos\alpha = m_1 g \operatorname{sen} 30^\circ$$

$$T\cos(90-\alpha) = m_2 g \cos 30^\circ \quad \text{e dividindo vem:}$$

$$\frac{\cos\alpha}{\cos(90-\alpha)} = \frac{m_1}{m_2}\frac{\operatorname{sen}30}{\cos 30} \;; \quad \frac{1}{\operatorname{tg}\alpha} = \frac{m_1}{m_2}\operatorname{tg}30^\circ$$

$$\operatorname{tg}\alpha = \frac{m_2}{m_1}\frac{1}{\operatorname{tg}30} = 3 \cdot 1,73 = 5,196 \qquad \alpha = 79,1^\circ$$

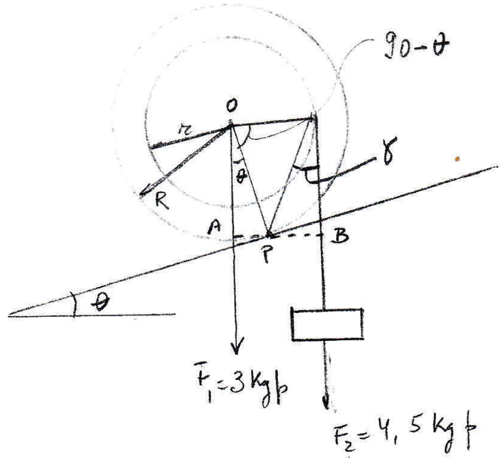
e ainda $T = m_1 g \dfrac{\operatorname{sen}30}{\cos 79,1} = 264,6$ gramas-peso

—— // —— // ——

2.25 2.25

$R = 6$

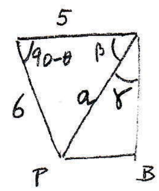
$r = 5$



No equilíbrio $\sum$ Momentos em relação a $P = 0$

$\overline{AP} = R\,\text{sen}\,\theta$

Cálculo de $\overline{PB}$



$$a^2 = 5^2 + 6^2 - 2\cdot 5\cdot 6\cos(90-\theta) = 61 - 60\,\text{sen}\,\theta$$

$$a = \sqrt{61 - 60\,\text{sen}\,\theta}$$

$$\frac{6}{\text{sen}\,\beta} = \frac{\sqrt{61-60\,\text{sen}\,\theta}}{\text{sen}(90-\theta)}$$

$$\gamma = 90 - \beta \qquad \text{sen}\,\beta = \frac{6\,\text{sin}(90-\theta)}{\sqrt{61-60\,\text{sen}\,\theta}} = \frac{6\cos\theta}{\sqrt{61-60\,\text{sen}\,\theta}} = \frac{6\cos\theta}{a}$$

$$\sum \vec{M} = 0$$

$$3\cdot R\cdot \text{sen}\,\theta - 4,5\cdot a\cdot \text{sin}\,\gamma = 0$$

$$3\cdot R\cdot \text{sen}\,\theta = 4,5\cdot\sqrt{61-60\,\text{sen}\,\theta}\cdot \text{sin}(90-\beta)$$

$$= 4,5\sqrt{61-60\,\text{sen}\,\theta}\cdot\cos\beta = 4,5\cdot a\cdot\sqrt{1-\left(\frac{6\cos\theta}{a}\right)^2} = 4,5\cdot a\cdot\frac{\sqrt{a^2-(6\cos\theta)^2}}{\sqrt{a^2}} =$$

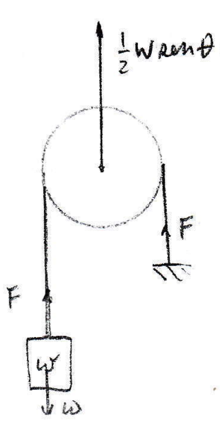
$$3R\,\text{sen}\,\theta = 4,5\cdot\sqrt{a^2-(6\cos\theta)^2} = 4,5\sqrt{61-60\,\text{sen}\,\theta - 36\cos^2\theta} = 4,5\sqrt{61-60\,\text{sen}\,\theta - 36 + 36\,\text{sen}^2\theta}$$

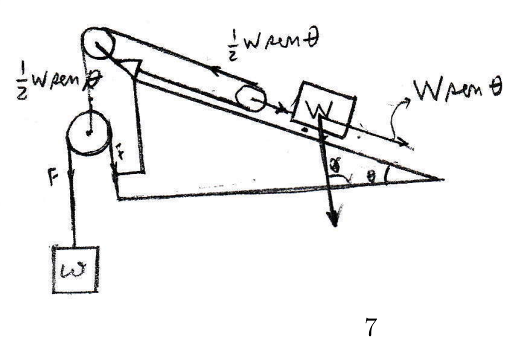
$$\frac{9R^2}{4,5^2}\,\text{sen}^2\theta = 25 - 60\,\text{sen}\,\theta + 36\,\text{sen}^2\theta\,; \quad \underbrace{\left(36 - \frac{9\cdot 36}{4,5^2}\right)}_{=20}\text{sen}^2\theta - 60\,\text{sen}\,\theta + 25 = 0$$

$\text{sen}^2\theta - 3\,\text{sen}\,\theta + 1,25 = 0$ cujas soluções são $\text{sen}\,\theta = 2,5$ impossível!

$\text{sen}\,\theta = \frac{1}{2}$ donde $\theta = 30^\circ$

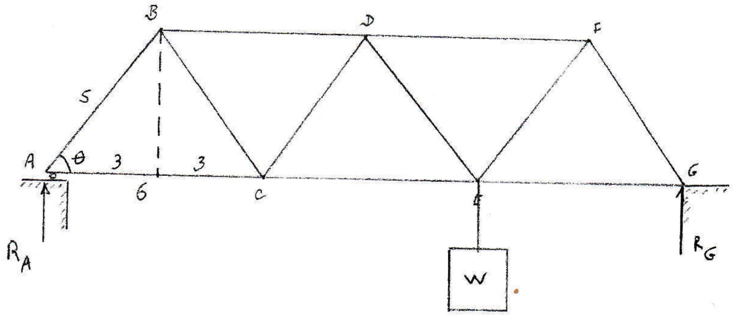
2.27 2.27





$$\frac{1}{2}W\,\text{sen}\,\theta = 2\cdot F = 2\cdot w$$

$$\boxed{W = \frac{4w}{\text{sen}\,\theta}}$$



$\cos\theta = \frac{3}{5}$

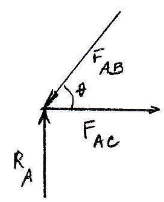
$\operatorname{sen}\theta = \sqrt{1-\left(\frac{3}{5}\right)^2} = \sqrt{1-\frac{9}{25}} = \frac{4}{5}$

Nota: cada nó é entendido como um ponto material livre e actuado por forças impostas pelas barras; se a barra está em compressão a força no nó aponta para o nó; em tracção será o contrário.

Cálculo das forças R_A e R_G: $R_A + R_G = W$ e $R_A \cdot 18 - 6W = 0$ donde $R_A = \frac{1}{3}W$ e $R_G = \frac{2}{3}W$

Equilíbrio de forças nos nós:

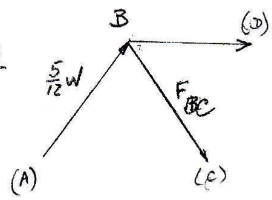
nó A:



$F_{AB}\cdot\operatorname{sen}\theta - R_A = 0$; $F_{AB} = \frac{1}{\operatorname{sen}\theta}R_A = \frac{5}{4}\frac{1}{3}W$ $\quad F_{AB} = \frac{5}{12}W$ compressão

$-F_{AB}\cos\theta + F_{AC} = 0$; $F_{AC} = F_{AB}\cos\theta = \frac{5}{12}\frac{3}{5}W = \frac{1}{4}W$ $\quad F_{AC} = \frac{1}{4}W$ tracção

nó B:

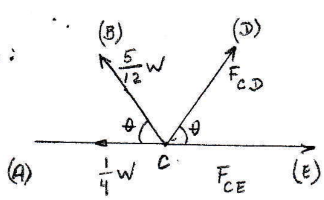


$F_{BC} = \frac{5}{12}W$ tracção

$\frac{5}{12}W\operatorname{sen}\theta - F_{BC}\operatorname{sen}\theta = 0$

$\frac{5}{12}W\cos\theta + F_{BC}\cos\theta + F_{BD} = 0$; $\frac{5}{12}\frac{3}{5}W + \frac{5}{12}\frac{3}{5}W = -F_{BD}$ $\quad F_{BD} = -\frac{1}{2}W$ compressão

nó C:



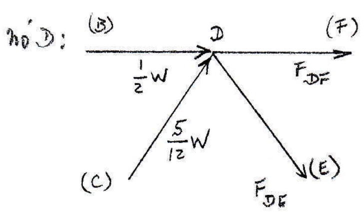
$F_{CD} = -\frac{5}{12}W$ compressão

$\frac{5}{12}W\operatorname{sen}\theta + F_{CD}\operatorname{sen}\theta = 0$

$-\frac{5}{12}W\cos\theta - \frac{1}{4}W + F_{CD}\cos\theta + F_{CE} = 0$; $-\frac{5}{12}\frac{3}{5}W - \frac{1}{4}W - \frac{5}{12}\frac{3}{5}W = -F_{CE}$; $\quad F_{CE} = \frac{3}{4}W$ tracção

$-\frac{1}{4} - \frac{1}{4} - \frac{1}{4}$

nó D:



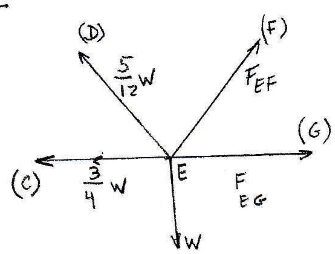
$F_{DE} = \frac{5}{12}W$ tracção

$\frac{5}{12}W\sin\theta - F_{DE}\sin\theta = 0$; $F_{DE} = \frac{5}{12}W$

$\frac{1}{2}W + \frac{5}{12}W\cos\theta + F_{DF} + F_{DE}\cos\theta = 0$; $\left(\frac{1}{2} + \frac{5}{12}\frac{3}{5} + \frac{5}{12}\frac{3}{5}\right)W = -F_{DF}$ $\quad F_{DF} = -W$ compressão

$\frac{1}{2} + \frac{1}{4} + \frac{1}{4} = 1$

nó E:



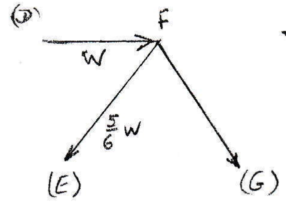
$\frac{5}{12}W\sin\theta + F_{EF}\sin\theta - W = 0$; $F_{EF} = \frac{1}{\sin\theta}\left(W - \frac{5}{12}W\sin\theta\right) = \frac{5}{4}\left(1 - \frac{5}{12}\frac{4}{5}\right)W$; $\quad F_{EF} = \frac{5}{6}W$ tracção

$\frac{5}{4} \quad \frac{2}{3}$

$-\frac{3}{4}W - \frac{5}{12}W\cos\theta + F_{EF}\cos\theta + F_{EG} = 0$; $\left(-\frac{3}{4} - \frac{5}{12}\frac{3}{5} + \frac{5}{6}\frac{3}{5}\right)W = -F_{EG}$; $\quad F_{EG} = \frac{1}{2}W$ tracção

$-\frac{3}{4} - \frac{1}{4} + \frac{1}{2} = -1 + \frac{1}{2} = -\frac{1}{2}$

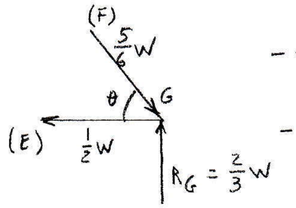
nó F:



$\frac{5}{6}W\sin\theta + F_{FG}\sin\theta = 0$; $F_{FG} = -\frac{5}{6}W$ compressão

Conhecemos agora o valor de todas as forças que actuam nas barras e se estas estão em compressão ou em tracção. Para verificação da correcção dos cálculos vamos calcular o equilíbrio das forças no nó G, que se deverá verificar se os valores calculados para as barras FG e EG estiverem correctos.

nó G:



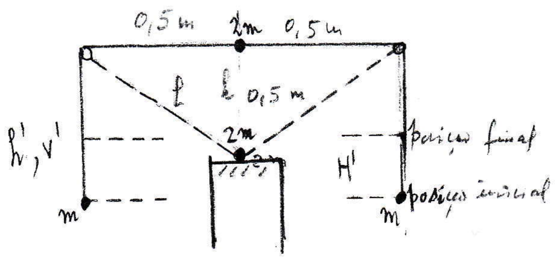
$$\left.\begin{aligned} -\frac{5}{6}W\sin\theta + R_G &= -\frac{5}{6}\frac{4}{5}W + \frac{2}{3}W = \left(-\frac{2}{3}+\frac{2}{3}\right)W = 0 \\ -\frac{1}{2}W + \frac{5}{6}W\cos\theta &= \left(-\frac{1}{2}+\frac{5}{6}\frac{3}{5}\right)W = \left(-\frac{1}{2}+\frac{1}{2}\right)W = 0 \end{aligned}\right\}$$ verifica-se o equilíbrio em G.

b) Todas as barras em tracção podem ser substituídas por cabos flexíveis.

São elas: AC, BC, CE, DE, EF, EG,

$F_{BD} = \frac{1}{2}W$ compressão e $F_{DE} = \frac{5}{12}W$ tracção

2.31



A perda de energia potencial da massa central 2m é transformada em energia cinética da massa central, energia cinética das duas massas laterais e em energia potencial das mesmas.

ou seja $2mgH = \frac{1}{2} 2m v^2 + 2 \cdot \frac{1}{2} m v'^2 + 2mgH'$

ora $l = \sqrt{\left(\frac{1}{2}\right)^2 + h^2} = \frac{1}{2}\sqrt{1+4h^2}$

$$v' = \frac{dh'}{dt} = \frac{dl}{dt} = \frac{1}{2}\,\frac{1}{2}\,\frac{8h}{\sqrt{1+4h^2}}\,\frac{dh}{dt} = \frac{2h}{\sqrt{1+4h^2}} \cdot v$$

Se $h = 0{,}5\,m$ vem $v' = \frac{1}{\sqrt{2}} v$

Cálculo de H': $l_{MAX} = \frac{1}{2}\sqrt{1+4\left(\frac{1}{2}\right)^2} = 0{,}5 \cdot \sqrt{2}$ e $l_{min} = 0{,}5$ e então

$$H' = l_{MAX} - l_{min} = 0{,}5(\sqrt{2} - 1) = 0{,}207\,m$$

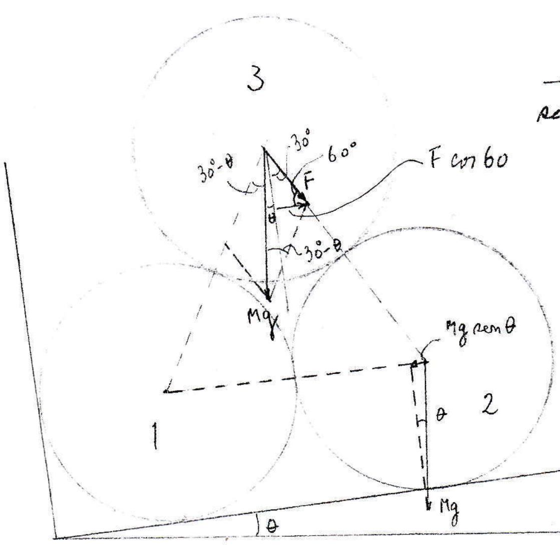
E substituindo na equação da energia, vem:

$$2mg \cdot H = \frac{1}{2} 2 m v^2 + 2\,\frac{1}{2} m \frac{1}{2} v^2 + 2mgH'$$

$\therefore\ v^2 + \frac{1}{2}v^2 = 2g(H - H')$; $\frac{3}{2}v^2 = 2g(0{,}5 - 0{,}207)$; $v^2 = \frac{4}{3}g(0{,}5 - 0{,}207)$

e vem $\underline{v = 1{,}96\,ms^{-1}}$

2.33



$$\frac{F}{sen(30-\theta)} = \frac{Mg}{sen\ 120}$$

$F \cos 60 = Mg\, sen\,\theta$ para que o cilindro 3 esteja em equilíbrio com o cilindro 2

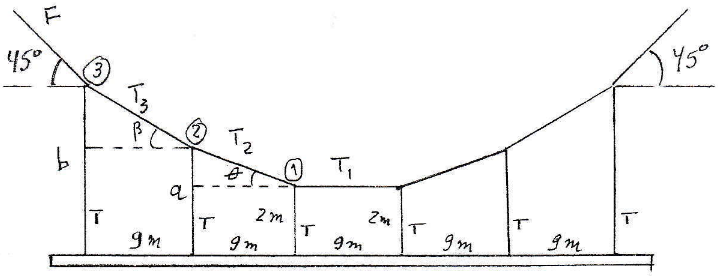
Eliminando F vem: $tg\,\theta = \frac{1}{3\sqrt{3}}$ ou $\theta = 10{,}9°$

$\frac{sen(30-\theta)}{sen\,60} \cos 60 = sen\,\theta$; $sen\,30 \cos\theta - sen\,\theta \cos 30 = tg\,60\, sen\,\theta$

$sen\,30 - \cos 30\, tg\,\theta = tg\,60\, tg\,\theta$; $sen\,30 = tg\,\theta(\cos 30 + tg\,60)$

$tg\,\theta = \frac{sen\,30}{\cos 30 + tg\,60} = \frac{1}{2}\,\frac{1}{\frac{\sqrt{3}}{2} + \sqrt{3}} = \frac{1}{3\sqrt{3}}$

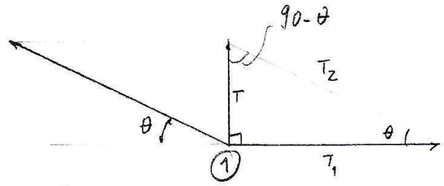
2.35 2.35



$$tg\,\theta = \frac{a-2}{9}$$

$$tg\,\beta = \frac{b-a}{9}$$

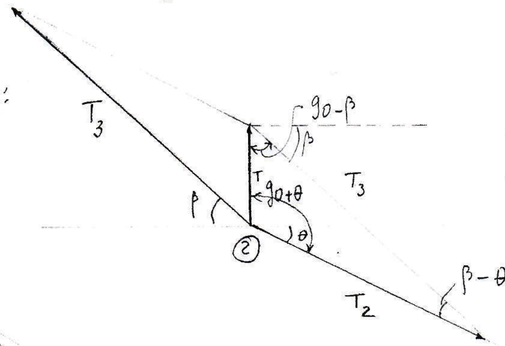
Equilíbrio de forças no nó 1:



$$\frac{T}{sen\,\theta} = \frac{T_1}{sen(90-\theta)} = \frac{T_2}{sen\,90°} \quad \text{donde}$$

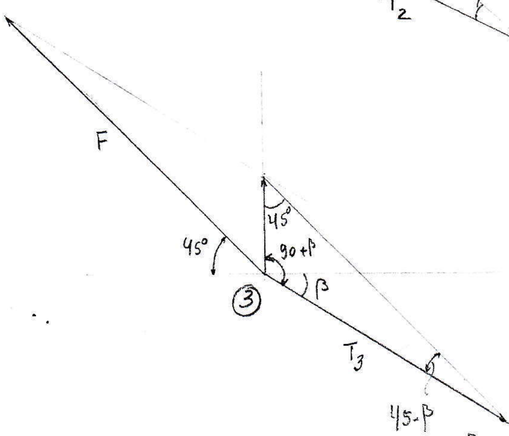
$$T_2 = \frac{T}{sen\,\theta} \quad e \quad T_1 = \frac{T}{tg\,\theta}$$

nó 2:



$$\frac{T}{sen(\beta-\theta)} = \frac{T_2}{sen(90-\beta)} = \frac{T_3}{sen(90+\theta)} \quad \text{donde:}$$

$$T_3 = \frac{sen(90+\theta)}{sen(90-\beta)}\,T_2 = \frac{\cos\theta}{\cos\beta}\,\frac{T}{sen\,\theta} = \frac{T}{tg\,\theta\,\cos\beta}$$



$$\frac{F}{sen(90+\beta)} = \frac{T}{sen(45-\beta)} = \frac{T_3}{sen\,45°}$$

Mas $F \cdot sen\,45° = 3 \cdot T$ ou $F = 3\sqrt{2}\,T$

e vem: $\dfrac{3\sqrt{2}\,T}{sen(90+\beta)} = \dfrac{T}{sen(45-\beta)}$ e simplificando:

$3\sqrt{2}\,sen(45-\beta) = sen(90+\beta)$; $3\sqrt{2}\left[sen\,45\cos\beta - sen\,\beta\cos 45\right] = \cos\beta$;

$sen\,45 - tg\,\beta\cos 45 = \dfrac{1}{3\sqrt{2}}$; $\cos 45°\left[tg\,45 - tg\,\beta\right] = \dfrac{1}{3\sqrt{2}}$; $1 - tg\,\beta = \dfrac{1}{3\sqrt{2}}\,\dfrac{1}{\cos 45°} = \dfrac{1}{3}$

$tg\,\beta = 1 - \frac{1}{3} = \frac{2}{3}$ ou seja $\dfrac{b-a}{9} = \dfrac{2}{3}$

Por outro lado: $T_3 = \dfrac{T}{tg\,\theta\,\cos\beta} = \dfrac{sen\,45°}{sen(90+\beta)} \cdot 3\sqrt{2}\,T$; $\dfrac{\cancel{T}}{tg\,\theta\,\cancel{\cos\beta}} = \dfrac{\sqrt{2}}{\cancel{2}}\,\dfrac{1}{\cancel{\cos\beta}}\,3\sqrt{2}\,\cancel{T}$; $\dfrac{1}{tg\,\theta} = 3$

$tg\,\theta = \dfrac{1}{3}$ ou seja $\dfrac{a-2}{9} = \dfrac{1}{3}$ ou $a = 5\,m$ e de $\dfrac{b-a}{9} = \dfrac{2}{3}$ dá $b - a = 6$ $b = 5+6 = 11\,m$

Assim: $a = 5\,m$ e $b = 11\,m$ e $\beta = 33{,}7°$ e $\theta = 18{,}4°$ e $F = 3\sqrt{2}\,T$ que dá:

$$F = 3\sqrt{2} \cdot \frac{4{,}8 \cdot 10^4}{6} = 34 \cdot 10^3\,kgf = 34\ ton\,f$$