7.7

i)

$$\vec{r}(t) = t\,\hat{i} + \left(t + \frac{t^2}{2}\right)\hat{j} - \frac{4}{\pi^2}\sin\left(\frac{\pi t}{2}\right)\hat{k}$$

$$\frac{d\vec{r}}{dt} = \hat{i} + (1+t)\hat{j} - \frac{4}{\pi^2}\,\frac{\pi}{2}\cos\left(\frac{\pi}{2}t\right)\hat{k}$$

$$\frac{d^2 r}{dt^2} = \hat{j} + \frac{2}{\pi}\,\frac{\pi}{2}\,\mathrm{sen}\left(\frac{\pi}{2}t\right)\hat{k}$$

$$T = \frac{1}{2}mv^2 = \frac{1}{2}\cdot 1\cdot\left[1 + (1+t)^2 - \left(\frac{2}{\pi}\right)^2\cos^2\left(\frac{\pi}{2}t\right)\right]$$

$$= \frac{1}{2}\left[2 + 2t + t^2 - \left(\frac{2}{\pi}\right)^2\cos^2\left(\frac{\pi}{2}t\right)\right]$$

	$t=0$	$t=1$
$\vec{r}(t)$	0	$\hat{i} + \frac{3}{2}\hat{j} - \frac{4}{\pi^2}\hat{k}$
$\vec{v}(t)$	$\hat{i} + \hat{j} - \frac{2}{\pi}\hat{k}$	$\hat{i} + 2\hat{j}$
$\vec{a}(t)$	$\hat{j}$	$\hat{j} + \hat{k}$
T	$1 - \frac{1}{2}\frac{4}{\pi^2}$	$\frac{5}{2}$

b) Raio de curvatura : $R = \dfrac{|v|^3}{|v \wedge \dot{v}|}$

$$|v|_{t=1} = \sqrt{5}$$

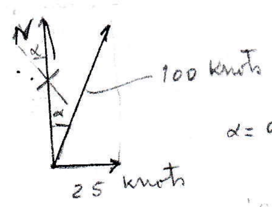
$$v \wedge \dot{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 0 \\ 0 & 1 & 1 \end{vmatrix} = 2\hat{i} - \hat{j} + \hat{k}$$

$$R = \frac{(\sqrt{5})^3}{\sqrt{6}} = \underline{4{,}56\ m}$$

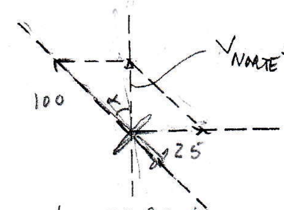
$$|v \wedge \dot{v}| = \sqrt{4+1+1} = \sqrt{6}$$

$$\vec{F} = m\cdot\vec{a} = 1\cdot\vec{a} = \hat{j} + \mathrm{sen}\left(\frac{\pi}{2}t\right)\hat{k}$$

7.8



$$\alpha = \arcsin\frac{25}{100} = 14{,}4^\circ$$

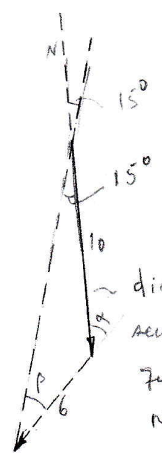


$V_{NORTE} = 100\cos 14{,}4 = 96{,}8$ Knots

$$T = \frac{100\ mi}{96{,}8\ \text{Knots} \times 1{,}15\ mi\ h^{-1}} = 0{,}898\ h = 53{,}9\ min$$

7.9

a)



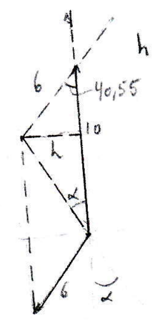
direcção do vento sentido pelo ciclista que se dirige para Norte, se não há vento adicional

$$\frac{6}{\mathrm{sen}\,15} = \frac{10}{\mathrm{sen}\,\beta} \Rightarrow \mathrm{sen}\,\beta = \frac{10}{6}\mathrm{sen}\,15 = 0{,}43136 \Rightarrow \beta = 25{,}55^\circ$$

$$\alpha = 180 - (180 - 15 - 25{,}55) = 40{,}55^\circ$$

O vento sopra de NE com $40{,}55^\circ$ seguinte

b)



$h = 6\,\mathrm{sen}\,40{,}55^\circ$

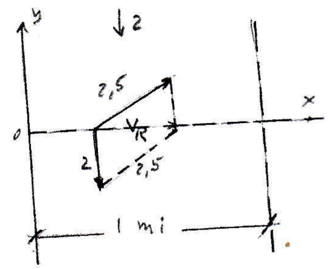
$(10 - 6\cos 40{,}55)\,\mathrm{tg}\,\alpha = 6\,\mathrm{sen}\,40{,}55$

$\alpha = 35{,}6^\circ$

O ciclista sente o vento a soprar de SE com $35{,}6^\circ$

7.10

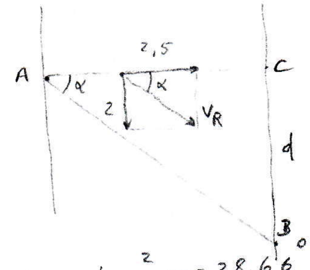
1° Método



$V_R = \sqrt{2,5^2 - 2^2} = 1,5 \ mi \ h^{-1}$

$T = \frac{1}{1,5} h = 40 min$

2° Método



$\alpha = \operatorname{arctg} \frac{2}{2,5} = 38,66°$

$d = 2,5 \cdot tg\,\alpha = 2,5 \frac{2}{2,5} = 2$

$V_R = \sqrt{2,5^2 + 2^2} = 3,2$

$\overline{AB} \cos\alpha = 1 \quad \overline{AB} = \frac{1}{\cos 38,66} = 1,28 \ mi$

$T_{AB} = \frac{\overline{AB}}{V_R} = \frac{1,28}{3,2} = 0,4 \ h = 24 \ min$

$\overline{BC} = \overline{AC}\, tg\,\alpha = 1 \cdot \frac{2}{2,5}$

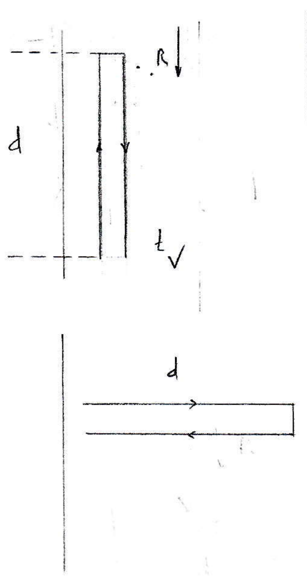
$T_{BC} = \frac{\frac{2}{2,5}}{4} = \frac{2}{10} = 0,2 \ h = 12 \ min$

$T_{total} = T_{AB} + T_{BC} = 24 + 12 = 36 min$

$\Delta T = T_{1° Método} - T_{2° Método} = 40 - 36 = \underline{4 \ min}$

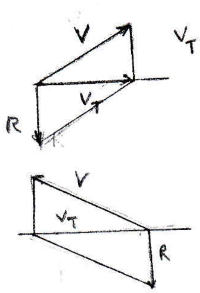
o método 2 é mais rápido 4 min do que o primeiro método!

7.11



$t_V = \frac{d}{V_{T_{subir}}} + \frac{d}{V_{T_{descer}}} = \frac{d}{V_{barco/água} - V_{água/Terra}} + \frac{d}{V_{barco/água} + V_{água/Terra}} =$

$= d \frac{2 V_{barco/água}}{V_{barco/água}^2 - V_{água/Terra}^2} = \frac{2 V_{b/a}}{V_{b/a}^2 - V_{a/T}^2} d$



$V_T = \sqrt{V^2 - R^2}$

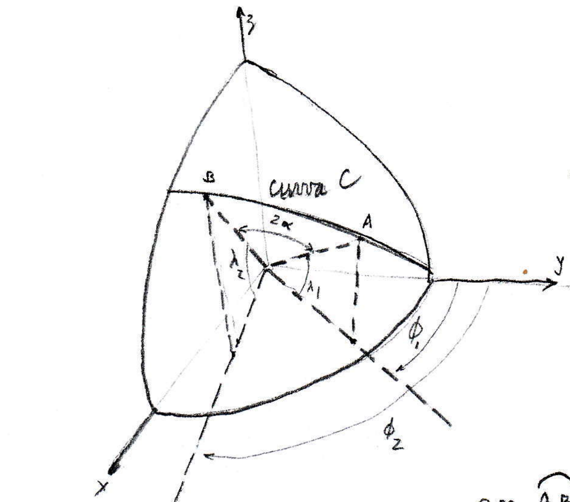
$t_A = \frac{2d}{V_T} = \frac{2d}{\sqrt{V_{barco/água}^2 - V_{água/Terra}^2}}$

$\frac{t_V}{t_A} = \frac{V_{b/a}}{\sqrt{V_{b/a}^2 - V_{a/T}^2}}$ e como $V_{b/a} > \sqrt{V_{b/a}^2 - V_{a/T}^2} \Rightarrow t_V > t_A$

No lago $(R=0)$ $\quad t_L = \frac{2d}{V}$ $\quad \frac{t_A}{t_L} = \frac{V_{b/a}}{\sqrt{V_{b/a}^2 - V_{a/T}^2}} > 1 \Rightarrow t_A > t_L$

Em conclusão: $t_V > t_A > t_L$

7.12 7.12

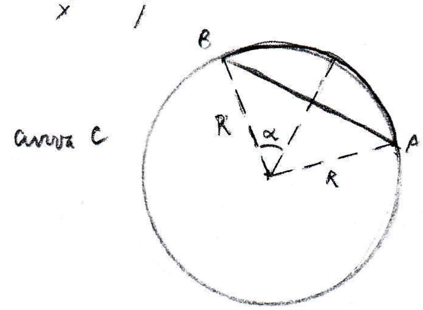


$$A: R\cos\lambda_1\cos\left(\frac{\pi}{2}-\phi_1\right)\hat{i} + R\cos\lambda_1\operatorname{sen}\left(\frac{\pi}{2}-\phi_1\right)\hat{j} + R\operatorname{sen}\lambda_1\hat{k}$$

$$B: R\cos\lambda_2\cos\left(\frac{\pi}{2}-\phi_2\right)\hat{i} + R\cos\lambda_2\operatorname{sen}\left(\frac{\pi}{2}-\phi_2\right)\hat{j} + R\operatorname{sen}\lambda_2\hat{k}$$

$$A: R\left[\cos\lambda_1\operatorname{sen}\phi_1\hat{i} + \cos\lambda_1\cos\phi_1\hat{j} + \operatorname{sen}\lambda_1\hat{k}\right]$$

$$B: R\left[\cos\lambda_2\operatorname{sen}\phi_2\hat{i} + \cos\lambda_2\cos\phi_2\hat{j} + \operatorname{sen}\lambda_2\hat{k}\right]$$



arco $\widehat{AB} = ?$

$$R\operatorname{sen}\alpha = \frac{1}{2}\overline{AB}\ ;\ \alpha = \operatorname{arc\,sen}\frac{\overline{AB}}{2R}$$

$$\widehat{AB} = 2R\alpha = 2R\operatorname{arc\,sen}\frac{\overline{AB}}{2R}$$

$$\overline{AB}^2 = R^2\left[\left(\cos\lambda_1\operatorname{sen}\phi_1 - \cos\lambda_2\operatorname{sen}\phi_2\right)^2 + \left(\cos\lambda_1\cos\phi_1 - \cos\lambda_2\cos\phi_2\right)^2 + \left(\operatorname{sen}\lambda_1 - \operatorname{sen}\lambda_2\right)^2\right]$$

$$= R^2\left[\cos^2\lambda_1\operatorname{sen}^2\phi_1 + \cos^2\lambda_2\operatorname{sen}^2\phi_2 - 2\cos\lambda_1\operatorname{sen}\phi_1\cos\lambda_2\operatorname{sen}\phi_2 + \cos^2\lambda_1\cos^2\phi_1 + \cos^2\lambda_2\cos^2\phi_2 - 2\cos\lambda_1\cos\phi_1\cos\lambda_2\cos\phi_2 + \operatorname{sen}^2\lambda_1 + \operatorname{sen}^2\lambda_2 - 2\operatorname{sen}\lambda_1\lambda_2\right] = R^2\left[\cos^2\lambda_1 + \cos^2\lambda_2 + \operatorname{sen}^2\lambda_1 + \operatorname{sen}^2\lambda_1 - 2\cos\lambda_1\cos\lambda_2\left(\operatorname{sen}\phi_1\operatorname{sen}\phi_2 + \cos\phi_1\cos\phi_2\right) - 2\operatorname{sen}\lambda_1\lambda_2\right] = R^2\left[2 - 2\cos\lambda_1\cos\lambda_2\cos(\phi_1-\phi_2) - 2\operatorname{sen}\lambda_1\lambda_2\right] =$$

$$= 2R^2\left[1 - \cos\lambda_1\cos\lambda_2\cos(\phi_1-\phi_2) - 2\operatorname{sen}\lambda_1\lambda_2\right]$$

$$\overline{AB} = R\sqrt{2}\sqrt{1-\cos\lambda_1\cos\lambda_2\cos(\phi_1-\phi_2) - \operatorname{sen}\lambda_1\operatorname{sen}\lambda_2}$$

$$\widehat{AB} = 2R\operatorname{arc\,sen}\frac{1}{2R}R\sqrt{2}\sqrt{1-\cos\lambda_1\cos\lambda_2\cos(\phi_1-\phi_2) - \operatorname{sen}\lambda_1\lambda_2} = 2R\operatorname{arc\,sen}\frac{1}{\sqrt{2}}\sqrt{1-\cos\lambda_1\lambda_2\cos(\phi_1-\phi_2) - \operatorname{sen}\lambda_1\operatorname{sen}\lambda_2}$$

Vamos simplificar esta expressão usando a relação: $\cos 2\alpha = \cos^2\alpha - \operatorname{sen}^2\alpha = 1 - 2\operatorname{sen}^2\alpha$

$$\operatorname{sen}^2\alpha = \frac{1}{2}(1-\cos 2\alpha)$$

Seja $\alpha = \operatorname{arc\,sen}\frac{1}{\sqrt{2}}\sqrt{1-\cos\lambda_1\cos\lambda_2\cos(\phi_1-\phi_2) - \operatorname{sen}\lambda_1\operatorname{sen}\lambda_2}$

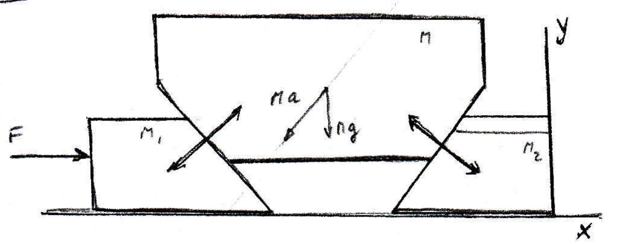
$$\operatorname{sen}^2\alpha = \frac{1}{2}\left(1-\cos\lambda_1\cos\lambda_2\cos(\phi_1-\phi_2) - \operatorname{sen}\lambda_1\operatorname{sen}\lambda_2\right)\ ;\ 1-2\operatorname{sen}^2\alpha = \cos\lambda_1\cos\lambda_2\cos(\phi_2-\phi_1) + \operatorname{sen}\lambda_1\operatorname{sen}\lambda_2 = \cos 2\alpha$$

$$2\alpha = \operatorname{arc\,cos}\left[\cos\lambda_1\cos\lambda_2\cos(\phi_1-\phi_2) + \operatorname{sen}\lambda_1\operatorname{sen}\lambda_2\right]$$

$$\widehat{AB} = 2R\alpha = \cancel{2}R\frac{1}{\cancel{2}}\left[\cos\lambda_1\cos\lambda_2\cos(\phi_1-\phi_2) + \operatorname{sen}\lambda_1\operatorname{sen}\lambda_2\right]$$

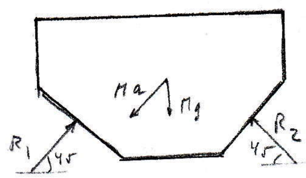
$$\boxed{\widehat{AB} = R\left[\cos\lambda_1\cos\lambda_2\cos(\phi_1-\phi_2) + \operatorname{sen}\lambda_1\operatorname{sen}\lambda_2\right]}$$

7.14



Diagramas de corpo livre:

a)



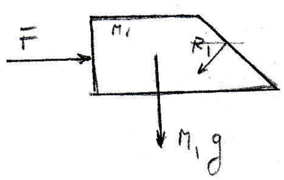
$$R_1 \cos 45 - R_2 \cos 45 = M\ddot{x}$$

$$R_1 \operatorname{sen} 45 + R_2 \operatorname{sen} 45 = M\ddot{y} + Mg$$

Mas $\ddot{x} = \ddot{y}$ pela geometria da figura

$$① \begin{cases} R_1 - R_2 = \sqrt{2}\, M\ddot{x} \\ R_1 + R_2 = \sqrt{2}\, M\ddot{x} + \sqrt{2}\, Mg \end{cases}$$

$$2R_1 = 2\sqrt{2}\, M\ddot{x} + \sqrt{2}\, Mg \; ; \quad R_1 = \sqrt{2}\, M\ddot{x} + \frac{1}{\sqrt{2}} Mg$$



$F - R_1 \cos 45 - M_1\ddot{x} = 0$ e, substituindo R_1 nesta equação, vem:

$$F - \frac{1}{\sqrt{2}}\left(\sqrt{2}\, M\ddot{x} + \frac{1}{\sqrt{2}} Mg\right) - M_1\ddot{x} = 0$$

$$F - M\ddot{x} - \frac{1}{2} Mg - M_1\ddot{x} = 0 \; ; \quad \ddot{x} = \frac{1}{M + M_1}\left(F - \frac{1}{2} Mg\right)$$

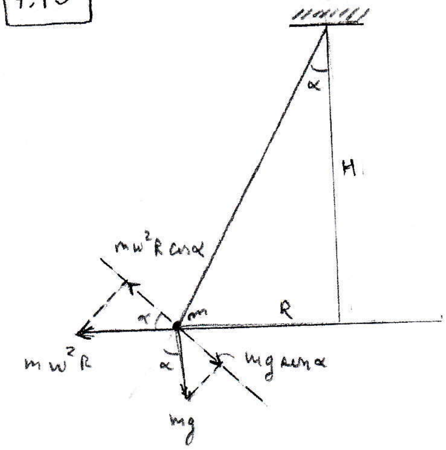
$$\ddot{x} = \frac{1}{384 + 8}\left(592 - \frac{1}{2} 384\right) g = 1\, g$$

b) $a_{M_2} = \sqrt{\ddot{x} + \ddot{y}} = \sqrt{2}\, \ddot{x} = \sqrt{2}\, g$

c) De ① vem, subtraindo, as 2 equações: $2R_2 = Mg\sqrt{2}$ ou $R_2 = \frac{1}{\sqrt{2}} Mg = \frac{384}{\sqrt{2}}$ kg-peso =

$R_2 = 272$ kg-peso

7.15 **7.15**



$$m\omega^2 R\cos\alpha = mg\,\text{sen}\,\alpha$$

$$R = H\,\text{tg}\,\alpha$$

$$\omega^2 H\frac{\text{sen}\,\alpha}{\cos\alpha}\cos\alpha = g\,\text{sen}\,\alpha$$

$$\omega^2 H = g \quad ; \quad \omega = \sqrt{\frac{g}{H}} = \frac{2\pi}{T} \qquad \boxed{T = 2\pi\sqrt{\frac{H}{g}}}$$

7.16 **7.16**

$$a = -9{,}8\,\hat{k}$$

$$r_a(0) = 7\hat{i} + 4{,}9\hat{k}$$

$$v_a(0) = 7\hat{i} + 3\hat{j}$$

$$v_a(t) = c_x\hat{i} + c_y\hat{j} + (-9{,}8t + c_3)\hat{k}$$

$$v_a(t=0) = c_x\hat{i} + c_y\hat{j} + c_3\hat{k} \quad \text{donde } c_x = 7,\ c_y = 3,\ c_3 = 0$$

$$v_a(t) = 7\hat{i} + 3\hat{j} - 9{,}8t\,\hat{k}$$

$$r_a(t) = (7t + c_x)\hat{i} + (3t + c_y)\hat{j} + \left(-\frac{9{,}8}{2}t^2 + c_3\right)\hat{k}$$

$$r_a(t=0) = c_x\hat{i} + c_y\hat{j} + c_3\hat{k} \qquad c_x = 7,\ c_y = 0,\ c_3 = 4{,}9$$

$$r_a(t) = (7t+7)\hat{i} + 3t\,\hat{j} + (-4{,}9t^2 + 4{,}9)\hat{k}$$

$$a = -9{,}8\,\hat{k}$$

$$r_b(0) = 49\hat{i} + 4{,}9\hat{k}$$

$$v_b(0) = -7\hat{i} + 3\hat{j}$$

$$v_b(t) = c_x\hat{i} + c_y\hat{j} + (-9{,}8t + c_3)\hat{k}$$

$$v_b(t=0) = c_x\hat{i} + c_y\hat{j} + c_3\hat{k} \qquad c_x = -7 \quad c_y = 3 \quad c_3 = 0$$

$$v_b(t) = -7\hat{i} + 3\hat{j} + (-9{,}8t)\hat{k}$$

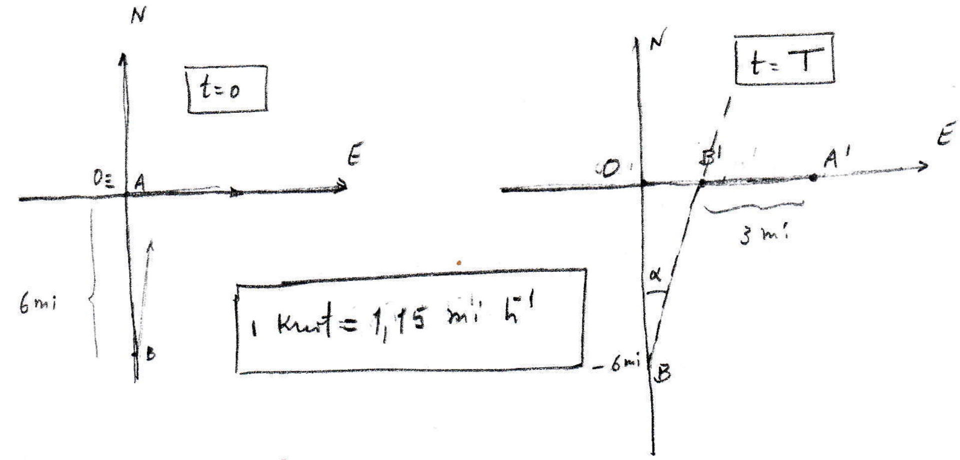
$$r_b(t) = (-7t + c_x)\hat{i} + (3t + c_y)\hat{j} + \left(-\frac{9{,}8}{2}t^2 + c_3\right)\hat{k}$$

$$r_b(t=0) = c_x\hat{i} + c_y\hat{j} + c_3\hat{k} \qquad c_x = 49,\ c_y = 0,\ c_3 = 4{,}9$$

$$r_b(t) = (-7t + 49)\hat{i} + 3t\,\hat{j} + (-4{,}9t^2 + 4{,}9)\hat{k}$$

As componentes segundo $\hat{j}$ e $\hat{k}$ são iguais. Quando $7t + 7 = -7t + 49$; $14t = 42$; $t = 3$ as componentes segundo $\hat{i}$ coincidem e as bolas chocam e como têm a mesma massa e ficam coladas e as componentes da velocidade, segundo $\hat{i}$ são iguais e opostas; $v_{a_x} = 7$ e $v_{b_x} = -7$, então a velocidade das partículas anula-se e a partir desse instante ($t \geq 3$) a componente da posição vai manter-se constante e

$$r_a(t) = r_b(t) = 28\hat{i} + 3t\,\hat{j} + (-4{,}9t^2 + 4{,}9)\hat{k}$$



$V_A \cdot T - V_B \operatorname{sen}\alpha \, T = 3$

$BB'\cos\alpha = 6 \qquad BB' = \frac{6}{\cos\alpha} \qquad \frac{BB'}{26\cdot 1,15} = T = \frac{1}{26\cdot 1,15}\,\frac{6}{\cos\alpha}$

$\left(V_A - V_B \operatorname{sen}\alpha\right)\frac{6}{\cos\alpha}\,\frac{1}{26\cdot 1,15} = 3$; $V_A - V_B \operatorname{sen}\alpha = \frac{3\cdot 26\cdot 1,15}{6}\cos\alpha$; $V_B \operatorname{sen}\alpha + \frac{3\cdot 26\cdot 1,15}{6}\cos\alpha = V_A$

Transformação de Trigonometria:

$a \operatorname{sen}\alpha + b\cos\alpha = a\left(\operatorname{sen}\alpha + \frac{b}{a}\cos\alpha\right)$ e se fizermos $tg\beta = \frac{b}{a}$ vem:

$= a\left(\operatorname{sen}\alpha + tg\beta\cos\alpha\right) = \frac{a}{\cos\beta}\left(\cos\beta \operatorname{sen}\alpha + \operatorname{sen}\beta\cos\alpha\right) = \frac{a}{\cos\beta}\operatorname{sen}(\alpha+\beta) =$

$= \frac{a}{\cos\beta}\left(\operatorname{sen}\left(\alpha + \operatorname{arctg}\frac{b}{a}\right)\right)$ e como $\cos\beta = \frac{1}{\sqrt{1+tg^2\beta}} = \frac{1}{\sqrt{1+\left(\frac{b}{a}\right)^2}} = \frac{a}{\sqrt{a^2+b^2}}$

$\therefore$ dá $\frac{a}{\cos\beta} = \sqrt{a^2+b^2}$ pelo que $a\operatorname{sen}\alpha + b\cos\alpha = \sqrt{a^2+b^2}\operatorname{sen}\left(\alpha + \operatorname{arctg}\frac{b}{a}\right)$

Aplicando isto à expressão $V_B \operatorname{sen}\alpha + A\cos\alpha = V_A$ com $V_A = 15\cdot 1,15$
$V_B = 26\cdot 1,15$
$A = 15$

$\sqrt{V_B^2 + A^2}\operatorname{sen}\left(\alpha + \operatorname{arctg}\frac{A}{V_B}\right) = V_A$; $\alpha + \operatorname{arctg}\frac{A}{V_B} = \operatorname{arcsin}\frac{V_A}{\sqrt{V_B^2+A^2}}$

$\alpha = -\operatorname{arctg}\frac{A}{V_B} + \operatorname{arcsin}\frac{V_A}{\sqrt{V_B^2+A^2}} = -26,64^\circ + 31,04^\circ = 4,4^\circ$

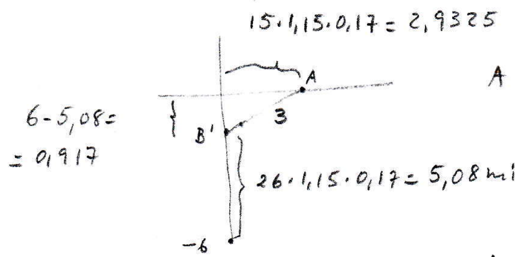
rumo: 4,4° Norte

b) $T = \frac{1}{26\cdot 1,15}\,\frac{6}{\cos 4,4^\circ} \simeq 0,2\ h = 12\ min$

Nota: na solução dá 0,17 h mas creio ser erro da solução, pois $26\cdot 1,15\cdot 0,17 = 5,08$ mi e o B nunca poderia ficar alinhado com A

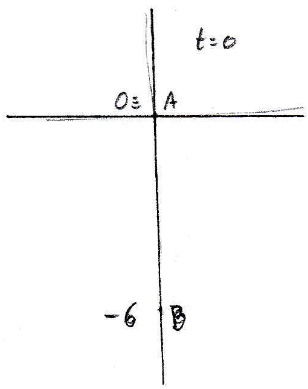
7.17 Contin. — Contin. 7.17

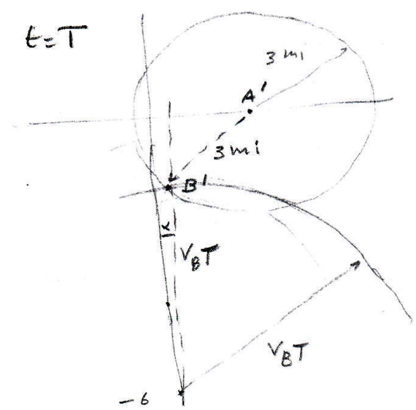
A solução $T=0{,}17h$ indica outra maneira de pensar o problema em que a distância de 3 mi não se dá para o alinhamento perfeito com o navio A.



A distância A'B' é $\sqrt{2{,}9325^2 + 0{,}917^2} = 3{,}07 \simeq 3$ mi

Seja então em geral (não há alinhamento!):





$\vec{r}_A = V_A T \hat{i}$

$\vec{r}_B = V_B T \,\mathrm{sen}\,\alpha\, \hat{i} + (-6 + V_B T \cos\alpha)\hat{j}$

$|\vec{r}_A - \vec{r}_B|^2 = (V_A T - V_B T \,\mathrm{sen}\,\alpha)^2 + (-6 + V_B T\cos\alpha)^2 = 3^2$

equação em 2 incógnitas T e α. Sem mais nenhuma informação adicional, $T = T(\alpha)$, e há uma infinidade soluções, uma das quais é a do livro $T = 0{,}17h$