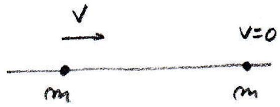
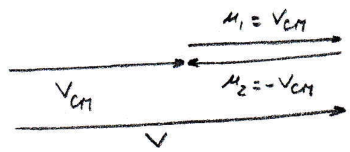


$$V_{CM} = \frac{m\vec{V}+m\vec{0}}{2m} = \frac{V}{2}$$

$$\vec{V} = \vec{V}_{CM} + \vec{\mu}_1 \qquad \vec{\mu}_1 = \vec{V}-\vec{V}_{CM} = \vec{V}-\frac{\vec{V}}{2} = \frac{\vec{V}}{2}$$

$$\vec{0} = \vec{V}_{CM}+\vec{\mu}_2 \qquad \vec{\mu}_2 = -V_{CM} = -\frac{\vec{V}}{2}$$

$$\vec{\mu}_2 = -\vec{\mu}_1 = -V_{CM}$$



Energia cinética antes da colisão:

$$E_a = \frac{1}{2} m_1 V_1^2 + \frac{1}{2} m_2 V_2^2 = \frac{1}{2} m (V_{CM}+\mu_1)^2 + \frac{1}{2} m (V_{CM}+\mu_2)^2 = \frac{1}{2} m\left[V_{CM}^2 + 2V_{CM}\mu_1 + \mu_1^2 + V_{CM}^2 + 2V_{CM}\mu_2 + \mu_2^2\right] =$$

$$= \frac{1}{2} m\left[2V_{CM}^2 + 2V_{CM}\mu_1 + \mu_1^2 - 2V_{CM}\mu_1 + \mu_1^2\right] = \frac{1}{2} m\left[2V_{CM}^2 + 2\mu_1^2\right] = mV_{CM}^2 + m\mu_1^2$$

Depois da colisão:

$$\vec{V}'_{CM} = \frac{m\vec{V}_3 + m\vec{V}_4}{2m} = \frac{m\vec{V}}{2m} = \frac{\vec{V}}{2} = \vec{V}_{CM} \qquad m_3\vec{\mu}_3 + m_4\vec{\mu}_4 = 0;\ m\vec{\mu}_3 + m\vec{\mu}_4 = 0;\ \vec{\mu}_3 = -\vec{\mu}_4$$

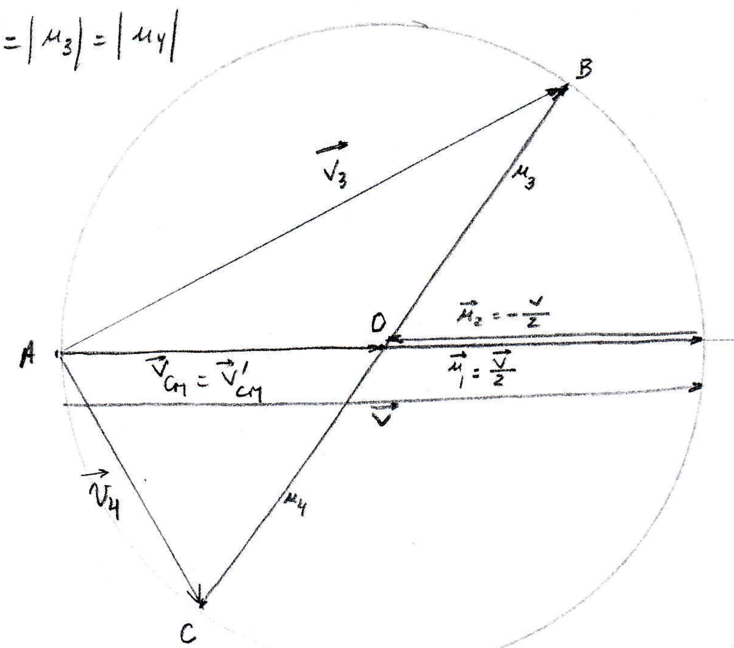
Energia cinética depois da colisão:

$$E_d = \frac{1}{2} m V_3^2 + \frac{1}{2} m V_4^2 = \frac{1}{2} m\left[(V_{CM}+\mu_3)^2 + (V_{CM}+\mu_4)^2\right] = \frac{1}{2} m\left[V_{CM}^2 + 2V_{CM}\mu_3 + \mu_3^2 + V_{CM}^2 + 2V_{CM}\mu_4 + \mu_4^2\right] =$$

$$= \frac{1}{2} m\left[2V_{CM}^2 + \cancel{2V_{CM}}\mu_3 - \cancel{2V_{CM}}\mu_3 + \mu_3^2 + \mu_3^2\right] = mV_{CM}^2 + m\mu_3^2$$

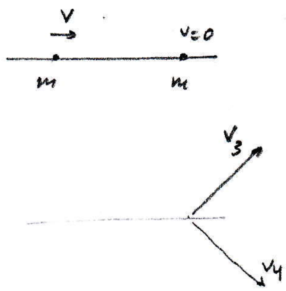
e a colisão é elástica, logo $E_a = E_d$, o que dá $mV_{CM}^2 + m\mu_1^2 = mV_{CM}^2 + m\mu_3^2 \Rightarrow |\mu_1| = |\mu_3|$

Assim $|\mu_1| = |\mu_2| = |\mu_3| = |\mu_4|$



e o ângulo formado por $\vec{V}_3$ e $\vec{V}_4$ é de 90° porque é metade do ângulo ao centro de 180°

2º Método: mais directo mas menos interessante.



$$\vec{V} = V\,\hat{i}$$

$$\vec{V}_3 = a\,\hat{i} + b\,\hat{j}$$

$$\vec{V}_4 = c\,\hat{i} + d\,\hat{j}$$

$$m\vec{V} = m\vec{V}_3 + m\vec{V}_4 \qquad \vec{V} = \vec{V}_3 + \vec{V}_4 \quad (1)$$

$$\frac{1}{2}mV^2 = \frac{1}{2}mV_3^2 + \frac{1}{2}mV_4^2 \qquad V^2 = V_3^2 + V_4^2 \quad (2)$$

de (1) vem: $V\hat{i} = (a+c)\,\hat{i} + (b+d)\,\hat{j}$. $\qquad V = a + c \qquad c = V - a$

$$0 = b + d \qquad d = -b$$

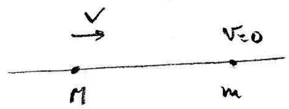
de (2) vem: $V^2 = a^2 + b^2 + c^2 + d^2 = a^2 + b^2 + (V-a)^2 + (-b)^2 = 2a^2 + 2b^2 - 2aV + V^2$

$$a^2 + b^2 - 2aV = 0 \quad ; \quad b = \sqrt{a(V-a)}$$

$$\vec{V}_3 = a\,\hat{i} + \sqrt{a(V-a)}\,\hat{j} \qquad \vec{V}_3 \cdot \vec{V}_4 = a(V-a) - a(V-a) = 0 \text{ e então } \vec{V}_3 \perp \vec{V}_4$$

$$\vec{V}_4 = (V-a)\,\hat{i} - \sqrt{a(V-a)}\,\hat{j}$$

Antes do choque:



$$\vec V_{CM} = \frac{MV + m\cdot 0}{M+m} = \frac{M}{M+m}\vec V$$

$$M V'_M + m V'_m = MV \qquad \vec V'_{CM} = \frac{MV'_M + mV'_m}{M+m} = \frac{MV}{M+m} = \vec V_{CM}$$

$$u_M = V - V_{CM} = V - \frac{M}{m+M}V = \frac{m}{m+M}V \qquad M u_M + m u_m = 0 \qquad M\vec u_M = -m\vec u_m$$

$$u_m = 0 - V_{CM} = -V_{CM} = -\frac{M}{M+m}V \qquad P = M|u_M| = -m|u_m| = \frac{Mm}{M+m}V$$

T_a = En. cinética antes $= \frac12 M u_M^2 + \frac12 m u_m^2 + \frac12(M+m)V_{CM}^2 = \frac12\frac{P^2}{M} + \frac12\frac{P^2}{m} + \frac12(M+m)V_{CM}^2 =$

$$= \frac12\left(\frac1M+\frac1m\right)P^2 + \frac12(M+m)V_{CM}^2 = \frac{P^2}{2m_r} + \frac12(M+m)V_{CM}^2$$

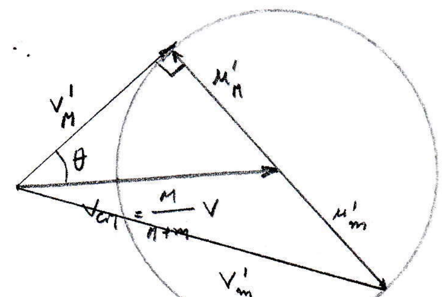
Depois do choque: $M\vec u'_M + m\vec u'_m = 0 \qquad M|u'_M| = -m|u'_m| = P'$

$T_d = \frac12\frac{P'^2}{m'_r} + \frac12(M+m)V'^2_{CM}$ Mas $V'_{CM}=V_{CM}$ e $m'_r = m_r$ porque as massas são iguais antes e depois do choque, pelo que

$P'=P$, isto é $M|u'_M| = m|u'_m| = M|u_M| = m|u_m|$ ou ainda

$$|u'_M| = |u_M| \quad \text{e} \quad |u'_m| = |u_m|$$

$$|u_M| = |u'_M| = \frac{m}{M+m}V$$



Cálculo do θ que é o máximo desvio possível.

$$V'_M \operatorname{sen}\theta = u'_M \operatorname{sen}(90-\theta) \qquad V'_M\operatorname{sen}\theta = \frac{m}{M+m}V\cos\theta$$

$$\operatorname{tg}\theta = \frac{m}{M+m}V\frac{1}{V'_M}$$

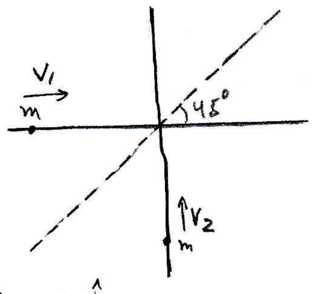
$$V'^2_M + u'^2_M = V^2_{CM} \;;\; V'^2_M = V^2_{CM} - u'^2_M = \frac{M^2}{(M+m)^2}V^2 - \frac{m^2}{(M+m)^2}V^2 =$$

$$V'^2_M = \frac{M^2-m^2}{(M+m)^2}V^2 = \frac{M-m}{M+m}V^2$$

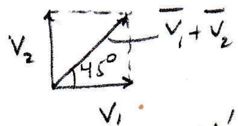
Então $\operatorname{tg}\theta_{MAX} = \frac{m}{M+m}V\frac{1}{\sqrt{\frac{M-m}{M+m}V^2}} = \frac{m}{\sqrt{(M+m)(M-m)}} = \frac{m}{\sqrt{M^2-m^2}} = \frac{1}{\sqrt{\left(\frac{M}{m}\right)^2-1}}$ ou ainda

tendo em conta que $\operatorname{sen}\theta = \frac{\operatorname{tg}\theta}{\sqrt{1+\operatorname{tg}^2\theta}} = \frac{m}{M}$ e finalmente $\theta_{MAX} = \arcsin\frac{m}{M}$

8.5 **8.5**



$$\overline{V}_{cm} = \frac{m_1\overline{V}_1 + m_2\overline{V}_2}{m_1+m_2} = \frac{\overline{V}_1+\overline{V}_2}{2} = \frac{1}{2}\, v(\hat{i}+\hat{j})$$

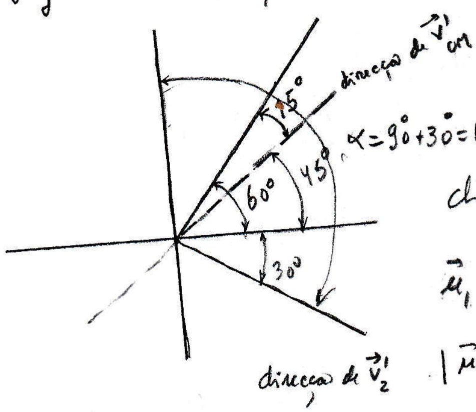


$$|V_{cm}| = \frac{\sqrt{2}}{2}v = \frac{v}{\sqrt{2}}$$

$\vec{V}_1 = v\,\hat{i}$

$\vec{V}_2 = v\,\hat{j}$

$$\vec{V}'_{cm} = \frac{m_1 V'_1 + m_2 V'_2}{m_1+m_2} = \frac{V'_1+V'_2}{2} = \vec{V}_{cm}$$



$\alpha = 90° + 30° = 120°$

$m\vec{\mu}_1 + m\vec{\mu}_2 = 0 \qquad |\mu_1| = |\mu_2| = P$

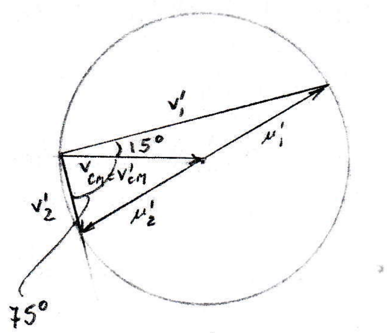
$m\mu'_1 + m\mu'_2 = 0 \qquad |\mu'_1| = |\mu'_2| = P'$

choque elástico $\;P = P' \qquad |\mu_1| = |\mu_2| = |\mu'_1| = |\mu'_2|$

$$\vec{\mu}_1 = \vec{V}_1 - \vec{V}_{cm} = v\,\hat{i} - \frac{v}{2}(\hat{i}+\hat{j}) = \left(v - \frac{v}{2}\right)\hat{i} - \frac{v}{2}\hat{j} = \frac{v}{2}\hat{i} - \frac{v}{2}\hat{j}$$

$$|\vec{\mu}_1| = \sqrt{\frac{v^2}{4}+\frac{v^2}{4}} = v\,\frac{1}{\sqrt{2}}$$

$$|\mu'_1| = |\mu_1| = \frac{v}{\sqrt{2}}$$



8.6 **8.6**

a) $$\vec{V}_{cm} = \frac{m\vec{V}_1 + m\vec{V}_2}{m+m} = \frac{\vec{V}_1+\vec{V}_2}{2} \qquad \vec{V}_1 = v_1\hat{i} \qquad \vec{V}_2 = -v_2\hat{j} \qquad \vec{V}_1+\vec{V}_2 = v_1\hat{i} - v_2\hat{j}$$

$$\vec{V}_{cm} = \frac{v_1}{2}\hat{i} - \frac{v_2}{2}\hat{j} = 4\hat{i} - 3\hat{j}$$

b) $$\vec{\mu}_1 = \vec{V}_1 - \vec{V}_{cm} = v_1\hat{i} - (4\hat{i}-3\hat{j}) = 4\hat{i}+3\hat{j}$$

$$\vec{\mu}_2 = \vec{V}_2 - \vec{V}_{cm} = -v_2\hat{j} - (4\hat{i}-3\hat{j}) = -4\hat{i}-3\hat{j}\;;\; |\mu_1| = |\mu_2| = \sqrt{4^2+3^2} = 5\ \mathrm{ms^{-1}}$$

choque elástico: $m\vec{\mu}_1 + m\vec{\mu}_2 = 0 \qquad \vec{\mu}_1 = -\vec{\mu}_2$

$m\vec{\mu}'_1 + m\vec{\mu}'_2 \qquad \vec{\mu}'_2 = -\vec{\mu}'_1$

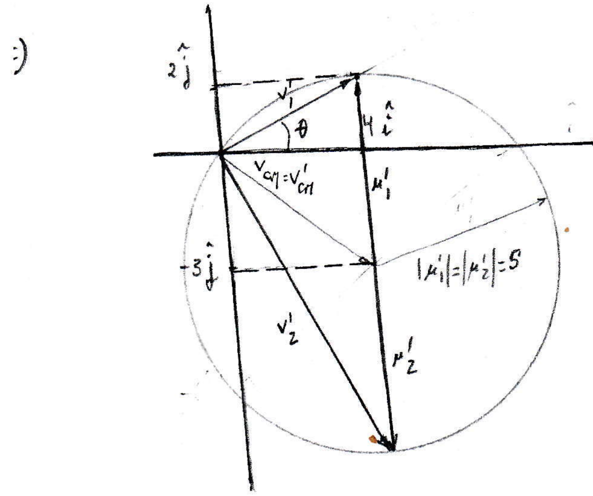
antes do choque: $$T_a = \tfrac{1}{2}m_1|\mu_1|^2 + \tfrac{1}{2}m_2|\mu_2|^2 + \tfrac{1}{2}(m_1+m_2)|V_{cm}|^2 = \frac{P^2}{2m_1} + \frac{P^2}{2m_2} + \tfrac{1}{2}(m_1+m_2)|V_{cm}|^2 = \frac{P^2}{2m_\mu} + \tfrac{1}{2}(m_1+m_2)|V_{cm}|^2$$

$$T_d = \tfrac{1}{2}m_1|\mu'_1|^2 + \tfrac{1}{2}m_2|\mu'_2|^2 + \tfrac{1}{2}(m_1+m_2)|V'_{cm}|^2 = \frac{P'^2}{2m'_\mu} + \tfrac{1}{2}(m_1+m_2)|V'_{cm}|^2;$$

Mas $T_a = T_d$, $V_{cm} = V'_{cm}$ e $m_\mu = m'_\mu$ facto que $P = P'$ isto é $m|\mu_1| = m|\mu_2| = m|\mu'_1| = \ldots$

Assim $|\mu_1| = |\mu_2| = 5$ e $|V_{cm}| = \sqrt{4^2+3^2} = 5$

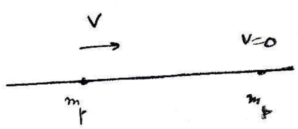
c)

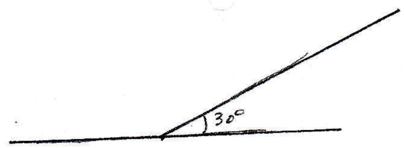


$tg\,\theta = \frac{2}{4} = \frac{1}{2}$

$\vec{v}_1{}' = 4\,\hat{i} + 2\,\hat{j}$

8.7



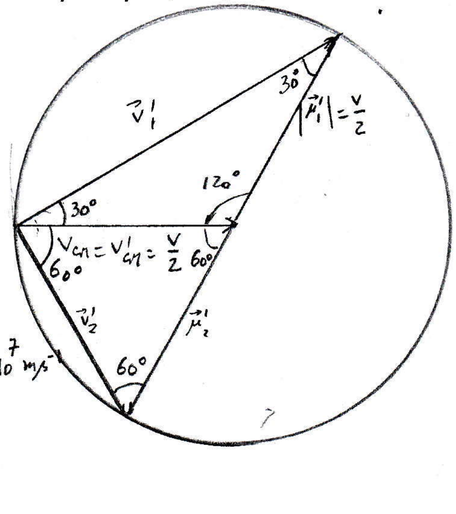


$\vec{V}_{cm} = \frac{m_p \vec{V} + m_p \cdot 0}{2\,m_p} = \frac{\vec{V}}{2}$

$\vec{\mu}_1 = \vec{V} - \vec{V}_{cm} = \vec{V} - \frac{\vec{V}}{2} = \frac{\vec{V}}{2}$

$\vec{\mu}_2 = 0 - \vec{V}_{cm} = -\frac{\vec{V}}{2}$

Sendo o choque elástico vimos no problema anterior que $|\mu_1| = |\mu_2| = |\mu_1'| = |\mu_2'| = \frac{V}{2}$



$\therefore$

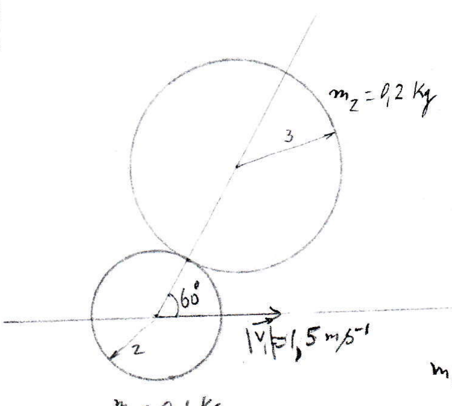
$v_1'^2 = v_{cm}'^2 + \mu_1'^2 - 2\,v_{cm}\,\mu_1' \cos 120°$

$v_1'^2 = v^2\left[\frac{1}{4} + \frac{1}{4} - 2\frac{1}{4}\left(-\frac{1}{2}\right)\right] = v^2\left[\frac{1}{2} + \frac{1}{4}\right] =$

$= v^2\frac{3}{4}$ donde $v_1' = \frac{\sqrt{3}}{2}\,v = \frac{\sqrt{3}}{2} \cdot 10^7\ m/s^{-1}$

$v_2' = |v_{cm}'| = \frac{1}{2}\,v = \frac{1}{2}\,10^7\ m/s^{-1}$

8.9



$\vec{V}_1 = V_1\,\hat{i}$

$\vec{V}_1{}' = a\hat{i} + b\hat{j}$

$\vec{V}_2{}' = c\hat{i} + d\hat{j}$

$m_1 \vec{V}_1 = m_1 \vec{V}_1{}' + m_2 \vec{V}_2{}'$

$m_1 V_1 = m_1 a + m_2 c \qquad c = \frac{m_1}{m_2}(V_1 - a)$

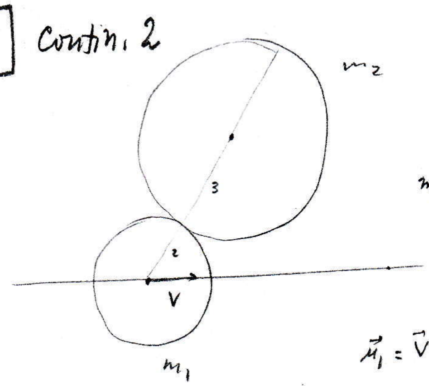
$0 = m_1 b + m_2 d \qquad d = -\frac{m_1}{m_2} b$

$\frac{1}{2} m_1 V_1^2 = \frac{1}{2} m_1 V_1'^2 + \frac{1}{2} m_2 V_2'^2$

$m_1 V_1^2 = m_1(a^2 + b^2) + m_2(c^2 + d^2)$

$m_1 V_1^2 = m_1 a^2 + m_1 b^2 + m_2\left(\frac{m_1^2}{m_2^2}(V_1 - a)^2 + \left(-\frac{m_1}{m_2} b\right)^2\right)$

8.9 Contin. 2 — Contin. 2 8.9



$$\frac{m_1\vec{V}}{m_1+m_2} = V_{CM} = \frac{m_1 v_1' + m_2 v_2'}{m_1+m_2} = V_{CM}'$$

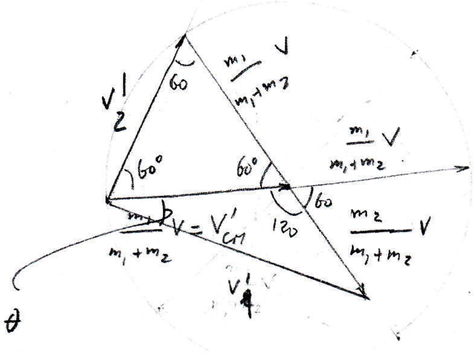
$$\vec{V}_{CM} = \vec{V}_{CM}'$$

$$\vec{\mu}_1 = \vec{V} - \vec{V}_{CM} = \vec{V} - \frac{m_1}{m_1+m_2}\vec{V} = \frac{m_2}{m_1+m_2}\vec{V}$$

$$\vec{\mu}_2 = 0 - \vec{V}_{CM} = \quad = -\frac{m_1}{m_1+m_2}\vec{V}$$

$$m_1|\mu_1| = m_2|\mu_2| = m_1|\mu_1'| = m_2|\mu_2'| \qquad |\mu_1'| = |\mu_1| = \frac{m_2}{m_1+m_2}V \qquad |\mu_1|+|\mu_2| = V$$

$$|\mu_2'| = |\mu_2| = \frac{m_1}{m_1+m_2}V = V_{CM}'$$



$$v_1'^2 = \left(\frac{m_1}{m_1+m_2}V\right)^2 + \left(\frac{m_2}{m_1+m_2}V\right)^2 - 2\frac{m_1}{m_1+m_2}\frac{m_2}{m_1+m_2}V^2\cos 120°$$

$$v_1'^2 = V^2\left[\frac{m_1^2+m_2^2}{(m_1+m_2)^2} + \frac{m_1 m_2}{(m_1+m_2)^2}\right] = V^2\frac{m_1^2+m_2^2+m_1m_2}{(m_1+m_2)^2}$$

$$v_1' = V\frac{\sqrt{m_1^2+m_2^2+m_1m_2}}{m_1+m_2} = 1{,}5\cdot\frac{\sqrt{0{,}1^2+0{,}2^2+0{,}1\cdot 0{,}2}}{0{,}1+0{,}2}$$

$$v_1' = 1{,}5 \underline{\qquad\qquad} = 1{,}32\ m\,s^{-1}$$

Cálculo de θ:
$$\frac{\frac{m_2}{m_1+m_2}V}{\operatorname{sen}\theta} = \frac{v_1'}{\operatorname{sen}120} \; ; \; \operatorname{sen}\theta = \frac{\frac{m_2}{m_1+m_2}V}{v_2'}\operatorname{sen}120 = \frac{\frac{0{,}2}{0{,}3}1{,}5}{1{,}32}\frac{\sqrt{3}}{2} =$$

$$\operatorname{sen}\theta = \frac{1}{1{,}32}\frac{\sqrt{3}}{2} = 0{,}656 \qquad \theta = 41°$$

Assim $\vec{v}_1'$ tem um módulo de $1{,}32\ m\,s^{-1}$ e faz um ângulo com a direcção de $\vec{V}$ (ou de $\vec{V}_{CM}$) de $-41°$.

Quanto a $\vec{v}_2'$ é fácil de ver pela geometria da figura que tem um módulo de $\frac{m_1}{m_1+m_2}V = \frac{0{,}1}{0{,}3}1{,}5 = 0{,}5\ m\,s^{-1}$ e faz um ∡ de $+60°$ com a direcção de $\vec{V}$

8.9 Contin. Contin. 8.9

$$m_1 V_1^2 = m_1 a^2 + m_1 b^2 + \frac{m_1^2}{m_2} V_1^2 - \frac{m_1^2}{m_2} 2 V_1 a + \frac{m_1^2}{m_2} a^2 + \frac{m_1^2}{m_2} b^2$$

$$m_1 V_1^2 = \left(m_1 + \frac{m_1^2}{m_2}\right) a^2 + \left(m_1 + \frac{m_1^2}{m_2}\right) b^2 - \frac{m_1^2}{m_2} 2 V_1 a + \frac{m_1^2}{m_2} V_1^2$$

$$V_1^2 = \frac{m_1+m_2}{m_2} a^2 + \frac{m_1+m_2}{m_2} b^2 - \frac{m_1}{m_2} 2Va + \frac{m_1}{m_2} V_1^2$$

$$\frac{m_1+m_2}{m_2} a^2 + \frac{m_1+m_2}{m_2} b^2 - \frac{m_1}{m_2} 2Va + \left(\frac{m_1}{m_2} - 1\right) V_1^2$$

$$a^2 + b^2 - \frac{m_1}{m_1+m_2} 2Va + \frac{m_1-m_2}{m_1+m_2} V_1^2 = 0 \qquad (1)$$

O ângulo com que o disco 2 se afastará será igual a 60°. Assim o argumento do vector $\vec{V}_2' = c\hat{i} + d\hat{j}$ é tal que $\operatorname{tg} 60° = \frac{d}{c} = \sqrt{3}$ ou $d = \sqrt{3}c$ e vem:

$-\frac{m_1}{m_2} b = \sqrt{3} \frac{m_1}{m_2}(V_1 - a)$ ou seja $-b = \sqrt{3}(V_1 - a)$ que substituindo na equação (1)

dá: $a^2 + 3(V_1 - a)^2 - \frac{m_1}{m_1+m_2} 2Va + \frac{m_1-m_2}{m_1+m_2} V_1^2 = 0$ e desenvolvendo e substituindo valores vem: $a^2 - 2{,}5a + 1{,}5 = 0$ cujas raízes são: $a = 1{,}5 \Rightarrow b = 0 \Rightarrow d = 0 \Rightarrow c = 0$ 1ª Sol.

2ª Sol.: $a = 1 \Rightarrow b = -\sqrt{3}(1{,}5 - 1) = -\frac{\sqrt{3}}{2} \Rightarrow$

$c = \frac{m_1}{m_2}(V_1 - a) = \frac{0{,}1}{0{,}3}(1{,}5 - 1) = 0{,}25 \Rightarrow d = -\frac{m_1}{m_2} b = \frac{\sqrt{3}}{4}$

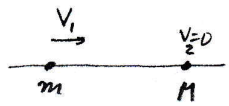
a 1ª Sol dá: $\vec{V}_1' = 1{,}5\hat{i}$

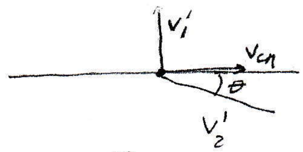
$\vec{V}_2' = 0$ isto é, o disco 1 "passa" pelo disco 2 sem lhe transmitir movimento.

a 2ª solução dá $V_1' = \hat{i} - \frac{\sqrt{3}}{2}\hat{j}$ pelo que $|V_1'| = \sqrt{1 + \frac{3}{4}} = \frac{\sqrt{7}}{2} = 1{,}32\ m s^{-1}$ e $\arg \vec{V}_1' = \operatorname{arctg}\left(-\frac{\sqrt{3}}{2}\right) = -41°$

$V_2' = 0{,}25\hat{i} + \frac{\sqrt{3}}{4}\hat{j}$ pelo que $|V_2'| = \sqrt{\frac{1}{16} + \frac{3}{16}} = \frac{1}{2}\ m s^{-1}$ e $\arg \vec{V}_2' = \operatorname{arctg} \frac{\frac{\sqrt{3}}{4}}{\frac{1}{4}} = \operatorname{arctg}\sqrt{3} = 60°$

8.11 8.11



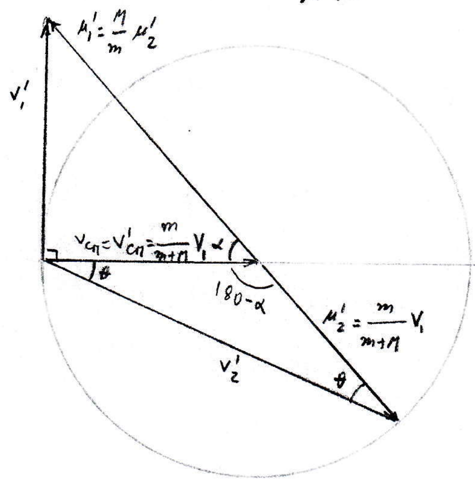


$\vec V_{CM} = \frac{m\vec V_1}{m+M} = \frac{m}{m+M}\vec V_1$ e de $m\vec V_1 = mV_1' + MV_2'$ resulta que $\vec V_{CM} = \vec V'_{CM}$

$\vec u_1 = \vec V_1 - \vec V_{CM} = \vec V_1\left(1-\frac{m}{m+M}\right) = \frac{M}{m+M}\vec V_1$

$\vec u_2 = 0 - \vec V_{CM} = \quad = -\frac{m}{m+M}\vec V_1$

O choque é elástico pelo que há conservação da energia cinética e viu-se em problemas anteriores que então: $P' = P = m|u_1| = m|u_1'| = M|u_2| = M|u_2'|$ e então $|u_1| = |u_1'|$ e $|u_2| = |u_2'|$ e neste caso $|u_2'| = \frac{m}{m+M}|\vec V_1| = |V_{CM}| = |V'_{CM}|$



$\cos\alpha = \frac{\frac{m}{m+M}V_1}{\frac{M}{m+M}V_1} = \frac{m}{M}$

$\theta = \frac{1}{2}\left(180-(180-\alpha)\right) = \frac{\alpha}{2}$ $\quad \alpha = 2\theta$

$\cos 2\theta = \frac{m}{M}$; $\boxed{\theta = \frac{1}{2}\arccos\frac{m}{M}}$

Vamos transformar $\cos 2\theta = \frac{m}{M}$ para obtermos a solução do livro.

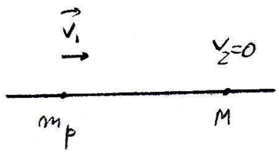
$\cos 2\theta = \frac{m}{M}$

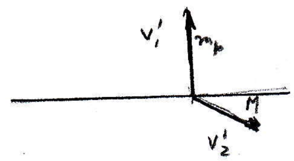
$\cos^2\theta - \operatorname{sen}^2\theta = \frac{m}{M}$; $1 - 2\operatorname{sen}^2\theta = \frac{m}{M}$ $\quad \operatorname{sen}^2\theta = \frac{1}{2}\left(1-\frac{m}{M}\right)$ $\quad \operatorname{sen}\theta = \frac{1}{\sqrt{2}}\sqrt{1-\frac{m}{M}}$

$2\cos^2\theta - 1 = \frac{m}{M}$ $\quad \cos^2\theta = \frac{1}{2}\left(1+\frac{m}{M}\right)$ $\quad \cos\theta = \frac{1}{\sqrt{2}}\sqrt{1+\frac{m}{M}}$

pelo que $\operatorname{tg}\theta = \frac{\operatorname{sen}\theta}{\cos\theta} = \sqrt{\frac{1-\frac{m}{M}}{1+\frac{m}{M}}}$ que é a solução apresentada no livro.

8.12 | 8.12





$m_p \vec{V}_1 + M\vec{V}_2 = m_p \vec{V}_1' + M\vec{V}_2'$ em que $\vec{V}_2 = 0$

Componentes horizontais e verticais: $\begin{cases} m_p v_{1_h} = \overbrace{m_p v'_{1_h}}^{=0} + M v'_{2_h} \\ \underbrace{m_p v_{1_v}}_{=0} = m_p v'_{1_v} + M v'_{2_v} \end{cases}$ $\begin{cases} m_p v_{1_h} = M v'_{2_h} \\ 0 = m_p v'_{1_v} + M v'_{2_v} \end{cases}$ (1)

Mas $E = \frac{1}{2} m_p v_1^2$; $2Em_p = m_p^2 v_1^2$; $m_p v_1 = \sqrt{2Em_p}$ e então $m_p v_{1_h} = \sqrt{2Em_p}$

$E' = \frac{1}{2} m_p v'^2_{1_v}$ pelo que $m_p v'_{1_h} = \sqrt{2E'm_p}$ e substituindo em (1) vem:

$\begin{cases} \sqrt{2Em_p} = M v'_{2_h} \\ 0 = \sqrt{2E'm_p} + M v'_{2_v} \end{cases}$ $\begin{cases} 2m_p E = M^2 v'^2_{2_h} \\ 2m_p E' = M^2 v'^2_{2_v} \end{cases}$ que somando dá $2m_p(E+E') = M^2(v'^2_{2_h} + v'^2_{2_v})$

mas $v'^2_{2_h} + v'^2_{2_v} = v'^2_2$

Como o choque é elástico há conservação de energia cinética e vem:

$E + 0 = E' + \frac{1}{2} M v_2'^2$ ou $E - E' = \frac{1}{2} M v_2'^2$ e então resulta: $E - E' = \frac{1}{2} M v_2'^2$

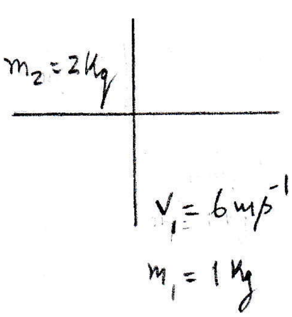
$2m_p(E+E') = M^2 v_2'^2$

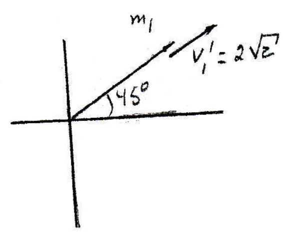
e dividindo as duas equações vem $\dfrac{2m_p(E+E')}{E-E'} = \dfrac{M^2 v_2'^2}{\frac{1}{2} M v_2'^2} = 2M$ pelo que

$M = m_p \dfrac{E+E'}{E-E'} = m_p \dfrac{1+0{,}8}{1-0{,}8} = \dfrac{1{,}8}{0{,}2} m_p = 9 m_p$ donde $\dfrac{M}{m_p} = 9$ //

8.13 | 8.13

$\vec{V}_1 = 6\hat{j}$ $\vec{V}_1' = 2\hat{i} + 2\hat{j}$





$\vec{V}_{cm} = \dfrac{m_1\vec{V}_1 + m_2 \cdot 0}{m_1 + m_2} = \dfrac{m_1}{m_1 + m_2}\vec{V} = \dfrac{\vec{V}}{3} = \vec{V}'_{cm}$

$\vec{\mu}_1 = \vec{V}_1 - \vec{V}_{cm} = \vec{V} - \dfrac{\vec{V}}{3} = \dfrac{2}{3}\vec{V}$ $|\mu_1| = \frac{2}{3} 6 = 4 \ ms^{-1}$

$\vec{\mu}_2 = 0 - \vec{V}_{cm} = -\dfrac{\vec{V}}{3} = -2\hat{j}$ $|\mu_2| = 2 \ ms^{-1}$

$m_1 \vec{V}_1 = m_1 \vec{V}_1' + m_2 \vec{V}_2'$

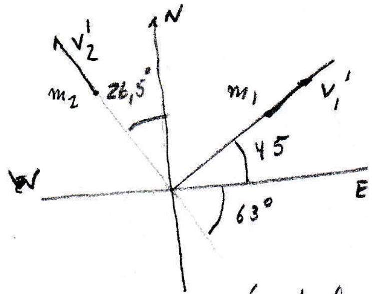
$1 \cdot 6\hat{j} = 1 \cdot (2\hat{i} + 2\hat{j}) + 2(a\hat{i} + b\hat{j})$

$6\hat{j} = (2 + 2a)\hat{i} + (2 + 2b)\hat{j}$

$\left.\begin{matrix} 2 + 2a = 0 & a = -1 \\ 2 + 2b = 6 & b = 2 \end{matrix}\right\} \Rightarrow V_2' = -i + 2\hat{j}$ $\begin{cases} |V_2'| = \sqrt{1^2 + 2^2} = \sqrt{5} \ ms^{-1} \\ \arg V_2' = \operatorname{arctg}\left(\frac{2}{-1}\right) = -63^\circ \end{cases}$

8.13 Contin. Contin. 8.13

Depois do choque



b) A en. cinética no sistema do CM é dada por:

antes do choque $T_a = \frac{1}{2} m_1 \mu_1^2 = \frac{1}{2} \cdot 1 \cdot 16 = 8$

depois do choque $T_d = \frac{1}{2} m_1 \mu_1'^2 + \frac{1}{2} m_2 \mu_2'^2 = \cancel{\frac{1}{2} \cdot 1 \cdot (2\sqrt{2})^2 + \frac{1}{2} \cdot 2 \cdot (\sqrt{5})^2}$

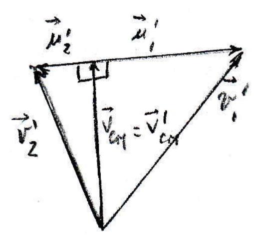
$\vec{\mu}_1' = \vec{V}_1' - \vec{V}_{CM}' = 2\hat{i} + 2\hat{j} - \frac{6}{3}\hat{j} = 2\hat{i}$ $\quad T_d = \frac{1}{2} 1 \cdot 2^2 + \frac{1}{2} \cdot 2 \cdot (-1)^2 = 2 + 1 = 3$

$\vec{\mu}_2' = \vec{V}_2' - V_{CM}' = -\hat{i} + 2\hat{j} - \frac{6}{3} j = -\hat{i}$ $\quad T_a - T_d = 8 - 3 = 5$ $\quad$ Perda $= \frac{5}{8}$

A perda de en. cinética no sistema do CM é de $\frac{5}{8}$.

Nota: a solução indica $\frac{3}{4}$!

c)



A trajectória da massa m_1 é deflectida de 90° em relação a $\vec{V}_{CM}'$, isto é, no sistema do Centro de Massa

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8.14 8.14

$\vec{V}_1 = 3\hat{i} + 2\hat{j} - \hat{k}$ $\quad \vec{V}_2 = -2\hat{i} + 2\hat{j} + 4\hat{k}$

$m_1 = 2$ kg; $m_2 = 3$ kg

$m_1\vec{v}_1 + m_2\vec{v}_2 = (m_1 + m_2)\vec{v}$; $\vec{v} = \frac{m_1}{m_1 + m_2}\vec{v}_1 + \frac{m_2}{m_1 + m_2}\vec{v}_2 = \frac{2}{5}(3i + 2j - k) + \frac{3}{5}(-2i + 2j + 4k) =$

8.14

$= 2\hat{j} + 2\hat{k}$

$|\vec{v}| = \sqrt{2^2 + 2^2} = 2\sqrt{2}$ m s^{-1}

$|v_1| = \sqrt{3^2 + 2^2 + (-1)^2} = \sqrt{14}$

$|v_2| = \sqrt{(-2)^2 + 2^2 + 4^2} = \sqrt{24} = 2\sqrt{6}$

$T_a = \frac{1}{2} m_1 |v_1|^2 + \frac{1}{2} m_2 |v_2|^2 = \frac{1}{2} \cdot 2 \cdot 14 + \frac{1}{2} \cdot 3 \cdot 24 = 14 + 36 = 50$ J que é a en. cinética no sistema laboratório

$\vec{V}_{CM} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2} = 2\hat{j} + 2\hat{k}$

$\vec{\mu}_1 = \vec{V}_1 - \vec{V}_{CM} = 3i + 2j - k - (2j + 2k) = 3\hat{i} - 3\hat{k}$ $\quad |\mu_1| = \sqrt{3^2 + 3^2} = \sqrt{18}$

$\vec{\mu}_2 = \vec{V}_2 - \vec{V}_{CM} = -2i + 2j + 4k - (2j + 2k) = -2\hat{i} + 2\hat{k}$ $\quad |\mu_2| = \sqrt{2^2 + 2^2} = 2\sqrt{2}$

$\boxed{8.14}$ Contin. Contin. $\boxed{8.14}$

A energia cinética no sistema do CM é:

$$T = \frac{1}{2} m_1 |u_1|^2 + \frac{1}{2} m_2 |u_2|^2 = \frac{1}{2} \cdot 2 \cdot 18 + \frac{1}{2} 3 \cdot 8 = 18 + 12 = 30 \text{ J}.$$