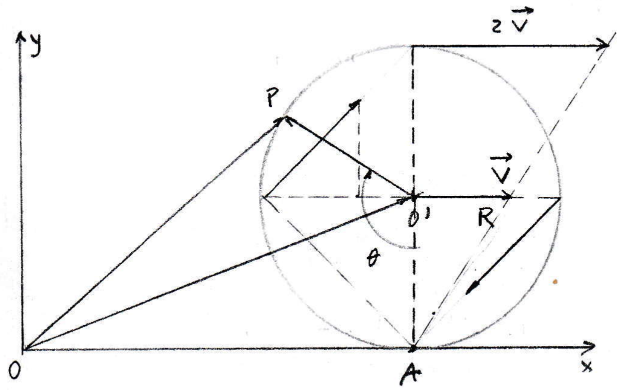
14.1



$$\vec{PO} = \vec{PO'} + \vec{O'O}$$

$$\vec{O'O} = R\theta\,\hat{i} + R\,\hat{j}$$

$$\vec{PO'} = -R\cos\left(\theta - \frac{\pi}{2}\right)\hat{i} + R\,\text{sen}\left(\theta - \frac{\pi}{2}\right)\hat{j} =$$

$$= -R\,\text{sen}\,\theta\,\hat{i} - R\cos\theta\,\hat{j}$$

$$\frac{d\vec{O'O}}{dt} = R\frac{d\theta}{dt}\hat{i} = V\hat{i} \quad \text{pois} \quad R\frac{d\theta}{dt} = V \text{ (não há escorregamento!)}$$

$$\frac{d\vec{PO'}}{dt} = -R\frac{d\theta}{dt}\cos\theta\,\hat{i} + R\frac{d\theta}{dt}\,\text{sen}\,\theta\,\hat{j} = -V\cos\theta\,\hat{i} + V\,\text{sen}\,\theta\,\hat{j}$$

$$\vec{v} = \frac{d\vec{PO}}{dt} = \frac{d\vec{PO'}}{dt} + \frac{d\vec{O'O}}{dt} = V(1-\cos\theta)\,\hat{i} + V\,\text{sen}\,\theta\,\hat{j}$$

Verificação: $\theta = 0$ $\quad \vec{v} = 0$. No ponto A a velocidade anula-se, Centro instantâneo de rotação

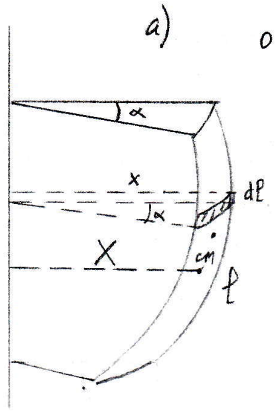
$\theta = \pi \quad \vec{v} = 2V\hat{i}$

$\theta = \frac{\pi}{2} \quad \vec{v} = V\hat{i} + V\hat{j}$

$\theta = \frac{3\pi}{2} \quad \vec{v} = V\hat{i} - V\hat{j}$

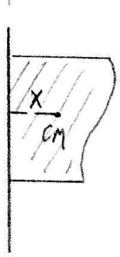
14.2

a) o elemento de área indicado é: $dA = x\alpha\, d\ell$



$A = \int dA = \alpha \int_\ell x\, d\ell$ mas $\int_\ell x\, d\ell = \ell \cdot X$ em que X é a coordenada do centro de massa e então:

$$A = \alpha\,\ell X = \alpha\, X\, \ell$$



$$\text{Volume} = \alpha \int_A x\, dA = \alpha\, A X$$

14.3 | 14.3

$\sum_i m_i \vec{r}_i = \vec{R} \sum m_i \qquad \sum m_i \ddot{\vec{r}}_i = \ddot{\vec{R}} \cdot \sum m_i \qquad \sum_i m_i \ddot{\vec{r}}_i = \ddot{\vec{R}} \cdot M$

ou seja $M \ddot{\vec{R}} = \sum_i f_i$ em que $f_i = m_i \ddot{\vec{r}}_i$

14.4 | 14.5

momento angular $\vec{L} = \vec{r} \times \vec{p}$; $\frac{d\vec{L}}{dt} = \frac{d\vec{r}}{dt} \times \vec{p} + \vec{r} \times \frac{d\vec{p}}{dt} = \vec{r} \times m \frac{d\vec{v}}{dt} = \vec{r} \times \vec{F} = \vec{\tau}$

em que $\frac{d\vec{r}}{dt} \times \vec{p} = \frac{d\vec{r}}{dt} \times m\vec{v} = \frac{d\vec{r}}{dt} \times m \frac{d\vec{r}}{dt} = 0$ pois os 2 vectores são colineares

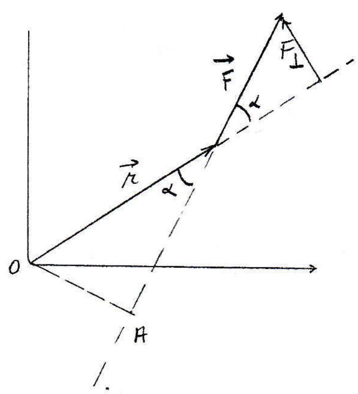
a)

$$\vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 8 & 6 & 0 \\ 30 & 40 & 0 \end{vmatrix} = (320 - 180)\hat{k} = 140 \cdot \hat{k}$$

$\vec{r} = 8\hat{i} + 6\hat{j}$

$\vec{F} = 30\hat{i} + 40\hat{j}$

b)



$|\vec{r} \times \vec{F}| = |r| \cdot |F| \cdot \operatorname{sen} \alpha$

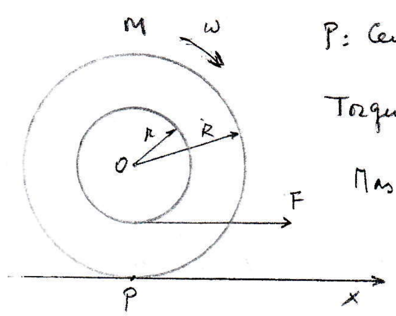
$\operatorname{sen} \alpha = \frac{\overline{OA}}{|\vec{r}|} \Rightarrow \overline{OA} = |\vec{r}| \operatorname{sen} \alpha$

$|\vec{r} \times \vec{F}| = |\vec{F}| |\vec{r}| \operatorname{sen} \alpha = |\vec{F}| |\overline{OA}|$

$\overline{OA} = \frac{|\vec{r} \times \vec{F}|}{|\vec{F}|} = \frac{140}{\sqrt{30^2 + 40^2}} = 2,8$ metro

c) $F_\perp = F \cdot \operatorname{sen} \alpha \qquad |\vec{r} \times \vec{F}| = |\vec{r}| \cdot F_\perp \qquad F_\perp = \frac{|\vec{r} \times \vec{F}|}{|\vec{r}|} = \frac{140}{\sqrt{8^2 + 6^2}} = 14$ N

14.5 | 14.5



P: Centro instantâneo de rotação. Não há escorregamento.

Torque em relação a P: $F(R-r)$

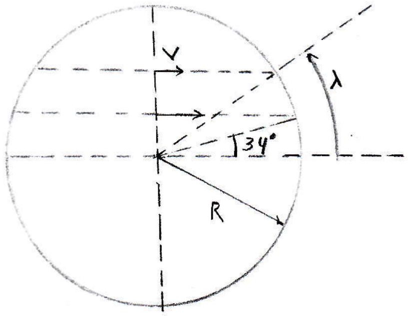
Mas $F(R-r) = I\ddot{\theta}$ com $I = I_0 + MR^2 = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$

$F(R-r) = \frac{3}{2}MR^2\ddot{\theta}$ e como $R \cdot \theta = x \Rightarrow R\ddot{\theta} = \ddot{x}$ e vem

$F(R-r) = \frac{3}{2}MR^2 \frac{1}{R}\ddot{x} \qquad F(R-r) = \frac{3}{2}MR\ddot{x} \Rightarrow \ddot{x} > 0$ isto é

a aceleração é positiva e dirigida no sentido positivo do eixo dos xx. Então o yo-yo move-se para a direita

14.6 | 14.6



$R = 6{,}38 \cdot 10^6\ m$

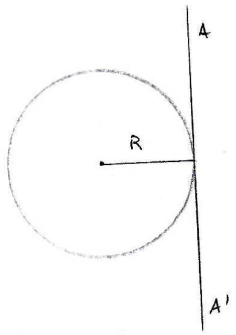
$V = R \cos\lambda\ \dot{\theta}$ $\qquad \dot{\theta} = \dfrac{d\theta}{dt} = \dfrac{2\pi}{24\,h}$

Em Los Ângeles $\quad V = 6{,}38 \cdot 10^6 \cdot \cos 34 \cdot \dfrac{2\pi}{24 \cdot 3600} = 385\ m\,s^{-1}$

$385 - 200 = 185 = R \cos\lambda\ \dfrac{2\pi}{24 \cdot 3600}$; $\cos\lambda = \dfrac{185 \cdot 24 \cdot 3600}{2\pi \cdot 6{,}38 \cdot 10^6} = 0{,}398$

$\lambda = 66{,}5^\circ\ N$

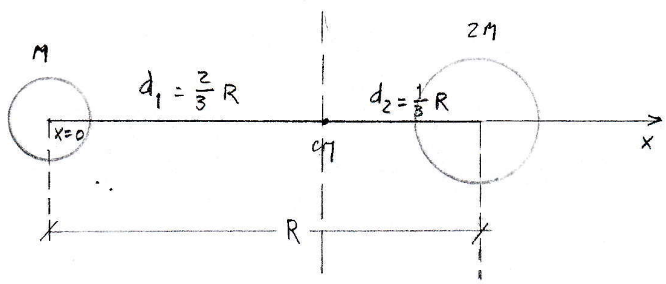
14.7 | 14.7



Volume do toro = Área do círculo x deslocamento do centro de massa

$\text{Volume} = \pi R^2 \cdot 2\pi R = 2\pi^2 R^3$

14.8 | 14.8



localização do CM:

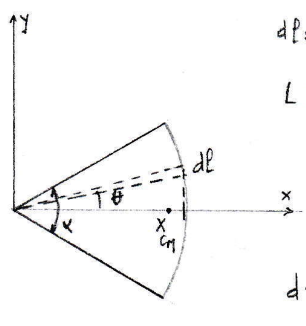
$M \cdot 0 + 2M\,R = 3M\,d_1$

$d_1 = \frac{2}{3}R \quad e \quad d_2 = \frac{1}{3}R$

En. cinética de rotação: $\quad T = \frac{1}{2} I \omega^2 \quad$ em que $\quad I = M\left(\frac{2}{3}R\right)^2 + 2M\left(\frac{1}{3}R\right)^2 = \left(\frac{4}{9} + \frac{2}{9}\right) M R^2 = \frac{2}{3} M R^2$

$T = \frac{1}{2}\ \frac{2}{3} M R^2 \omega^2 = \frac{1}{3} M R^2 \omega^2$

14.9 | 14.9



$d\ell = R\,d\theta$

$L X_{CM} = \int_\ell x\,d\ell = \int_{-\frac{\alpha}{2}}^{\frac{\alpha}{2}} R\cos\theta \cdot R\,d\theta = R^2 \operatorname{sen}\theta \Big|_{-\frac{\alpha}{2}}^{\frac{\alpha}{2}} = 2R^2 \operatorname{sen}\frac{\alpha}{2}$; $\quad X_{CM} = \dfrac{2R^2}{L} \operatorname{sen}\dfrac{\alpha}{2}$

Por curiosidade façamos este cálculo em coordenadas cartesianas;

$d\ell = \sqrt{dx^2 + dy^2} = dx\sqrt{1 + \left(\dfrac{dy}{dx}\right)^2} \qquad x^2 + y^2 = R^2\ ;\ 2x\,dx + 2y\,dy = 0 \qquad \dfrac{dy}{dx} = -\dfrac{x}{y}$

$d\ell = dx\sqrt{1 + \dfrac{x^2}{y^2}} = dx\sqrt{\dfrac{x^2 + y^2}{y}} = dx\,\dfrac{R}{y} = \dfrac{R}{y}\,dx$

14.9 contin. contin. 14.9

$$d\ell = \frac{R}{y}\,dx \quad \text{mas} \quad y = \pm\sqrt{R^2 - x^2} \qquad d\ell = \frac{R}{\pm\sqrt{R^2-x^2}}\,dx$$

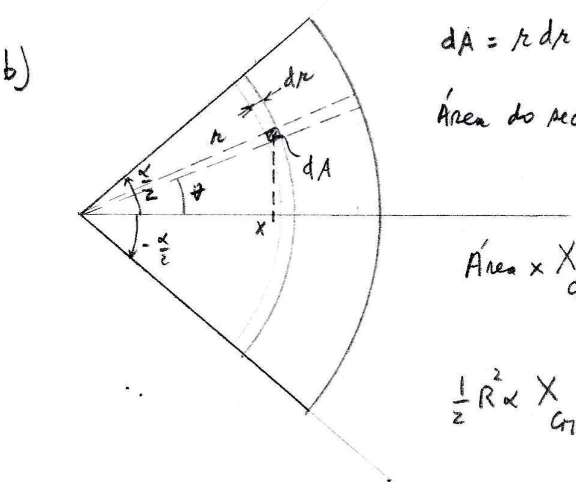
$$L\,X_{CM} = \int_\ell x\,d\ell = \int \frac{Rx}{\pm\sqrt{R^2-x^2}}\,dx$$

$$X_{CM} = \frac{R}{L}\left[\int_R^{R\cos\frac{\alpha}{2}} \frac{x}{-\sqrt{R^2-x^2}}\,dx + \int_{R\cos\frac{\alpha}{2}}^{R} \frac{x}{\sqrt{R^2-x^2}}\,dx\right] = \frac{R}{L}\,2\int_{R\cos\frac{\alpha}{2}}^{R} \frac{x}{\sqrt{R^2-x^2}}\,dx =$$

$$= \frac{2R}{L}\left[-\sqrt{R^2-x^2}\,\Big|_{R\cos\frac{\alpha}{2}}^{R}\right] = \frac{2R}{L}\left[-0+\sqrt{R^2-R^2\cos^2\frac{\alpha}{2}}\right] = \frac{2R}{L}\,R\sqrt{1-\cos^2\frac{\alpha}{2}} = \frac{2R^2}{L}\,\text{sen}\,\frac{\alpha}{2}$$

que dá, claro, o mesmo resultado que o obtido com coordenadas polares mas é mais trabalhoso.

b)



$$dA = r\,dr\,d\theta$$

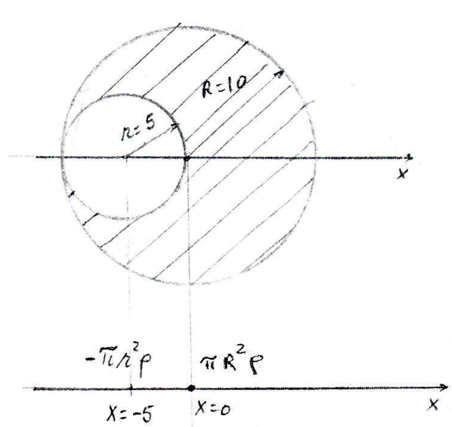
$$\text{Área do sector circular} = \frac{\pi R^2}{2}\,\frac{\alpha}{\pi} = \frac{1}{2}R^2\alpha$$

$$\text{Área} \times X_{CM} = \int_A x\,dA \qquad \frac{1}{2}R^2\alpha \cdot X_{CM} = \iint r\cos\theta\, r\,dr\,d\theta =$$

$$\frac{1}{2}R^2\alpha\, X_{CM} = \int_0^R r^2\,dr \int_{-\frac{\alpha}{2}}^{\frac{\alpha}{2}} \cos\theta\,d\theta = \frac{R^3}{3}\cdot \text{sen}\,\theta\,\Big|_{-\frac{\alpha}{2}}^{\frac{\alpha}{2}} = \frac{R^3}{3}\,2\,\text{sen}\,\frac{\alpha}{2}$$

$$X_{CM} = \frac{4}{3}\frac{R}{\alpha}\,\text{sen}\,\frac{\alpha}{2}$$

14.10 14.10

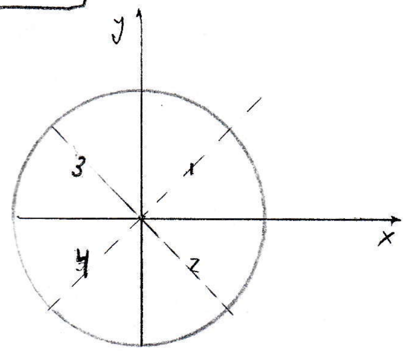


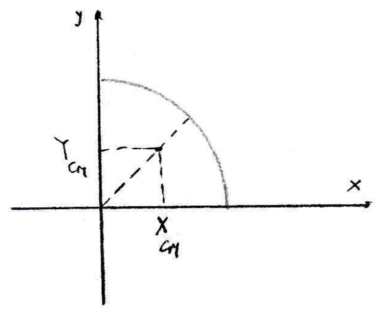
$$\left(\pi R^2\rho - \pi r^2\rho\right)X_{CM} = (-5)(-\pi r^2\rho) + 0\cdot\pi R^2\rho$$

$$X_{CM} = \frac{5r^2}{R^2-r^2} = \frac{5\cdot 5^2}{10^2-5^2} = \frac{125}{75} = \frac{5}{3} = 1\frac{2}{3} \simeq 1{,}66\ldots$$

$$Y_{CM} = 0$$

 14.11





Cálculo do CM do quadrante:

Volume criado pela rotação em torno do eixo dos yy é:

$$\text{Volume} = \frac{1}{2}\,\frac{4}{3}\pi R^3$$

Área do quadrante: $A = \frac{1}{4}\pi R^2$

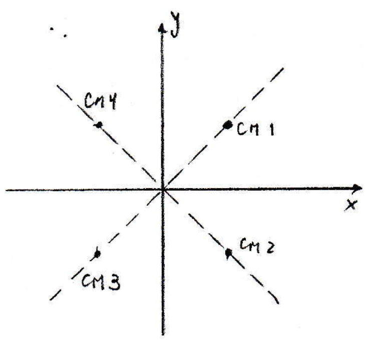
Comprimento do caminho percorrido pelo CM quando faz uma rotação completa:

$$2\pi X_{CM}$$

Foi demonstrado que $2\pi X_{CM}\cdot \text{Area} = \text{Volume}$ e então $2\pi X_{CM} = \dfrac{\frac{1}{2}\frac{4}{3}\pi R^3}{\frac{1}{4}\pi R^2} = \dfrac{\frac{2}{3}R}{\frac{1}{4}} = \frac{8}{3}R$

pelo que $X_{CM} = \dfrac{8R}{6\pi} = \dfrac{4R}{3\pi}$

Por simetria $Y_{CM} = \dfrac{4R}{3\pi}$



	Coordenadas	massa
CM1	$\left(\frac{4R}{3\pi}, \frac{4R}{3\pi}\right)$	$\frac{1}{4}\pi R^2\rho_1$
CM2	$\left(\frac{4R}{3\pi}, -\frac{4R}{3\pi}\right)$	$\frac{1}{4}\pi R^2\rho_2 = \frac{1}{4}\pi R^2\, 2\rho_1$
CM3	$\left(-\frac{4R}{3\pi}, -\frac{4R}{3\pi}\right)$	$\frac{1}{4}\pi R^2\rho_3 = \frac{1}{4}\pi R^2\, 4\rho_1$
CM4	$\left(-\frac{4R}{3\pi}, \frac{4R}{3\pi}\right)$	$\frac{1}{4}\pi R^2\rho_4 = \frac{1}{4}\pi R^2\, 3\rho_1$

Cálculo do CM conjunto: $m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + m_4\vec{r}_4 = (m_1+m_2+m_3+m_4)\vec{R}$ e vem

$$\cancel{\tfrac{1}{4}\pi R^2}(\rho_1+2\rho_1+3\rho_1+4\rho_1)\cdot X_{CM_c} = \cancel{\tfrac{1}{4}\pi R^2}\rho_1\cdot\frac{4R}{3\pi} + \cancel{\tfrac{1}{4}\pi R^2}2\rho_1\cdot\frac{4R}{3\pi} + \cancel{\tfrac{1}{4}\pi R^2}4\rho_1\left(-\frac{4R}{3\pi}\right) + \cancel{\tfrac{1}{4}\pi R^2}3\rho_1\left(-\frac{4R}{3\pi}\right)$$

$$10\rho_1 X_{CM_c} = \rho_1(1+2-4-3)\frac{4R}{3\pi}\ ;\quad X_{CM_c} = \frac{1}{10}(-4)\frac{4R}{3\pi} = -\frac{8R}{15\pi}$$

$$\cancel{\tfrac{1}{4}\pi R^2}(\rho_1+2\rho_1+3\rho_1+4\rho_1)Y_{CM_c} = \cancel{\tfrac{1}{4}\pi R^2}\rho_1\frac{4R}{3\pi} + \cancel{\tfrac{1}{4}\pi R^2}2\rho_1\left(-\frac{4R}{3\pi}\right) + \cancel{\tfrac{1}{4}\pi R^2}4\rho_1\left(-\frac{4R}{3\pi}\right) + \cancel{\tfrac{1}{4}\pi R^2}3\rho_1\left(\frac{4R}{3\pi}\right)$$

$10\,Y_{CM_c} = \dfrac{4R}{3\pi}(1-2-4+3)\ ;\quad Y_{CM_c} = \dfrac{1}{10}\,\dfrac{4R}{3\pi}\cdot(-2) = -\dfrac{4R}{15\pi}$. Assim a recta que →

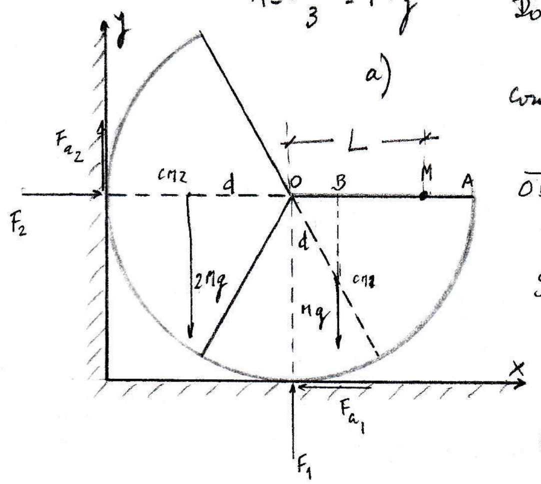
14.11 contin. Contin 14.11

a origem das coordenadas com o CM_c, cujas coordenadas são: $X_{CM_c} = -\frac{8R}{15\pi}$ e $Y_{CM_c} = -\frac{4R}{15\pi}$

é $y = \frac{Y_{CM_c}}{X_{CM_c}} x = \frac{-\frac{4R}{15\pi}}{-\frac{8R}{15\pi}} x = \frac{1}{2} x$

14.12 14.12

$M = \frac{3\,Kg}{3} = 1\,Kg$



Do problema 14.9 vem: $d = \frac{4R}{3\alpha}\sin\frac{\alpha}{2}$ que, neste caso, com $R = \pi$ e $\alpha = 120° = \frac{2\pi}{3}$ dá: $d = \frac{4\pi}{3\,\frac{2\pi}{3}}\sin 60° = \sqrt{3}$

$\overline{OB} = d\cos 60° = \frac{\sqrt{3}}{2}$

Somatório dos momentos em relação a O: $\sum \vec{M}_O = 0$

$2Mg\sqrt{3} - Mg\frac{\sqrt{3}}{2} - F_{a_2}\pi - F_{a_1}\pi = 0$ mas $F_{a_2} = 0$ (do enunciado)

$(2 - \frac{1}{2})\sqrt{3}\,Mg - F_{a_1}\pi = 0$; $\frac{3\sqrt{3}}{2\pi}Mg = F_{a_1} = \frac{3\sqrt{3}}{2\pi}\,1\,kgf = \frac{3\sqrt{3}}{2\pi}\,kgf$

Somatório das forças verticais: $\sum F_y = 0$: $-2Mg + F_1 - Mg = 0$; $F_1 = 3Mg = 3\,kgf$

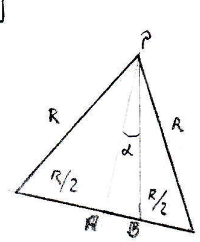
" " " horizontais: $\sum F_x = 0$: $F_2 - F_{a_1} = 0$; $F_2 = F_{a_1} = \frac{3\sqrt{3}}{2\pi}\,kgf = \underline{\underline{8,1\ N}}$

b) É colocada a massa M à distância L de O e retirada a parede. Calcular L

$\sum \vec{M}_O = 0$: $2Mg\sqrt{3} - Mg\frac{\sqrt{3}}{2} - F_{a_1}\pi - MgL = 0$. Mas neste caso $F_{a_1} = 0$ (bem como F_{a_2})

$(2 - \frac{1}{2})\sqrt{3}\,Mg = MgL$ donde $L = \frac{3\sqrt{3}}{2} = \underline{\underline{2,6\ cm}}$

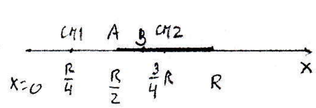
14.13 14.13



$tg\,\alpha = \frac{\overline{AB}}{\overline{AP}} = \frac{\frac{R}{12}}{R\frac{\sqrt{3}}{2}} = \frac{1}{6\sqrt{3}}$ ou $\alpha = 5,49°$

$\overline{AP} = \sqrt{R^2 - \left(\frac{R}{2}\right)^2} = R\cdot\frac{\sqrt{3}}{2}$

$\overline{AB} = \frac{R}{12}$ (calculada abaixo)

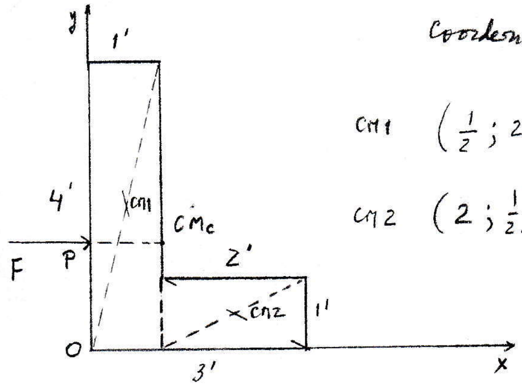


	Coordenadas dos CM	Massa
CM1	$(\frac{R}{4};0)$	M
CM2	$(\frac{3}{4}R;0)$	2M

$(M+2M)X_{CM_c} = \frac{R}{4}M + \frac{3}{4}R\,2M$; $X_{CM} = \frac{7}{12}R$

Distância $\overline{AB} = \frac{7}{12}R - \frac{R}{2} = \frac{R}{12}$

14.14 | 14.14



Coordenadas dos CM: | Massa nos CM:

CM1 $\left(\frac{1}{2}; 2\right)$ em inches $\quad 1 \cdot 4 \cdot \rho$

CM2 $\left(2; \frac{1}{2}\right)$ " $\quad 1 \cdot 2 \cdot \rho$

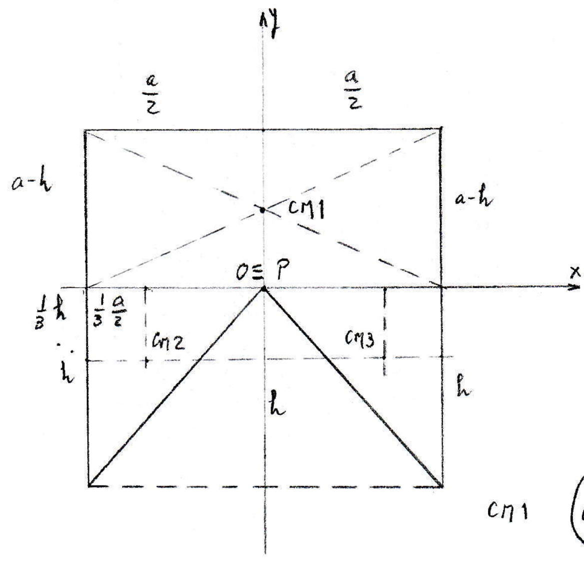
Cálculo do CM conjunto: CM_c

$$(4\rho+2\rho)X_{CM_c} = \frac{1}{2}4\rho + 2\cdot 2\rho;\quad 6X_{CM_c} = 2+4;\quad X_{CM_c} = 1'$$

$$(4\rho+2\rho)Y_{CM_c} = 2\cdot 4\rho + \frac{1}{2}2\rho;\quad 6Y_{CM_c} = 8+1;\quad Y_{CM_c} = \frac{9}{6} = \frac{3}{2} = 1{,}5'$$

O impulso deve ser aplicado em P à distância 1,5' de O para que o conjunto deslize sem rodar ao longo de x.

14.15 | 14.15



Para que o equilíbrio seja indiferente é preciso que Y_{CM_t}, centro de Massa total, coincida com o vertice P do corte e, para o sistema de eixos da figura, com a origem das coordenadas.

Coordenadas dos CM: | Massas nos CM:

CM1 $\left(0; \frac{1}{2}(a-h)\right)$ $\quad a(a-h)$

CM2 $\left(-\frac{2}{3}\frac{a}{2}; -\frac{1}{3}h\right)$ $\quad \frac{1}{2}\frac{a}{2}h = \frac{ah}{4}$

CM3 $\left(\frac{2}{3}\frac{a}{2}; -\frac{1}{3}h\right)$ $\quad \frac{ah}{4}$

Cálculo do CM total: $X_{CM_t} = 0$ porque a simetria da figura assim o impõe

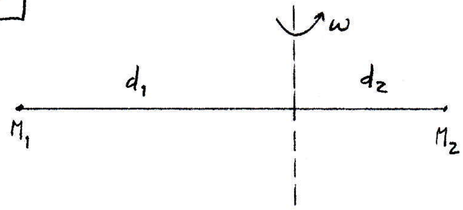
$$\left(a(a-h) + \frac{ah}{4} + \frac{ah}{4}\right)Y_{CM_t} = \frac{1}{2}(a-h)\,a(a-h) - \frac{1}{3}h\frac{ah}{4} - \frac{1}{3}h\frac{ah}{4}$$ que é zero porque

se impõe que $Y_{CM_t} = 0 \Rightarrow Y_{CM_t} = 0$ e então $\frac{1}{2}a(a-h)^2 - \frac{2ah^2}{12} = 0;\quad (a-h)^2 = \frac{h^2}{3}$

$$a^2 - 2ah + h^2 = \frac{h^2}{3};\quad \frac{2}{3}h^2 - 2ah + a^2 = 0;\quad h^2 - 3ah + \frac{3}{2}a^2 = 0;\quad h = \frac{1}{2}\left(3a \pm \sqrt{9a^2 - 6a^2}\right) = \frac{a}{2}\left(3 \pm \sqrt{3}\right)$$

E a única solução possível será $h = a \cdot \frac{3-\sqrt{3}}{2}$

14.16 **14.16**



$$d_1+d_2=L$$

$$W=\frac{1}{2}I\omega^2$$

$$I=M_1 d_1^2+M_2 d_2^2$$

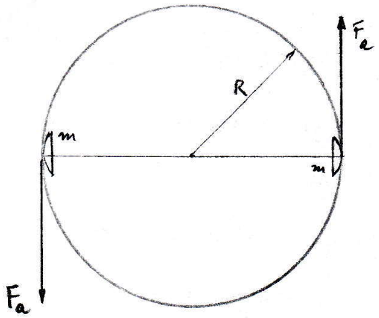
$$W=\frac{1}{2}\left(M_1(L-d_2)^2+M_2 d_2^2\right)\omega^2$$

. Qual é o valor de d_2 que minimiza a energia de rotação?

$$\frac{dW}{dd_2}=\frac{1}{2}\left[M_1 2(L-d_2)(-1)+2M_2 d_2\right]\omega^2=0;\ -M_1 2(L-d_2)+2M_2 d_2=0;\ -2M_1 L+2M_1 d_2+2M_2 d_2=0$$

$$(M_1+M_2)d_2=M_1 L \qquad d_2=\frac{M_1}{M_1+M_2}L$$

14.17 **14.17**

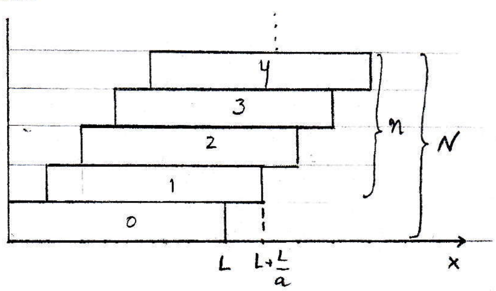


$$F_a=\mu m\frac{v^2}{R}=\mu m\omega^2 R$$

$$\tau_a=2F_a\cdot R=2\mu m\omega^2 R^2$$

$$P=\tau\cdot\omega=2\mu m\omega^3 R^2=\mu m\, 2\cdot(2\pi)^3 f^3 R^2=16\pi^3\mu m R^2 f^3$$

14.18 **14.18**



Para que a pilha não desmorone é necessário que a coordenada X_{cm} seja inferior a L, e assim:

$$P\left(\frac{L}{2}+\frac{L}{a}\right)+P\left(\frac{L}{2}+2\frac{L}{a}\right)+P\left(\frac{L}{2}+3\frac{L}{a}\right)+\dots+P\left(\frac{L}{2}+n\frac{L}{a}\right)=nPX_{cm}$$

$$n\frac{L}{2}+\frac{L}{a}(1+2+\dots+n)=nX_{cm}$$

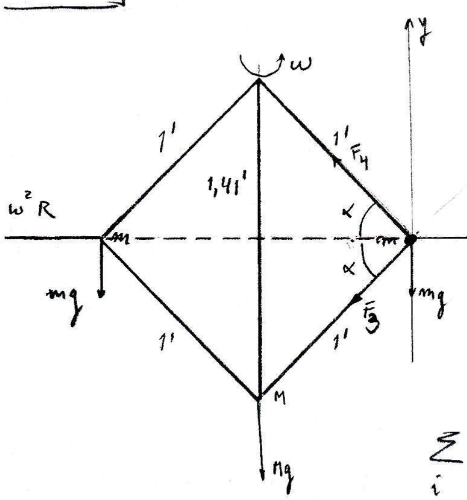
$$n\frac{L}{2}+\frac{L}{a}\frac{n+1}{2}n=nX_{cm}\ ;\ \frac{L}{2}+\frac{L}{a}\frac{n+1}{2}=X_{cm}$$

Se $X_{cm}>L$ a pilha cai e assim:

$$\frac{L}{2}+\frac{n+1}{2}\frac{L}{a}>L\ ;\ \frac{n+1}{2}\frac{L}{a}>\frac{L}{2}\ ;\ \frac{n+1}{2}>\frac{a}{2}\quad n+1>a$$

$n>a-1$ ou $\underline{\underline{N>a}}$

14.19



$$\operatorname{sen}\alpha = \frac{\frac{1,41}{2}}{1} = 0,705 \qquad \alpha \simeq 45^\circ$$

$$F_1 = m\omega^2 R\,\hat{i}$$

$$F_2 = -mg\,\hat{j}$$

$$F_3 = k_3\left(\cos\alpha\,\hat{i} + \operatorname{sen}\alpha\,\hat{j}\right) = \frac{k_3}{\sqrt{2}}\left(\hat{i}+\hat{j}\right)$$

$$F_4 = k_4\left(\cos\alpha\,\hat{i} - \operatorname{sen}\alpha\,\hat{j}\right) = \frac{k_4}{\sqrt{2}}\left(\hat{i}-\hat{j}\right)$$

$$\sum_i F_i = 0 \qquad m\omega^2 R + \frac{k_3}{\sqrt{2}} + \frac{k_4}{\sqrt{2}} = 0\;;\; k_3 + k_4 = -\sqrt{2}\,m\omega^2 R$$

$$-mg + \frac{k_3}{\sqrt{2}} - \frac{k_4}{\sqrt{2}} = 0\;;\; k_3 - k_4 = +\sqrt{2}\,mg$$

$$2k_3 = -\sqrt{2}\,m\omega^2 R + \sqrt{2}\,mg \qquad k_3 = \frac{-1}{\sqrt{2}}\left(m\omega^2 R - mg\right) \qquad k_4 = \frac{-1}{\sqrt{2}}\left(-m\omega^2 R - mg\right)$$

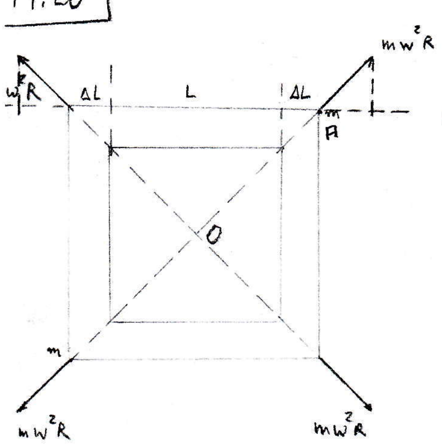
Mas $2F_{3_y} = -Mg\;;\; 2\frac{k_3}{\sqrt{2}} = -Mg\;;\; k_3 = -\frac{1}{\sqrt{2}}Mg = -\frac{1}{\sqrt{2}}m\left(\omega^2 R - g\right)$

$m\left(\omega^2 R - g\right) = Mg \qquad m = \dfrac{Mg}{\omega^2 R - g}$ em que $R = \sqrt{1 - \left(\frac{1,41}{2}\right)^2} = 0,709\ ft = 0,2156\ m$

e $\omega = \dfrac{120}{60} \times 2\pi = 4\pi\ rad\ s^{-1}$ e $M = 10\ lb = 4,536\ kg$ pelo que:

$$m = \frac{4,536 \cdot 9,8}{(4\pi)^2 \cdot 0,2156 - 9,8} = \frac{44,453}{24,24} = 1,8338\ kg = 4\ lb$$

14.20



$$m\omega^2 R\cos 45^\circ = k\,2\Delta L\;;\quad \Delta L = \frac{m\omega^2 R}{2\sqrt{2}\,k}$$

$$\overline{OA} = \left(L + 2\Delta L\right)\cos 45^\circ = R\;;\quad R = \frac{1}{\sqrt{2}}\left(L + 2\Delta L\right)$$

$$\Delta L = \frac{m\omega^2}{2\sqrt{2}\,k}\,\frac{1}{\sqrt{2}}\left(L + 2\Delta L\right);\quad \Delta L\left(1 - \frac{m\omega^2}{2k}\right) = \frac{m\omega^2}{4k}L$$

$$\Delta L = \frac{m\omega^2}{4k}\,\frac{2k}{2k - m\omega^2}L = \frac{m\omega^2}{2\left(2k - m\omega^2\right)}$$

e o alongamento é $2\Delta L = \dfrac{m\omega^2}{2k - m\omega^2}$

b) $2k > m\omega^2$ para alongamento ser finito, isto é, para ω: $2k - m\omega^2 = 0 \Rightarrow 2\Delta L = \infty$!