13.1 $\boxed{13.1}$

$E=\gamma m_e c^2 \quad m_e c^2 = 0{,}511\cdot 10^6\ eV \quad \gamma = \dfrac{10^9}{0{,}511\cdot 10^6} = 1957; \quad \dfrac{1}{\sqrt{1-\beta^2}} = 1957; \quad \beta = \dfrac{v}{c} = 0{,}999999869$

$\dfrac{c-\beta c}{c} = 1-\beta = 1{,}3\cdot 10^{-7}$ e então $\dfrac{1}{1-\beta} = 7{,}7\cdot 10^6 \simeq 8\cdot 10^6$

Isto é, a velocidade v difere de c de 1 parte em $8\cdot 10^6$ partes

---

3.2 $\boxed{13.2}$

$E=\sqrt{(pc)^2+(mc^2)^2} = T + mc^2 \;;\; (pc)^2+(mc^2)^2 = (T+mc^2)^2 \;;\; (pc)^2 + \cancel{(mc^2)^2} = T^2 + 2mc^2T + \cancel{(mc^2)^2}$

$pc = \sqrt{T^2+2mc^2T} = T\sqrt{1+2\dfrac{mc^2}{T}}$

Se $T = mc^2$ então $pc = T\sqrt{1+2} = \sqrt{3}\,T = \sqrt{3}\,mc^2$

$pc = \gamma \cancel{m}\mu \cancel{c} = \sqrt{3}\,\cancel{m}c^{\cancel{2}} \;;\; \gamma\mu = \sqrt{3}\,c \quad \dfrac{\frac{\mu}{c}}{\sqrt{1-\left(\frac{\mu}{c}\right)^2}} = \sqrt{3} \;;\; \dfrac{\beta}{\sqrt{1-\beta^2}} = \sqrt{3} \;;\; \beta^2 = 3-3\beta^2 \;;\; \beta = \dfrac{\sqrt{3}}{2}$

$\beta = \dfrac{\mu}{c} = \dfrac{\sqrt{3}}{2} = 0{,}866 \qquad \underline{\underline{\mu = 0{,}866\,c}}$

3.3 $\boxed{13.3}$

$E = \gamma m_p c^2 \quad \gamma = \dfrac{10^{10}\cdot 10^9}{938\cdot 10^6} = 1{,}066\cdot 10^{10} \;;\; \gamma = \dfrac{1}{\sqrt{1-\left(\frac{\mu}{c}\right)^2}} = 1{,}066\cdot 10^{10} \Rightarrow \mu \simeq c$

No referencial fixo o tempo necessário para percorrer 10^5 anos-luz à velocidade da luz é, claro, 10^5 anos.

No referencial do protão o tempo necessário é de $\dfrac{10^5\text{ anos}}{1{,}066\cdot 10^{10}} = \dfrac{10^5}{1{,}066\cdot 10^{10}}\cdot 365\cdot 24\cdot 60\ \text{min} = \underline{\underline{4{,}93\ \text{min}}}$

13.4 $\boxed{13.4}$

$\dfrac{mv^2}{R} = qvB \;;\; mv = qBR \qquad p = qBR$ com p em $Kg\times ms^{-1}$, q em Coulombs, B em Tesla, R em metro

Mas $1T = 10^4$ Gauss

$1\ Kg\ ms^{-1} = 1\ kg\ \dfrac{ms^{-2}m}{ms^{-1}} = 1\ \dfrac{\text{Joule}}{ms^{-1}} = \dfrac{1}{1{,}6\cdot 10^{-19}}\ \dfrac{eV}{ms^{-1}} = \dfrac{c}{1{,}6\cdot 10^{-13}}\ \dfrac{MeV}{c}$

$p = \dfrac{3\cdot 10^8}{1{,}6\cdot 10^{-13}}\ \dfrac{1{,}6\cdot 10^{-19}}{10^4}\ \dfrac{q}{e}\ B\,R = 3\cdot 10^{-2}\ Z\,B\,R$ com p em $\dfrac{MeV}{c}$, B em Gauss, R em metro

13.4 Contin. Contin 13.4

b) $T = 60 \cdot 10^9\, eV = 60 \cdot 10^3\, MeV$; $E_{total} = T + m_p c^2 \simeq T$

$m_p c^2 = 938 \cdot MeV$

logo $T >> m_p c^2$ e resulta $E_{total} = pc = 3 \cdot 10^{-2}\, Z \cdot B \cdot R = 3 \cdot 10^{-2} \cdot 0{,}3 \cdot R$

$$R = \frac{60 \cdot 10^3\ MeV}{3 \cdot 10^{-2} \cdot 1 \cdot 0{,}3} = 6{,}7 \cdot 10^3\ Km$$ com $Z = 1$

13,5 13,5

$E_K = T = \frac{1}{2} m_p v^2 = \frac{p^2}{2 m_p}$; $p = \sqrt{2 m_p T} = \sqrt{2 \cdot 938 \cdot 150 \left(\frac{MeV}{c}\right)^2} = \frac{1}{c}\, 530\ MeV$

$pc = 530\ MeV$ mas $p \left[\frac{MeV}{c}\right] = 3 \cdot 10^{-2} \cdot Z \cdot B[Gauss] \cdot R[metro]$ pelo que

$$R = \frac{530}{3 \cdot 10^{-2} \cdot 1 \cdot 10^4} = 1{,}8\ m$$

b) $\frac{m v^2}{R} = q v B$; $\frac{m v}{R} = q B$; $m \omega = q B$; $\omega = \frac{qB}{m}$; $2\pi f = \frac{qB}{m}$; $f = \frac{1}{2\pi} \frac{qB}{m}$

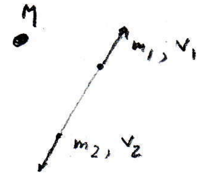
$$f = \frac{1}{2\pi} \frac{1{,}6 \cdot 10^{-19} \cdot 1}{1{,}67 \cdot 10^{-27}}$$ em que se fez B em Tesla $= 10^4$ Gauss e também $m_p = 1{,}67 \times 10^{-27}$ Kg. Com estes valores o valor da frequência (nº de voltas completas por unidade de tempo) é de $\underline{f = 15{,}25\ MHz}$

13.6

13.6

Antes:

Depois:



A conservação da energia permite escrever:

$$Mc^2 = E_{k_1} + E_{0_1} + E_{k_2} + E_{0_2}$$

$$E_1 = E_{k_1} + E_{0_1} = \sqrt{(p_1c)^2 + (m_1c^2)^2} \qquad E_2 = E_{k_2} + E_{0_2} = \sqrt{(p_2c)^2 + (m_2c^2)^2}$$

A conservação do momento permite escrever que $p_1 = p_2$

Fazendo, para facilitar, $p_1c = p_2c = a$, vem:

$$Mc^2 = \sqrt{a^2 + (m_1c^2)^2} + \sqrt{a^2 + (m_2c^2)^2} \; ; \; M^2c^4 = a^2 + (m_1c^2)^2 + a^2 + (m_2c^2)^2 + 2\sqrt{(a^2 + m_1^2c^4)(a^2 + m_2^2c^4)}$$

$$(M^2 - m_1^2 - m_2^2)c^4 - 2a^2 = 2\sqrt{a^4 + (m_1^2 + m_2^2)a^2c^4 + m_1^2m_2^2c^8}$$

$$\left[(M^2 - m_1^2 - m_2^2)c^4 - 2a^2\right]^2 = 4\left[a^4 + (m_1^2 + m_2^2)a^2c^4 + m_1^2m_2^2c^8\right]$$

$$(M^2 - m_1^2 - m_2^2)^2c^8 - 4(M^2 - m_1^2 - m_2^2)a^2c^4 + \cancel{4a^4} = \cancel{4a^4} + 4(m_1^2 + m_2^2)a^2c^4 + 4m_1^2m_2^2c^8$$

Dividindo tudo por c^4 e agrupando em c^4 vem:

$$\underbrace{\left[(M^2 - m_1^2 - m_2^2)^2 - 4m_1^2m_2^2\right]}_{B}c^4 = \left[\underbrace{4(M^2 - m_1^2 - m_2^2) + 4(m_1^2 + m_2^2)}_{4M^2}\right]a^2$$

$$B = (M^2 - m_1^2 - m_2^2)^2 - 4m_1^2m_2^2 = M^4 - 2M^2(m_1^2 + m_2^2) + \underbrace{(m_1^2 + m_2^2)^2 - 4m_1^2m_2^2}_{(m_1^2 - m_2^2)^2} =$$

$$= M^4 - 2M^2(m_1^2 + m_2^2) + (m_1^2 - m_2^2)^2$$

$$a^2 = (p_1c)^2 = (p_2c)^2 = \frac{M^4 - 2M^2(m_1^2 + m_2^2) + (m_1^2 - m_2^2)^2}{4M^2}c^4$$

$$E_1 = E_{k_1} + E_{0_1} = \sqrt{\frac{M^4 - 2M^2(m_1^2 + m_2^2) + (m_1^2 - m_2^2)^2}{4M^2}c^4 + (m_1c^2)^2} = \frac{\sqrt{M^4 - 2M^2(m_1^2 + m_2^2) + (m_1^2 - m_2^2)^2 + 4M^2m_1^2}}{2M}c^2$$

$$= \frac{\sqrt{M^4 - 2M^2m_1^2 - 2M^2m_2^2 + (m_1^2 - m_2^2)^2 + 4M^2m_1^2}}{2M}c^2 = \frac{\sqrt{M^4 + 2M^2m_1^2 - 2M^2m_2^2 + (m_1^2 - m_2^2)^2}}{2M}c^2 =$$

$$= \frac{\sqrt{M^4 + 2M^2(m_1^2 - m_2^2) + (m_1^2 - m_2^2)^2}}{2M}c^2 = \frac{\sqrt{(M^2 + m_1^2 - m_2^2)^2}}{2M}c^2 = \frac{M^2 + m_1^2 - m_2^2}{2M}c^2$$

Pelo que: $E_{k_1} = E_1 - E_{0_1} = \left[\frac{M^2 + m_1^2 - m_2^2}{2M} - m_1\right]c^2 = \frac{M^2 + m_1^2 - m_2^2 - 2Mm_1}{2M}c^2 = \frac{(M - m_1)^2 - m_2^2}{2M}c^2$

Para calcular E_{k_2} basta trocar os índices e vem: $E_{k_2} = \frac{(M - m_2)^2 - m_1^2}{2M}c^2$

13.7 **13.7**

Viu-se que: $E_{k_1} = \dfrac{(M-m_1)^2 - m_2^2}{2M}\, c^2$ em que, neste caso, $M = m_\pi = 273\, m_e$

$m_1 = m_\mu = 207\, m_e$

$m_2 = m_\nu = 0$

e assim $E_{k_1} = \dfrac{(273-207)^2\, m_e^2}{2\cdot 273\cdot m_e}\, c^2 = \dfrac{(273-207)^2}{2\cdot 273}\, m_e c^2 =$

$= 7{,}98\cdot 0{,}511\ \text{MeV} = 4{,}07\ \text{MeV}$

Por outro lado: $E_{k_2} = \dfrac{(M-m_2)^2 - m_1^2}{2M}\, c^2 = \dfrac{M^2 - m_1^2}{2M}\, c^2 = \dfrac{273^2 m_e^2 - 207^2 m_e^2}{2\cdot 273\cdot m_e}\, c^2 = \dfrac{273^2 - 207^2}{2\cdot 273}\, m_e c^2 =$

$= 58\cdot 0{,}511\cdot \text{MeV} = 29{,}6\ \text{MeV}$

$E_\mu = E_K + E_0 = 4{,}07 + 207\cdot m_e c^2 = 109{,}8\ \text{MeV}$

$E_\mu = \sqrt{(pc)^2 + E_{0_\mu}^2} \Rightarrow \sqrt{E_\mu^2 - E_{0_\mu}^2} = pc = \sqrt{109{,}8^2 - (207\cdot m_e c^2)^2} = 29{,}6\ \text{MeV}$

No caso do neutrino: $E_{total_\nu} = E_{k_\nu} = 29{,}6\ \text{MeV} = p_\nu c =$

Finalmente temos: $p_\nu = p_\mu = 29{,}6\ \dfrac{\text{MeV}}{c}$ como se poderia concluir logo após se conhecer $E_{k_2} = 29{,}6\ \text{MeV}$

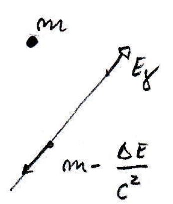
13.8 **13.8**

A conservação de energia permite escrever:

$$mc^2 = E_\gamma + E_{K_{at}} + \underbrace{mc^2 - \Delta E}_{\text{en. em repouso do átomo depois de emitir um fotão}} \; ; \quad E_\gamma = \Delta E - E_{K_{at}}$$

Antes:

Depois:



$$E_{K_{at}} = E_{total} - E_{repouso} = \sqrt{(p_{at}c)^2 + \left[\left(m - \frac{\Delta E}{c^2}\right)c^2\right]^2} - \left(m - \frac{\Delta E}{c^2}\right)c^2$$

$$E_\gamma = \cancel{\Delta E} - \sqrt{(p_{at}c)^2 + (mc^2 - \Delta E)^2} + mc^2 - \cancel{\Delta E} = mc^2 - \sqrt{(p_{at}c)^2 + (mc^2-\Delta E)^2}$$

A conservação do momento: $p_{at}c = p_\gamma c = E_\gamma$ pelo que $E_\gamma = mc^2 - \sqrt{E_\gamma^2 + (mc^2 - \Delta E)^2}$ ou

$$mc^2 - E_\gamma = \sqrt{E_\gamma^2 + (mc^2-\Delta E)^2} \; ; \quad (mc^2 - E_\gamma)^2 = E_\gamma^2 + (mc^2 - \Delta E)^2 \; ; \quad (mc^2)^2 - 2(mc^2)E_\gamma + \cancel{E_\gamma^2} =$$
$$= \cancel{E_\gamma^2} + (mc^2 - \Delta E)^2$$

$$\cancel{(mc^2)^2} - 2(mc^2)E_\gamma = \cancel{(mc^2)^2} - 2mc^2\Delta E + (\Delta E)^2$$

$$E_\gamma = \Delta E - \frac{(\Delta E)^2}{2mc^2} = \Delta E\left(1 - \frac{\Delta E}{2mc^2}\right)$$

13.9 13.9

$m_1 = m_2 = m$

$pc = \gamma m v c$

$E = \gamma m c^2$ e dividindo dá:

$$\frac{pc}{E} = \frac{v}{c}$$

$$\frac{p_1 c}{E_1} = \frac{4}{5} \qquad p_1 c = \frac{4}{5} E_1$$

$$E_1 = \sqrt{\left(\frac{4}{5}E_1\right)^2 + (m_1 c^2)^2} \; ; \; E_1^2 = \left(\frac{4}{5}\right)^2 E_1^2 + (m_1 c^2)^2 \; ; \; \left[1 - \left(\frac{4}{5}\right)^2\right] E_1^2 = (m_1 c^2)^2 \; ; \; \left[1 - \frac{16}{25}\right] E_1^2 = (m_1 c^2)^2$$

$$E_1^2 = \frac{25}{9}(m_1 c^2)^2 \; ; \; E_1 = \frac{5}{3} m_1 c^2 \qquad p_M c = p_1 c = \frac{4}{5} E_1 = \frac{4}{5}\,\frac{5}{3} m_1 c^2 = \frac{4}{3} m_1 c^2 = \frac{4}{3} m c^2$$

$E_2 = m_2 c^2$ porque a en. cinética de m_2 é zero

$$E_1 + E_2 = E_{\text{total depois do choque}} = \sqrt{(p_M c)^2 + (M c^2)^2} = E_{\text{total de } M} = \frac{5}{3} m_1 c^2 + m_2 c^2 = \left(\frac{5}{3} + 1\right) m c^2$$

$p_M c = p_1 c$ pelo cons. do momento e então $\frac{V_M}{c} = \frac{p_M c}{E_{\text{total de } M}} = \frac{p_1 c}{\left(\frac{5}{3}+1\right) m c^2} = \frac{\frac{4}{5} E_1}{\frac{8}{3} m c^2} =$

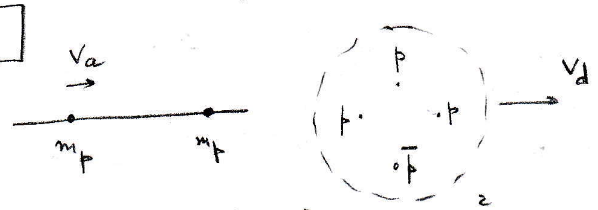
a) $$\frac{V_M}{c} = \frac{\frac{4}{5}\,\frac{5}{3} m c^2}{\frac{8}{3} m c^2} = \frac{4}{8} = \frac{1}{2} \quad \text{donde } \boxed{V_M = \frac{c}{2}}$$

b) $$\sqrt{(p_M c)^2 + (M c^2)^2} = \sqrt{\left(\frac{4}{3} m c^2\right)^2 + (M c^2)^2} = \left(\frac{5}{3} + 1\right) m c^2 \; ; \; \left(\frac{4}{3}\right)^2 (m c^2)^2 + (M c^2)^2 = \left(\frac{8}{3}\right)^2 (m c^2)^2$$

$$M^2 = \left[\left(\frac{8}{3}\right)^2 - \left(\frac{4}{3}\right)^2\right] m^2 \; ; \; M^2 = \frac{64-16}{9} m^2 = \frac{48}{9} m^2 \; ; \; M = \frac{\sqrt{48}}{3} m \; ; \; \boxed{M = \frac{4\sqrt{3}}{3} m = \frac{4}{\sqrt{3}} m}$$

// //

13.10 13.10



Consev. do momento:

$$p_a c = p_M c$$

Consev. da energia

$$E_a = E_d$$

Relação do problema anterior: $\frac{p_a c}{E_1} = \frac{V_a}{c}$

antes: $E_a = E_{K_1} + m c^2 + m c^2 = E_{K_1} + 2 m c^2$

depois: $E_d = \sqrt{(p_M c)^2 + (4 m c^2)^2}$

$$E_{K_1} = \gamma_1 m c^2 - m c^2 = (\gamma_1 - 1) m c^2$$

então: $(\gamma_1 - 1) m c^2 + 2 m c^2 = \sqrt{(p_a c)^2 + (4 m c^2)^2}$; $(\gamma+1) m c^2 = \sqrt{\left(E_1 \frac{V_a}{c}\right)^2 + 16 (m c^2)^2}$ em que

$E_1 = E_{K_1} + m c^2 = (\gamma_1 - 1) m c^2 + m c^2 = \gamma_1 m c^2$ e então $(\gamma+1)^2 (m c^2)^2 = \left(\gamma_1 m c^2 \frac{V_a}{c}\right)^2 + 16 (m c^2)^2$

$(\gamma+1)^2 = \gamma_1^2 \left(\frac{V_a}{c}\right)^2 + 16$; $\gamma^2 + 2\gamma + 1 = \gamma_1^2 \left(\frac{V_a}{c}\right)^2 + 16$; $\gamma^2\left[1 - \left(\frac{V_a}{c}\right)^2\right] + 2\gamma = 15$. Mas $\gamma^2\left[1 - \left(\frac{V_a}{c}\right)^2\right] = 1$

e vem $1 + 2\gamma = 15$; $\gamma = 7$ pelo que $E_{K_1} = (\gamma_1 - 1) m c^2 = (7-1) m c^2 = 6 m_p c^2 = 5{,}6$ GeV

13.11 $\qquad$ 13.11

$$e^- + e^- \longrightarrow e^- + e^- + p^- + p^+$$

antes: $E_a = E_{K_1} + 2 m_e c^2$

depois: $E_d = \sqrt{(p_M c)^2 + (M c^2)^2}$ $\qquad p_a c = \frac{v_a}{c} E_1$ e $p_a c = p_M c$ (cons. do momento)

$M = 2(m_e + m_p) \qquad E_1 = \gamma_1 m_e c^2$

$E_{K_1} = (\gamma_1 - 1) m_e c^2$

$$(\gamma_1 - 1) m_e c^2 + 2 m_e c^2 = \sqrt{\left(\gamma_1 m_e c^2\right)^2 \left(\frac{v_a}{c}\right)^2 + \left(2(m_e + m_p) c^2\right)^2}$$

$$(\gamma_1 + 1)^2 (m_e c^2)^2 = \gamma_1^2 \left(\frac{v_a}{c}\right)^2 (m_e c^2)^2 + 4\left((m_e + m_p) c^2\right)^2$$

$$(\gamma_1 + 1)^2 = \gamma_1^2 \left(\frac{v_a}{c}\right)^2 + \frac{4\left((m_e + m_p) c^2\right)^2}{(m_e c^2)^2} \ ; \ \gamma_1^2 + 2\gamma_1 + 1 = \gamma_1^2 \left(\frac{v_a}{c}\right)^2 + \frac{4\left((m_e + m_p) c^2\right)^2}{(m_e c^2)^2}$$

$$\underbrace{\gamma_1^2 \left[1 - \left(\frac{v_a}{c}\right)^2\right]}_{=1} + 2\gamma_1 + 1 = 4\left(\frac{m_e + m_p}{m_e}\right)^2 \ ; \ 2\gamma_1 + 2 = 4\left(\frac{m_e + m_p}{m_e}\right)^2 ; \ \gamma_1 + 1 = 2\left(\frac{\cancel{0{,}5} + 10^3}{0{,}5}\right)^2$$

$$\gamma_1 = 2\,\frac{10^6}{\frac{1}{4}} \cong 8 \cdot 10^6$$

Por fim: $E_{K_1} = (\gamma_1 - 1) m_e c^2 \cong 8 \cdot 10^6 \cdot 0{,}5 \text{ MeV} = \underline{4000 \text{ GeV}}$

13.12 $\qquad$ 13.12

$$\gamma + p \longrightarrow p + p + \bar{p}$$

$E_a = E_\gamma + m_p c^2 \qquad p_a c = E_\gamma = p_M c \qquad M = 3 m_p$

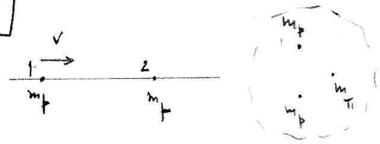
$E_d = \sqrt{(p_M c)^2 + (M c^2)^2} = \sqrt{E_\gamma^2 + (3 m_p c^2)^2}$ $\qquad$ Mas $E_a = E_d$ pelo que vem, igualando e elevando ao quadrado:

$$(E_\gamma + m_p c^2)^2 = E_\gamma^2 + 9 (m_p c^2)^2$$

$$\cancel{E_\gamma^2} + 2 E_\gamma m_p c^2 + (m_p c^2)^2 = \cancel{E_\gamma^2} + 9 (m_p c^2)^2 \ ; \ 2 E_\gamma m_p c^2 = 8 (m_p c^2)^2$$

$$E_\gamma = 4 m_p c^2 = 4 \cdot 938 \text{ MeV} = 3{,}8 \cdot 10^3 \text{ MeV} = 3{,}8 \text{ GeV}$$

13.13 | 13.13



a) 1) <u>Cálculo relativista.</u> Energia total antes = Energia total depois
Momento antes = Momento depois

$$E_{\text{total antes}} = E_{\text{total de }1} + m_p c^2 = \gamma m_p c^2 + m_p c^2 = (\gamma+1) m_p c^2$$

Por outro lado o momento total antes: $pc = \gamma m v c$
a energia total de 1 antes: $E_{t_1} = \gamma m c^2$ e dividindo: $\dfrac{pc}{E_{t_1}} = \dfrac{v}{c}$ pelo que

$$pc = \frac{v}{c} E_{t_1} = \frac{v}{c}\gamma m_p c^2 = p_M c$$ que é o momento total depois.

Energia total depois: $E_{t_d} = \sqrt{(p_M c)^2 + (M c^2)^2} = E_{t_a} = (\gamma+1) m_p c^2$ e, substituindo $p_M c$, vem:

$$\sqrt{\left(\frac{v}{c}\gamma m_p c^2\right)^2 + \left((2m_p + m_\pi)c^2\right)^2} = (\gamma+1) m_p c^2$$ e vamos extrair o valor de γ

$$(\gamma+1)^2 m_p^2 c^4 = \left(\frac{v}{c}\gamma m_p c^2\right)^2 + \left((2m_p + m_\pi)c^2\right)^2$$ e dividindo por c^4 vem:

$$(\gamma+1)^2 m_p^2 = \left(\frac{v}{c}\gamma\right)^2 m_p^2 + (2m_p + m_\pi)^2$$

$$(\gamma^2 + 2\gamma + 1) m_p^2 - \gamma^2\left(\frac{v}{c}\right)^2 m_p^2 = 4m_p^2 + 4m_p m_\pi + m_\pi^2$$

$$\underbrace{\gamma^2\left[1 - \left(\frac{v}{c}\right)^2\right]}_{=1} m_p^2 + 2\gamma m_p^2 + m_p^2 = 4m_p^2 + 4m_p m_\pi + m_\pi^2$$

$$2\gamma m_p^2 = 2m_p^2 + 4m_p m_\pi + m_\pi^2 \; ; \; \gamma = 1 + 2\frac{m_\pi}{m_p} + \frac{m_\pi^2}{2m_p^2}$$

Assim: a energia cinética antes $= E_{K_1} = E_{\text{total de }1} - m_p c^2 = (\gamma - 1) m_p c^2 = \left(2\dfrac{m_\pi}{m_p} + \dfrac{m_\pi^2}{2m_p^2}\right) m_p c^2$

ou $E_{K_1} = \left(2m_\pi + \dfrac{m_\pi^2}{2m_p}\right)c^2 = 2m_\pi c^2 + \dfrac{m_\pi^2}{2m_p}c^2 = m_\pi\left(2 + \dfrac{m_\pi}{2m_p}\right)c^2$ e se $m_\pi \ll m_p$ $2 \gg \dfrac{m_\pi}{2m_p}$ e

vem $E_{K_1} \simeq 2 m_\pi c^2$

2) <u>Cálculo não relativista</u> $\quad m_p \cdot v = (2m_p + m_\pi)V \quad V = \dfrac{m_p}{2m_p + m_\pi} v$

$E_{K_1} + 2m_p c^2 = 2\cdot\frac{1}{2} m_p V^2 + \frac{1}{2} m_\pi V^2 + (2m_p + m_\pi)c^2$ ou, substituindo, vem:

$E_{K_1} = -2m_p c^2 + \frac{1}{2}(2m_p + m_\pi)\cdot\dfrac{m_p^2}{(2m_p + m_\pi)^2} v^2 + (2m_p + m_\pi)c^2$ mas $E_{K_1} = \frac{1}{2} m_p v^2$ $\; v^2 = \frac{2}{m_p} E_{K_1}$

$$E_{K_1} = \frac{1}{\cancel{2}}\frac{m_p^{\cancel{2}}}{2m_p + m_\pi}\frac{\cancel{2}}{\cancel{m_p}} E_{K_1} + m_\pi c^2 = \frac{m_p}{2m_p + m_\pi} E_{K_1} + m_\pi c^2$$

13.13 Contin. Contin. 13.13

$$\left(1-\frac{m_p}{2m_p+m_\pi}\right)E_{k_1}=m_\pi c^2 \ ; \quad \frac{2m_p+m_\pi-m_p}{2m_p+m_\pi}E_{k_1}=m_\pi c^2 \ ; \quad E_{k_1}=\frac{2m_p+\cancel{m_\pi}}{m_p+\cancel{m_\pi}}m_\pi c^2$$ e se $m_\pi << m_p$

$$E_{k_1}\simeq 2m_\pi c^2$$

b) Qual a energia cinética do mesão π?

Cálculo não-relativista: $p_a=p_d \qquad m_p V_a=(2m_p+m_\pi)V_d$

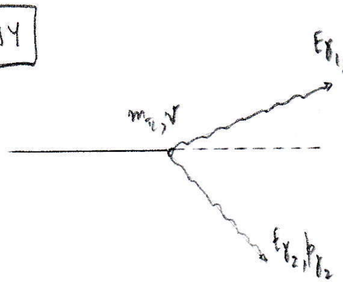
$E_{k_\pi}=\frac{1}{2}m_\pi V_d^2=\frac{1}{2}m_\pi\frac{m_p^2}{(2m_p+m_\pi)^2}V_a^2$. Na alínea anterior viu-se que $E_{k_a}=2m_\pi c^2$

e então: $2m_\pi c^2=\frac{1}{2}m_p V_a^2$ pelo que $V_a^2=4\frac{m_\pi}{m_p}c^2$ e substituindo vem:

$E_{k_\pi}=\frac{1}{2}\frac{m_\pi m_p^{\cancel{2}}}{(2m_p+m_\pi)^2}4\frac{m_\pi}{\cancel{m_p}}c^2=2\frac{m_\pi^2 m_p}{(2m_p+\cancel{m_\pi})^2}c^2$ e se $m_\pi << m_p$ dá: $E_{k_\pi}=2\frac{m_\pi^2 m_p}{4m_p^2}c^2$

pelo que $E_{k_\pi}=\frac{m_\pi^2}{2m_p}c^2$

13.14 13.14



energia total antes: $E_a=\sqrt{(p_\pi c)^2+(m_\pi c^2)^2}=\gamma m_\pi c^2$

" " depois: $E_{\gamma_1}+E_{\gamma_2}$ e $E_a=E_{\gamma_1}+E_{\gamma_2}$ (1)

Conservação do momento:

$p_{\gamma_1}\cos\theta_1+p_{\gamma_2}\cos\theta_2=p_\pi$; $\frac{E_{\gamma_1}}{c}\cos\theta_1+\frac{E_{\gamma_2}}{c}\cos\theta_2=\gamma m_\pi v$ (2)

(3)

$p_{\gamma_1}\operatorname{sen}\theta_1+p_{\gamma_2}\operatorname{sen}\theta_2=0$ em que $\gamma=\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

de (1), (2) e (3) vem:
$$\begin{cases} E_{\gamma_1}+E_{\gamma_2}=\gamma mc^2 \\ E_{\gamma_1}\cos\theta_1+E_{\gamma_2}\cos\theta_2=\gamma m_\pi vc \\ E_{\gamma_1}\operatorname{sen}\theta_1+E_{\gamma_2}\operatorname{sen}\theta_2=0 \end{cases}$$
, que é um sistema de 3 equações a 4 incógnitas: $E_{\gamma_1}, E_{\gamma_2}, \theta_1, \theta_2$

Vamos explicitar E_{γ_1} em função de θ_1. Podemos escrever:

$E_{\gamma_2} = \gamma mc^2 - E_{\gamma_1}$ e então $E_{\gamma_1} \operatorname{sen}\theta_1 + (\gamma mc^2 - E_{\gamma_1}) \operatorname{sen}\theta_2 = 0$ ou $E_{\gamma_1}(\operatorname{sen}\theta_1 - \operatorname{sen}\theta_2) + \gamma mc^2 \operatorname{sen}\theta_2 = 0$

donde $\operatorname{sen}\theta_2 = \dfrac{E_{\gamma_1}\operatorname{sen}\theta_1}{E_{\gamma_1} - \gamma mc^2}$.

Por outro lado temos: $E_{\gamma_1}\cos\theta_1 + (\gamma mc^2 - E_{\gamma_1})\sqrt{1 - \left(\dfrac{E_{\gamma_1}\operatorname{sen}\theta_1}{E_{\gamma_1} - \gamma mc^2}\right)^2} = \gamma m v c$ que

simplificando $E_{\gamma_1}\cos\theta_1 - \sqrt{(E_{\gamma_1} - \gamma mc^2)^2 - (E_{\gamma_1}\operatorname{sen}\theta_1)^2} = \gamma m v c$ ou ainda:

$$E_{\gamma_1}\cos\theta_1 - \sqrt{E_{\gamma_1}^2 - 2E_1\gamma mc^2 + (\gamma mc^2)^2 - E_{\gamma_1}^2\operatorname{sen}^2\theta_1} = \gamma m v c$$

$$E_{\gamma_1}\cos\theta_1 - \gamma m v c = \sqrt{E_{\gamma_1}^2\cos^2\theta_1 - 2E_{\gamma_1}\gamma mc^2 + (\gamma mc^2)^2}$$

$$\cancel{E_{\gamma_1}^2\cos^2\theta_1} - 2E_{\gamma_1}\cos\theta_1\,\gamma m v c + (\gamma m v c)^2 = \cancel{E_{\gamma_1}^2\cos^2\theta_1} - 2E_{\gamma_1}\gamma mc^2 + (\gamma mc^2)^2$$

$$2E_{\gamma_1}(\gamma mc^2 - \gamma m v c\cos\theta_1) = (\gamma mc^2)^2 - (\gamma m v c)^2$$

$$E_{\gamma_1} = \frac{(\gamma mc^2)^2 - (\gamma m v c)^2}{2(\gamma mc^2 - \gamma m v c\cos\theta_1)} = \frac{\gamma^2 m^2 (c^4 - v^2c^2)}{2\gamma m(c^2 - vc\cos\theta_1)} = \frac{\gamma m c^4\left(1 - \frac{v^2}{c^2}\right)}{2c^2\left(1 - \frac{v}{c}\cos\theta_1\right)} = \frac{\gamma mc^2\left(1 - \frac{v^2}{c^2}\right)}{2\left(1 - \frac{v}{c}\cos\theta_1\right)}$$

e então finalmente $E_{\gamma_1} = \dfrac{mc^2}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}\left(1 - \dfrac{v^2}{c^2}\right)\cdot\dfrac{1}{2\left(1 - \frac{v}{c}\cos\theta_1\right)} = \dfrac{mc^2\sqrt{1 - \left(\frac{v}{c}\right)^2}}{2\left(1 - \frac{v}{c}\cos\theta\right)}$

$\therefore$

e fazendo $\beta = \dfrac{v}{c}$ vem $E_{\gamma_1} = \dfrac{mc^2\sqrt{1-\beta^2}}{2(1 - \beta\cos\theta_1)}$

Para E_{γ_2} virá: $E_{\gamma_2} = \gamma mc^2 - E_{\gamma_1} = mc^2\left[\gamma - \dfrac{\sqrt{1-\beta^2}}{2(1-\beta\cos\theta_1)}\right]$ e ~~tambem~~:

$$\operatorname{sen}\theta_2 = \frac{E_{\gamma_1}\operatorname{sen}\theta_1}{E_{\gamma_1} - \gamma mc^2} = \frac{\operatorname{sen}\theta_1}{1 - \dfrac{\gamma mc^2}{E_{\gamma_1}}} = \frac{\operatorname{sen}\theta_1}{1 - \gamma \cancel{mc^2}\dfrac{2(1-\beta\cos\theta_1)}{\cancel{mc^2}\sqrt{1-\beta^2}}} = \frac{\operatorname{sen}\theta_1}{1 - \dfrac{1}{\sqrt{1-\beta^2}}\cdot\dfrac{2(1-\beta\cos\theta_1)}{\sqrt{1-\beta^2}}}$$ ou

$$\operatorname{sen}\theta_2 = \frac{\operatorname{sen}\theta_1}{1 - \dfrac{2(1-\beta\cos\theta_1)}{1-\beta^2}}$$

Isto é, para cada ângulo de desvio θ_1 do fotão, este tem uma energia E_{γ_1}, o outro fotão gama terá uma energia E_{γ_2} e um desvio θ_2.

b) Se $\theta_1 = 0^\circ$ então $E_{\gamma_{1_{0^\circ}}} = \dfrac{mc^2\sqrt{1-\beta^2}}{2(1-\beta)} = mc^2\dfrac{\sqrt{(1+\beta)(1-\beta)}}{2(1-\beta)} = \dfrac{mc^2}{2}\sqrt{\dfrac{1+\beta}{1-\beta}}$

Se $\theta_1 = 90^\circ$ então $E_{\gamma_{1_{90^\circ}}} = \dfrac{mc^2}{2}\sqrt{1-\beta^2}$

Dividindo, vem: $\dfrac{E_{\gamma_{1_{0^\circ}}}}{E_{\gamma_{1_{90^\circ}}}} = \dfrac{\dfrac{\sqrt{1+\beta}}{\sqrt{1-\beta}}}{\sqrt{1-\beta^2}} = \dfrac{\sqrt{1+\beta}}{\sqrt{1-\beta}\sqrt{1+\beta}\sqrt{1-\beta}} = \dfrac{1}{1-\beta^2} > 1$ pois $0<\beta<1$

e então $E_{\gamma_{1_{0^\circ}}} > E_{\gamma_{1_{90^\circ}}}$ e $E_{\gamma_{1_{0^\circ}}}$ é máximo e $E_{\gamma_{1_{90^\circ}}}$ é mínimo

No caso de $v=0$, ou seja $\beta = 0$ vem: $E_{\gamma_1} = \dfrac{mc^2}{2}$; $E_{\gamma_2} = mc^2\left[1-\dfrac{1}{2}\right] = \dfrac{mc^2}{2}$ e

$\operatorname{sen}\theta_2 = \dfrac{\operatorname{sen}\theta_1}{1-2} = -\operatorname{sen}\theta_1$ o que implica $\theta_2 = -\theta_1$