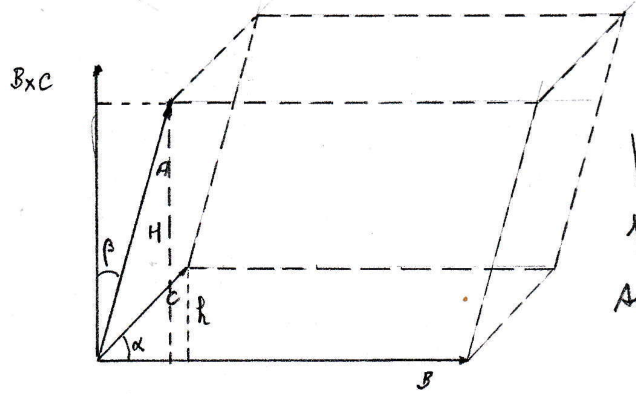
16.1 16.1



$|B \times C| = |B| \cdot |C| \operatorname{sen}\alpha = |B| \cdot h =$ Área da base

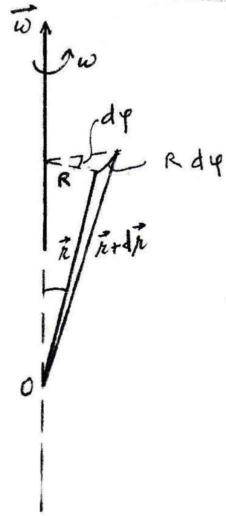
$|A \cdot (B \times C)| = |A| \cdot |B \times C| \cdot \cos\beta =$ Área da base • Altura H

pois $|A| \cos\beta = H$

Assim: o volume é dado por $|A \cdot (B \times C)|$

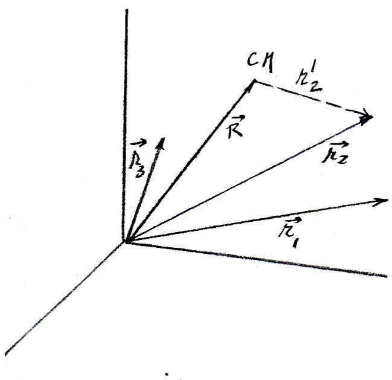
16.2 16.2

16.3



$$|\vec{\omega} \times \vec{r}| = \frac{d\varphi}{dt}\, r \,\mathrm{sen}\,\theta = \frac{d\varphi}{dt} R = \frac{ds}{dt}$$

16.4



$$\vec{r}_i = \vec{R} + \vec{r}'_i \qquad v_i = \frac{d\vec{R}}{dt} + \frac{dr'_i}{dt}$$

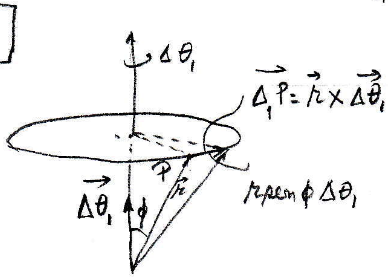
$$\sum_i m_i \vec{r}_i = \vec{R} \sum_i m_i \quad \text{e} \quad \sum_i m_i r'_i = 0 \quad \text{e} \quad \sum m_i \frac{dr'_i}{dt} = 0$$

$$L = \sum_i m_i \vec{r}_i \times \vec{v}_i = \sum_i m_i (\vec{R} + \vec{r}'_i) \times \left(\frac{d\vec{R}}{dt} + \frac{d\vec{r}'_i}{dt} \right) =$$

$$= \sum_i m_i \vec{R} \times \frac{d\vec{R}}{dt} + \underbrace{\sum_i m_i r'_i \times \frac{d\vec{R}}{dt}}_{=0} + \underbrace{\sum_i m_i \vec{R} \times \frac{dr'_i}{dt}}_{=0} + \underbrace{\sum_i m_i \vec{r}'_i \times \frac{dr'_i}{dt}}_{L_{CM}} =$$

$$= M R_{CM} \times V_{CM} + L_{CM}$$

16.5



$$\overrightarrow{\Delta_1 P} = \vec{r} \times \overrightarrow{\Delta\theta_1} \qquad \overrightarrow{\Delta_2 P} = \vec{r} \times \overrightarrow{\Delta\theta_2}$$

$$\overrightarrow{\Delta P} = \overrightarrow{\Delta_1 P} + \overrightarrow{\Delta_2 P} = \vec{r} \times \left(\overrightarrow{\Delta\theta_1} + \overrightarrow{\Delta\theta_2} \right)$$

$$\frac{\overrightarrow{\Delta P}}{\Delta t} = \vec{r} \times \left(\frac{\overrightarrow{\Delta\theta_1}}{\Delta t} + \frac{\overrightarrow{\Delta\theta_2}}{\Delta t} \right) = \vec{r} \times (\vec{\omega}_1 + \vec{\omega}_2) = \vec{r} \times \vec{\omega} \quad \text{com } \vec{\omega} = \vec{\omega}_1 + \vec{\omega}_2$$

16.6 $\vec{A} = (10, -5, 3)$ $\quad \vec{B} = (3, -4, 7)$ $\quad \vec{C} = (-5, -6, 3)$

$$V = |\vec{A} \cdot (\vec{B} \times \vec{C})| \qquad \vec{B} \times \vec{C} = \begin{vmatrix} i & j & k \\ 3 & -4 & 7 \\ -5 & -6 & 3 \end{vmatrix} = \left[(-4)\cdot 3 - (-6)\cdot 7 \right] i - \left[3\cdot 3 - (-5)\cdot 7 \right] j + \left[3\cdot(-6) - (-5)\cdot(-4) \right] =$$

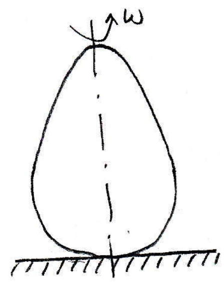
$$= (-12+42)\hat{i} - (9+35)\hat{j} + (-18-20)\hat{k} = 30\hat{i} - 44\hat{j} - 38\hat{k}$$

$$\vec{A} \cdot (\vec{B} \times \vec{C}) = (10i - 5j + 3k)(30i - 44j - 38k) = 300 + 220 - 114 = 406 \ m^3$$

16.7 | 16.7

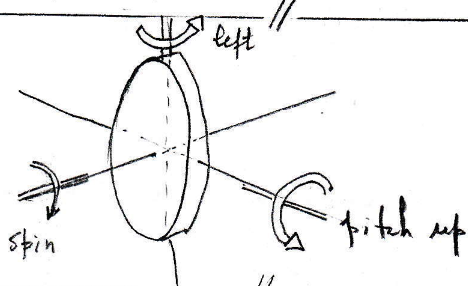
Se o gelo dos pólos derrete o momento de inércia da Terra em relação ao eixo aumenta pois a massa que estava concentrada nos pólos espalha-se pelos oceanos. Como o momento angular $L = I\omega$ é constante, pois não há torques externos aplicados, então se I aumenta ω diminui. Como $T = \frac{2\pi}{\omega}$ então o período de rotação da Terra T aumenta.

16.8 | 16.8

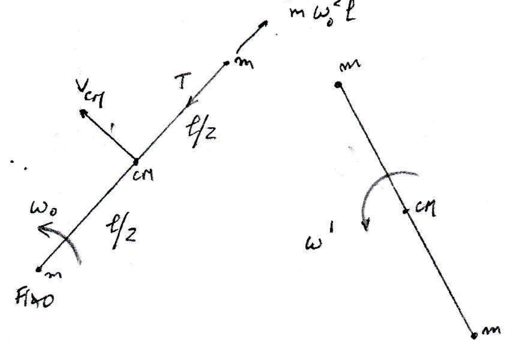


O ovo cozido pode rodar em torno do eixo indicado na figura qual pião. O ovo não cozido não o faz.

16.9 | 16.9



16.10 | 16.10



ANTES: DEPOIS:

$V_{cm} = \frac{\ell\omega_0}{2}$ $\qquad$ $V_{cm} = \frac{\ell\omega_0}{2}$

$E_{K_{rot}} = \frac{1}{2} m \ell^2 \omega_0^2$ $\qquad$ $E_{K_{total}} = E_{K_{cm}} + E_{K_{rot}} = \frac{1}{2}(2m)\left(\frac{\ell\omega_0}{2}\right)^2 + \frac{1}{2}\left[m\left(\frac{\ell}{2}\right)^2 + m\left(\frac{\ell}{2}\right)^2\right]\omega'^2 = \frac{1}{4}m\ell^2\omega_0^2 + \frac{1}{4}m\ell^2\omega'^2$

Conservação da energia: $E_{K_{antes}} = E_{K_{depois}}$ ou seja $\frac{1}{2}m\ell^2\omega_0^2 = \frac{1}{4}m\ell^2\omega_0^2 + \frac{1}{4}m\ell^2\omega'^2$; $\frac{1}{2}\omega_0^2 = \frac{1}{4}\omega_0^2 + \frac{1}{4}\omega'^2$

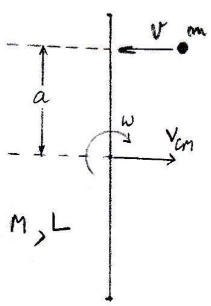
$\frac{1}{4}\omega_0^2 = \frac{1}{4}\omega'^2$ pelo que $\omega' = \omega_0$

Tensão no fio antes: $T_a = m\omega_0^2\ell$

Tensão no fio depois: $T_d = m\omega_0^2\frac{\ell}{2} = \frac{1}{2}T_a$. Então a tensão no fio é maior antes de ser largado o conjunto, pelo que poderá quebrar antes.

16.11 16.11

16.12 16.12



	ANTES DO IMPACTO	DEPOIS DO IMPACTO
MOM. ANGULAR	$I_{CM}\omega - mva$	$mv'a$
ENERGIA CIN.	$\frac{1}{2}MV_{CM}^2 + \frac{1}{2}I_{CM}\omega^2 + \frac{1}{2}mv^2$	$\frac{1}{2}mv'^2$
MOM. LINEAR	$MV_{CM} - mv$	mv'

com $I_{CM} = \frac{1}{12}ML^2$

Depois do impacto a massa m retrocede com velocidade v' e a barra fica imóvel. As leis de conservação permitem escrever:

(1) $I_{CM}\omega - mva = mv'a$

(2) $\frac{1}{2}mv'^2 = \frac{1}{2}MV_{CM}^2 + \frac{1}{2}I_{CM}\omega^2 + \frac{1}{2}mv^2$

(3) $MV_{CM} - mv = mv'$

De (3) vem $v' = \frac{M}{m}V_{CM} - v$; que substituido em (1) dá: $I_{CM}\omega - mva = m\left(\frac{M}{m}V_{CM} - v\right)a$;

$I_{CM}\omega - \cancel{mv}a = MV_{CM}a - \cancel{mva}$; $I_{CM}\omega = MV_{CM}a$ donde $a = \frac{I_{CM}\omega}{MV_{CM}} = \frac{\omega}{MV_{CM}}\frac{1}{12}ML^2 = \frac{L^2\omega}{12V_{CM}}$

Por outro lado substituindo v' em (2) vem:

$m\left(\frac{M}{m}V_{CM} - v\right)^2 = MV_{CM}^2 + I_{CM}\omega^2 + \frac{1}{2}mv^2$; $\left(\frac{M}{m}V_{CM}\right)^2 - 2\frac{M}{m}V_{CM}v + \cancel{v^2} = \frac{M}{m}V_{CM}^2 + \frac{I_{CM}}{m}\omega^2 + \cancel{v^2}$

$\left(\frac{M}{m}V_{CM}\right)^2 - \frac{M}{m}V_{CM}^2 - \frac{I_{CM}}{m}\omega^2 = 2\frac{M}{m}V_{CM}v$; $\frac{M^2}{m^2}V_{CM} - \frac{M}{m}V_{CM} - \frac{I_{CM}}{mV_{CM}}\omega^2 = 2\frac{M}{m}v$

$v = \frac{1}{2}\frac{M}{m}V_{CM} - \frac{1}{2}V_{CM} - \frac{1}{2}\frac{m}{M}\frac{1}{12}ML^2\frac{\omega^2}{mV_{CM}} = \frac{1}{2}\left(\frac{M}{m} - 1\right)V_{CM} - \frac{L^2\omega^2}{24V_{CM}}$

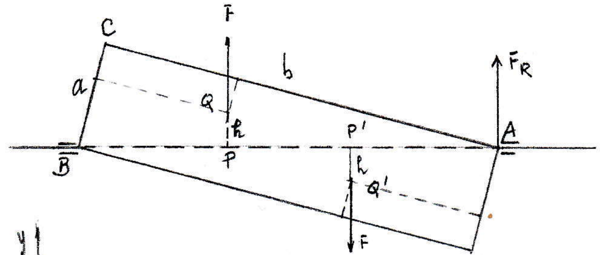
Em conclusão: $v = \frac{1}{2}\left(\frac{M}{m} - 1\right)V_{CM} - \frac{L^2\omega^2}{24V_{CM}}$ e $a = \frac{L^2\omega}{12V_{CM}}$

16.32

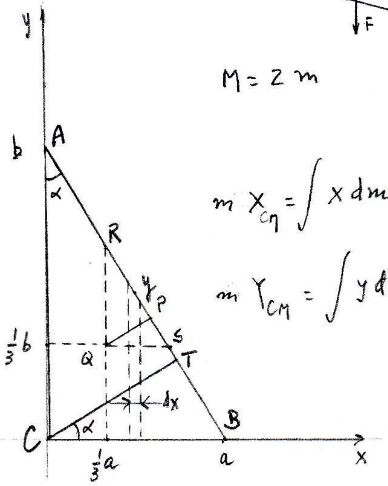
16.32

Para calcular a força exercida nos rolamentos vamos proceder assim:

- Calcular a localização do CM do triângulo ABC
- Calcular a força devido à rotação exercida nos CM
- Fazer a soma dos momentos das forças em acção, incluindo a reacção do apoio em relação a um ponto qualquer que é, por exemplo, o ponto B



$M = 2m$



$$m X_{CM} = \int x\,dm$$

$$m Y_{CM} = \int y\,dm$$

$$dm = \frac{m}{\text{área}} \cdot y\,dx = \frac{2m}{ab}\,y \cdot dx \qquad \text{Mas } y = -\frac{b}{a}x + b \qquad dm = \frac{2m}{ab}\left(-\frac{b}{a}x + b\right)dx$$

$$m X_{CM} = \int_0^a \frac{2m}{ab}\left(-\frac{b}{a}x + b\right)x\,dx \; ; \; X_{CM} = \frac{2}{ab}\left[\left(-\frac{b}{a}\right)\int_0^a x^2 dx + b\int_0^a x\,dx\right] = \frac{2}{ab}\left[\left(-\frac{b}{a}\right)\frac{a^3}{3} + b\frac{a^2}{2}\right] = \frac{2}{ab}\,\frac{-2a^3b + 3a^3b}{6a} =$$

$$= \frac{2}{ab}\,\frac{a^2b}{6a} = \frac{a}{3} \qquad \text{e então } X_{CM} = \frac{a}{3}$$

Cálculo semelhante dá $Y_{CM} = \frac{b}{3}$

É necessário calcular a distância $\overline{QP}$. O triângulo ABC e RSQ são semelhantes

e então: $\frac{a}{b} = \frac{\overline{QS}}{\overline{QR}}$; $\overline{QS} = \frac{a}{b}\overline{QR}$

e também: $\frac{\overline{QR} + \frac{b}{3}}{\frac{2}{3}a} = \frac{\frac{2}{3}b}{\frac{1}{3}a + \overline{QS}}$ e, substituindo nesta expressão $\overline{QS}$ por $\frac{a}{b}\overline{QR}$ e

resolvendo em ordem a $\overline{QR}$, vem: $QR = \frac{1}{3}b$ e $\overline{QS} = \frac{1}{3}a$

Por outro lado $\overline{CT} = H = a\cos\alpha = \frac{ab}{c}$ com $c = \sqrt{a^2+b^2}$ e $\frac{\overline{QS}}{a} = \frac{\overline{QP}}{H}$ e então

$\overline{QP} = H\frac{\overline{QS}}{a} = \frac{1}{3}H$ pelo que $\overline{QP} = h = \frac{1}{3}\frac{ab}{c}$

A força $F = m\omega_0^2 h = \frac{M}{2}\omega_0^2 \frac{ab}{3c}$

16.30 Contin. Contin. **16.30**

ou seja $0 = -g T_u + \dfrac{J \,\mathrm{sen}\, 45^\circ}{M}$ donde $T_u = \dfrac{J \,\mathrm{sen}\, 45^\circ}{Mg}$ e o tempo total de subida e de descida é $T_2 = 2 T_u = \dfrac{2 J \,\mathrm{sen}\, 45^\circ}{Mg}$

Por último os tempos calculados T_1 e T_2 devem ser iguais, ou seja:

$$\frac{2\pi}{6}\,\frac{ML}{J\cos 45^\circ} = \frac{2J\,\mathrm{sen}\,45^\circ}{Mg} \quad \text{donde} \quad J^2 = \frac{2\pi}{6}\,\frac{ML}{\cos 45^\circ}\,\frac{Mg}{2\,\mathrm{sen}\,45^\circ} = \frac{2\pi}{6}\,\frac{M^2 L g}{\mathrm{sen}(2\cdot 45^\circ)} = \frac{2\pi}{6} M^2 L g$$

pelo que $J = M\sqrt{\dfrac{\pi L g}{3}}$

16.31 **16.31**

a) $(I_0 + mR^2)\,\omega_0 = (I_0 + m r^2)\,\omega$ da conserv. do mom. angular, pelo que:

$$\omega = \frac{I_0 + mR^2}{I_0 + m r^2}\,\omega_0$$

b) A força centrípeta $F_c = m\omega^2 r$. O trabalho realizado é: $\displaystyle\int_R^r F_c\,dr = m\int_R^r \left(\frac{I_0+mR^2}{I_0+mr^2}\right)^2 \omega_0^2\, r\,dr =$

$$= m(I_0+mR^2)^2\omega_0^2 \int_R^r \frac{r\,dr}{(I_0+mr^2)^2} = m(I_0+mR^2)^2\omega_0^2\left[-\frac{1}{2m(I_0+mr^2)}\Bigg|_R^r\right] =$$

$$= m(I_0+mR^2)^2\omega_0^2\left[-\frac{1}{2m(I_0+mr^2)} + \frac{1}{2m(I_0+mR^2)}\right] = m(I_0+mR^2)^2\omega_0^2\,\frac{1}{2m}\left[\frac{-I_0-mR^2+I_0+mr^2}{(I_0+mR^2)(I_0+mr^2)}\right] =$$

$$= \frac{1}{2}(I_0+mR^2)^2\omega_0^2\,\frac{m(r^2-R^2)}{(I_0+mR^2)(I_0+mr^2)} = \frac{1}{2}(I_0+mR^2)\,\frac{m(r^2-R^2)}{I_0+mr^2}\,\omega_0^2$$

Calculemos agora a diferença das energias cinéticas de rotação:

$$\Delta W = \frac{1}{2}(I_0+mR^2)\omega_0^2 - \frac{1}{2}(I_0+mr^2)\left(\frac{I_0+mR^2}{I_0+mr^2}\right)^2\omega_0^2 = \frac{1}{2}(I_0+mR^2)\left[1-\frac{I_0+mR^2}{I_0+mr^2}\right]\omega_0^2 =$$

$$= \frac{1}{2}(I_0+mR^2)\,\frac{I_0+mr^2-I_0-mR^2}{I_0+mr^2}\,\omega_0^2 = \frac{1}{2}(I_0+mR^2)\,\frac{m(r^2-R^2)}{I_0+mr^2}$$

que é, como se vê, precisamente igual ao trabalho realizado pela força centrípeta.

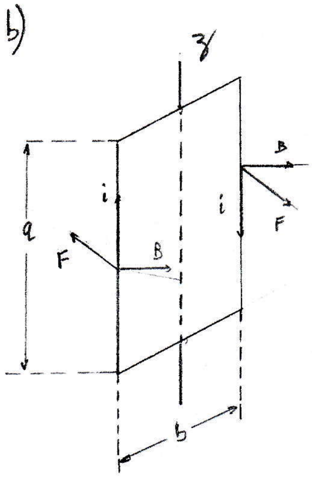
c) No instante em que o cabo é solto, a velocidade é zero. Todo o trabalho realizado transforma-se em energia cinética: $\frac{1}{2}mv^2 = \frac{1}{2}m(I_0+mR^2)\dfrac{-(r^2-R^2)}{I_0+mr^2}\omega_0^2$

pelo que $v = \omega_0\sqrt{\dfrac{I_0+mR^2}{I_0+mr^2}(R^2-r^2)}$ quando passa por R

16.29

a) $\tau = -K\theta \quad \frac{dU}{d\theta} = -\tau \; ; \; \frac{dU}{d\theta} = K\theta \; ; \; U = \frac{1}{2} K\theta^2$

b)



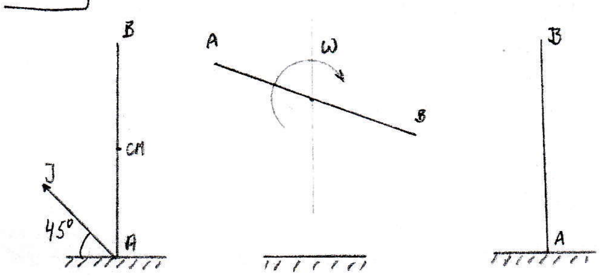
$F = n\,a\,i\,B$

$\tau = 2\,n\,a\,i\,B\,\frac{b}{2} = n\,a\,b\,i\,B = n\,A\,i\,B \qquad A = \text{Área}$

$n A B i = I \ddot{\theta} \qquad n A B \frac{dq}{dt} = I \frac{d\dot{\theta}}{dt} \qquad n A B\, dq = I\, d\dot{\theta}$

$n A B \int_0^Q dq = I \int_0^{V_0} d\dot{\theta} \qquad n A B Q = I V_0 \qquad V_0 = \frac{n A B}{I} Q$ em que I é o momento de inércia em relação ao eixo dos zz.

16.30



A componente horizontal do impulso J provoca um torque em torno do CM que faz rodar a barra; a componente vertical de J faz projectar a barra na vertical. O tempo que demora uma rotação completa deverá ser igual ao tempo de subida e de descida da barra para que ela "aterre" sobre a extremidade A com que partiu.

Cálculo do tempo necessário para efectuar uma rotação T_1:

$F \cdot \cos 45 \cdot \frac{L}{2} = I \ddot{\theta} \; ; \; \frac{L}{2} \cos 45 \int F dt = I \int \ddot{\theta} dt \; ; \; \frac{L}{2} \cos 45° \cdot J = \frac{1}{12} M L^2 \dot{\theta}$ pelo que

$$\dot{\theta} = \frac{6 J \cos 45°}{ML} \qquad T_1 = \frac{2\pi}{\dot{\theta}} = \frac{2\pi}{6} \cdot \frac{ML}{J \cos 45°}$$

Cálculo do tempo de subida e de descida:

- o tempo de subida é igual ao tempo de descida (não há resistência do ar)
- " " " " pode ser obtido calculando o tempo para o qual a velocidade vertical se anula

$\ddot{y} = -g \; ; \; \dot{y} = -gt + C_1$

$\ddot{y}(0) = \frac{F \operatorname{sen} 45°}{M} \; ; \; \dot{y}(0) = \frac{\int F \operatorname{sen} 45\, dt}{M} = \frac{J \operatorname{sen} 45°}{M}$ e então $C_1 = \dot{y}(t=0) = \frac{J \operatorname{sen} 45°}{M}$ e vem

$\dot{y} = -gt + \frac{J \operatorname{sen} 45°}{M}$ e para $t = T_M$ a velocidade vertical anula-se,

16.27 Contin. ⊖ ⊖ Contin **16.27**

Cálculo de I_{capa}: $I_{capa} = \frac{2}{5} m' \frac{r_2^5 - r_1^5}{r_2^3 - r_1^3}$ e $m' = \rho \frac{4}{3}\pi (r_2^3 - r_1^3)$ com $\rho = 10^3\ kg/m^3$

$$I_{capa} = \frac{2}{5}\,\frac{4}{3}\pi\rho(r_2^3 - r_1^3)\frac{r_2^5 - r_1^5}{r_2^3 - r_1^3} = \frac{8}{15}\pi\rho\left(r_2^5 - r_1^5\right) = \frac{8}{15}\pi\rho r_1^5\left[\left(\frac{r_2}{r_1}\right)^5 - 1\right]$$

$200\ ft = 61\ m$ $\quad r_2 = r_1 + 61$ $\quad \frac{r_2}{r_1} = \frac{r_1 + 61}{r_1} = 1 + \frac{61}{6370\cdot 10^3} = 1{,}000009576$

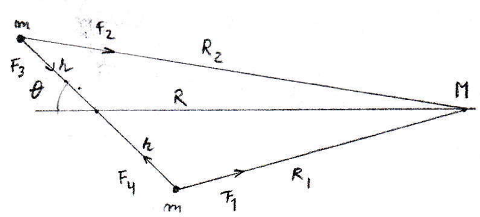
$\left(\frac{r_2}{r_1}\right)^5 = 1{,}000047882$; $\quad I_{capa} = \frac{8}{15}\pi\, 10^3 \left(6370\cdot 10^3\right)^5\left[1{,}000047882 - 1\right] = \frac{8}{10}\pi\, 10^3\left(6{,}37\cdot 10^6\right)^5\cdot 4{,}7882\cdot 10^{-5} =$

$$I_{capa} = 0{,}8414\cdot 10^{33}\ kg\ m^2$$

$$\Delta T = \frac{I_{capa}}{I_1} T_1 = \frac{0{,}84\cdot 10^{33}}{8{,}11\cdot 10^{37}} = 1{,}01\cdot T_1 = 0{,}87\,s \qquad \text{com } T_1 = 24\cdot 3600 = 86400\,s$$

Nota: a solução do livro indica $\Delta T \simeq 1\ ms$! Este valor obtinha-se se a massa específica da água fosse $\rho = 1\ kg/m^3$ e não $\rho = 10^3\ kg/m^3$.

16.28 **16.28**



$F_1 = GMm\frac{1}{R_1^2}$

$F_2 = GMm\frac{1}{R_2^2}$

$F_3 = Gmm\frac{1}{(2r)^2} \simeq 0$ face a F_2

$F_4 = Gmm\frac{1}{(2r)^2} \simeq 0$ face a F_1

Torque devido a F_1 : $\vec{\tau}_1 = \vec{r}\times\vec{F}_1$ $\qquad \tau_1 = r\,sen\,\theta\, GMm\frac{1}{R_1^2}$

" " " F_2 : $\vec{\tau}_2 = \vec{r}\times\vec{F}_2$ $\qquad \tau_2 = r\,sen\,\theta\, GMm\frac{1}{R_2^2}$

Torque total : $\tau = \tau_1 - \tau_2$ porque têm sentidos opostos, e vem $\tau = r\,sen\,\theta\, GMm\left(\frac{1}{R_1^2} - \frac{1}{R_2^2}\right)$

Mas $R_1 \simeq R - r\cos\theta$ e $R_2 \simeq R + r\cos\theta$ e substituindo e simplificando vem:

$$\frac{1}{R_1^2} - \frac{1}{R_2^2} = \frac{R_2^2 - R_1^2}{R_1^2 R_2^2} = \frac{(R + r\cos\theta)^2 - (R - r\cos\theta)^2}{\left[(R + r\cos\theta)(R - r\cos\theta)\right]^2} = \frac{\cancel{R^2} + 2rR\cos\theta + \cancel{r^2\cos^2\theta} - \cancel{R^2} + 2Rr\cos\theta - \cancel{r^2\cos^2\theta}}{\left(R^2 - r^2\cos^2\theta\right)^2} = \frac{4Rr\cos\theta}{\left(R^2 - r^2\cos^2\theta\right)^2}$$

$$\tau = r\,sen\,\theta\, GMm\frac{4Rr\cos\theta}{\left(R^2 - r^2\cos^2\theta\right)^2} \simeq r\,sen\,\theta\, GMm\frac{4Rr\cos\theta}{R^4} = GMm\frac{r^2\, 4\, sen\,\theta\cos\theta}{R^3} = GMm\frac{r^2}{R^3}2\,sen\,2\theta$$

$$\tau \simeq 2GMm\frac{r^2}{R^3}sen\,2\theta$$

What Would Happen to Earth's Rotation if the Ice Caps Melted?

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Some Physical Data We're Going to Need

- Mass of the earth 5.9736 E+24 kg
- Surface Area - Land: 148,000,000 sq. km
- Surface Area - Water: 362,000,000 sq. km
- Surface Area - Total: 510,000,000 sq. km
- Mean Radius 6371.01 km
- Volume of the Earth: 1.0832 E+12 km^3
- Mass of the Oceans: 1.3370 E+21 kg
- Volume of the Oceans: 1.377 E+09 km^3
- Mass of the Cryosphere (Ice) 2.6 E+19 kg
- Volume of the Cryosphere 29,000,000 km^3

Simple (Inaccurate) Approach

The mass of the world's ice is 1/200,000 of the total mass of the earth. If we take that mass from the pole and put it along the equator, we'd expect it to slow the earth's rotation by about 1/200,000. Since there are 60 x 60 x 24 = 86,400 seconds in a day, the slowdown would be 0.37 seconds.

This is simple and intuitive, but not very accurate. Nevertheless, it suggests that the effect of melting the ice caps wouldn't be very great. The reason it's not very accurate is that rotation depends greatly on how mass is distributed around the rotation axis.

A Bit More Technical Approach

Rotating objects possess a quantity called *moment of inertia* that plays much the same role in rotational motion that mass does in linear motion. For example, the linear momentum of a moving object is p = mv, where m is the mass, v is velocity and p is linear momentum. For a rotating object, the formula is J = Iω, where I is moment of inertia, ω is angular velocity in radians per second (there are 2 pi radians in 360 degrees) and J is angular momentum. Similarly, for an object moving in a straight line, kinetic energy is given by K = 1/2 mv^2. For a rotating object, K = 1/2 I ω^2.

Obviously I is going to depend on the mass of the object and how it's spinning. It takes a lot less energy to get a rod spinning at 100 rpm around its axis than perpendicular to it. It takes a lot less effort to get a playground carousel spinning if the passengers are all in the center than if they're on the outside circumference. For almost all objects, I = kmr^2, where m is the mass, r is the radius and k is some constant. Some values of k we will find useful:

- For a uniform sphere, k = 2/5 = 0.4. This is a bit counterintuitive, since we'd expect a fundamental shape like a sphere to have some nice neat value like 1 or 1/2, but there it is.
- The earth's moment of inertia enters into formulas for precession, so we can determine the value of k for the real earth. Since the earth is denser near the center, k is smaller than 0.4. It's 0.33, or a hair less than 1/3.
- For a thin spherical shell, k = 2/3
- For a flat disk, k=1/2

We're going to want the moment of inertia of the earth and its rotational velocity. We can calculate them, but the figures are already tabulated.

- Equatorial moment of inertia: 8.0095 E+37 kg m^2
- Rotational velocity 7.27220 E-5 radians/sec

Since J = Iω, we have J = 5.624 E+33 kg m^2 radians/sec. Don't worry about the weird looking units.

Now, we melt the cryosphere and add it to the oceans. We add 26,000,000 cubic kilometers of water (because ice has a density of 0.92, remember) and add it to 362,000,000 sq. km of ocean. That comes out to 72 meters of water. It will actually be less, because as sea level rises, the oceans will cover larger areas. Also, any ice below sea level (a pretty large amount in Antarctica) won't contribute to sea level rise at all. Exact modeling of sea level change will have to include the isostatic depression of oceanic crust and the thermal expansion or contraction of sea water. It gets very hairy.

For our purposes, we won't care about the exact rise of sea level, for reasons that will become clear. We care that we're taking ice out of the polar regions and creating a thin global shell of water. Since the only thing that matters to the earth's rotation is how far the ice is from the pole, we can lump Greenland and Antarctica together and approximate them as a disk of ice 2500 km in radius. So the moment of inertia of this spinning disk of ice is 1/2 x (2,500,000 m)^2 x 2.6 E+19 kg = 8.125 E+31 kg m^2. That's about a millionth of the total earth's angular momentum.

We take that ice, melt it and create a thin spherical shell of water (the gaps created by the continents don't affect the result very much). The shell has a radius of 6,371,010 meters and a mass of 2.6 E+19 kg, so its moment of inertia is 2/3 x (6,371,010 m)^2 x 2.6 E+19 kg = 7.03 E+32 kg m^2 or over eight times that of the polar ice. Talk about bang for the buck. The increase comes from taking all that mass and redistributing a lot of it at low latitudes. The net increase in the earth's moment of inertia is 7.03 E+32 kg m^2 gained - 8.125 E+31 kg m^2 lost or 6.22 E+32 kg m^2 gained.

From here on, it's simple proportion. Since J = Iω, and the earth's angular momentum stays constant as long as there is no outside disturbance, any increase in I will be offset by the exact same decrease in ω. The change in I, 6.22 E+32 kg m^2, compared to the total moment of inertia of the earth, 8.0095 E+37 kg m^2, amounts to an increase of 7.77 x 10^{-6}. So to conserve angular momentum, we need to slow the earth down by the same amount. There are 60 x 60 x 24 = 86,400 seconds in a day, so a decrease of 7.77 x 10^{-6} amounts to 0.67 seconds or about 2/3 of a second. The day will become about 2/3 of a second longer. We'll have more time to get stuff done. Like pile sandbags along the coasts. Yay. The actual figure will probably be less because ice below sea level doesn't contribute any effect - it just melts and is replaced by sea water.

Bottom Line

If the ice caps melt, changes in the earth's rotation will be very far down on our list of concerns.

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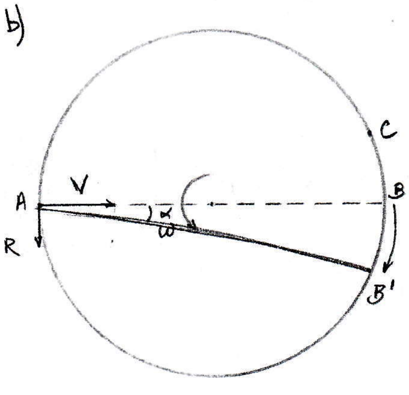
16.25

16.25

$$V \times \dot{V} = V_M \times a_{c_M} = \begin{bmatrix} \hat{i}' & \hat{j}' & \hat{k}' \\ V\cos\theta & -V\operatorname{sen}\theta & 0 \\ 2\omega V\operatorname{sen}\theta & 2\omega V\cos\theta & 0 \end{bmatrix} = \left(2\omega V^2\cos^2\theta + 2\omega V^2\operatorname{sen}^2\theta\right)\hat{k}' = 2\omega V^2\,\hat{k}'$$

Raio de curvatura: $R = \dfrac{V^3}{2\omega V^2} = \dfrac{V}{2\omega}$

b)



Tempo para percorrer a distância $2R$: $t = \dfrac{2R}{V}$

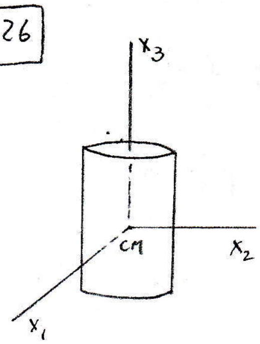
Rotação no tempo t : $\theta = \omega t = \dfrac{2R}{V}\omega$

$\dfrac{2R}{V}\omega$ Admitindo que $V >> \omega R$ então a curva AB' tem um grande raio de curvatura e podemos considerá-la uma recta. Assim para que a bola chegue, de facto, a B deve ser dirigida no início segundo um ângulo aproximadamente $\dfrac{2R}{V}\omega$ dirigido para o ponto C

c) A trajectória vista pelo observador estacionário é uma recta, como é claro.

16.26

16.26



O momento angular inicial, $I_3\omega_0$, vai passar a ser $I_1\omega_f$ quando o satélite passar a rodar em torno do eixo x_1

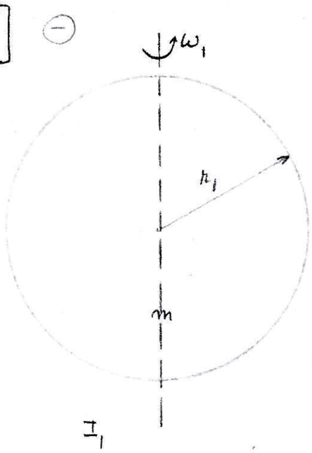
$I_3 = \frac{1}{2}ma^2$

$I_1 = \dfrac{1}{12}m\left(3a^2 + (6a)^2\right) = \dfrac{39}{12}ma^2 = \dfrac{13}{4}ma^2$

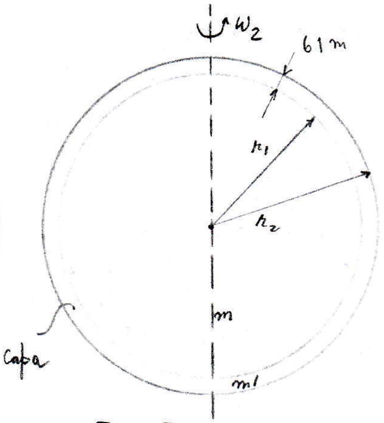
$I_3\omega_0 = I_1\omega_f \qquad \omega_f = \dfrac{I_3}{I_1}\omega_0 = \dfrac{\frac{1}{2}}{\frac{13}{4}}\omega_0 = \dfrac{2}{13}\omega_0$

16.27

16.27



I_1



$I_2 = I_1 + I_{capa}$

Conservação do momento angular:

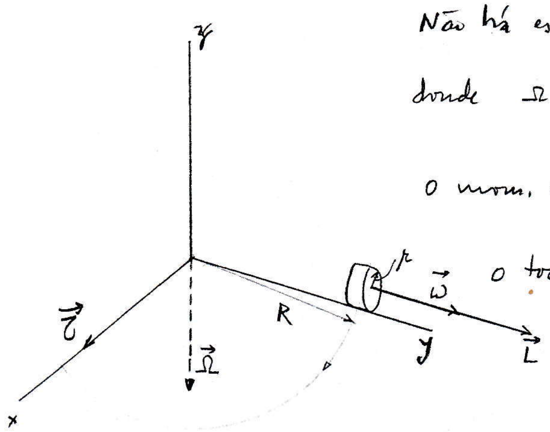
$I_1\omega_1 = I_2\omega_2 \quad ; \quad I_1\dfrac{2\pi}{T_1} = I_2\dfrac{2\pi}{T_2}$

$\dfrac{T_2}{T_1} = \dfrac{I_2}{I_1} \qquad T_2 - T_1 = \Delta T = \left(\dfrac{I_2}{I_1} - 1\right)T_1$

$\Delta T = \left(\dfrac{I_1 + I_{capa}}{I_1} - 1\right)T_1 = \dfrac{I_{capa}}{I_1}T_1$

com $T_1 = 24 \times 3600$ seg

16.23 | 16.23



Não há escorregamento e então: $2\pi r\omega = 2\pi R\Omega$

donde $\Omega = \frac{r}{R}\omega$ e $\vec{\Omega} = -\frac{r}{R}\omega\,\hat{k}$

o mom. angular $\vec{L} = I\cdot\vec{\omega}$ com $I = \frac{1}{2}Mr^2$

o torque é $\vec{\tau} = \vec{\Omega}\times\vec{L} = \begin{vmatrix} i & j & k \\ 0 & 0 & -\frac{r}{R}\omega \\ 0 & I\omega & 0 \end{vmatrix} = \frac{r}{R}\omega I\omega\,\hat{i}$

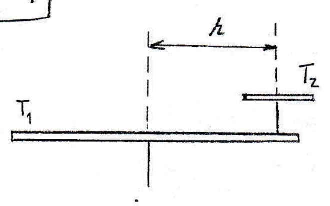
Este torque provoca uma força adicional na mesa de $\tau = F_z \cdot R = \frac{r}{R}\omega^2 I$

ou $F_z = \frac{r}{R^2}\omega^2\,\frac{1}{2}Mr^2 = \frac{1}{2}M\frac{r^3}{R^2}\omega^2$

Com os valores dados a força exercida na mesa é $F_{z_t} = Mg + \frac{1}{2}M\frac{r^3}{R^2}\omega^2$ ou

$$F_{z_t} = 1\cdot 9{,}8 + \frac{1}{2}\,1\cdot\frac{0{,}1^3}{0{,}5^2}\left(\frac{12\,000}{60}\right)^2 = 9{,}8 + 80 = 89{,}8\ N$$

16.24 | 16.24



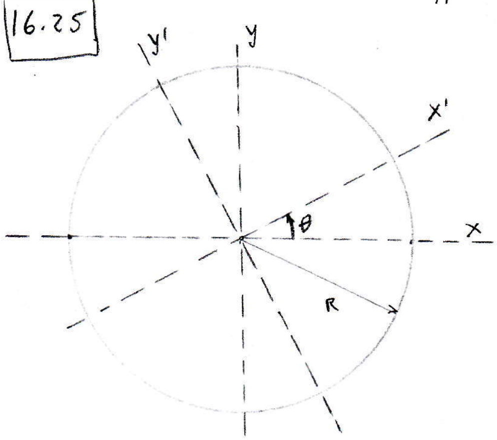
T_1 : M_1, I_1

T_2 : M_2, I_2

Há conservação do momento angular e vem: $I_t\,\omega_1 = I_2\,\omega_2$

em que $I_t = I_1 + I_2 + M_2 r^2$ pelo que $\omega_1 = \frac{I_2}{I_1 + I_2 + M_2 r^2}\,\omega_2$

16.25 | 16.25



$v_{fixo} = V\cdot i$

$$\begin{pmatrix} i' \\ j' \end{pmatrix} = \begin{pmatrix} \cos\theta & \operatorname{sen}\theta \\ -\operatorname{sen}\theta & \cos\theta \end{pmatrix}\begin{pmatrix} i \\ j \end{pmatrix}$$

$$\begin{pmatrix} i \\ j \end{pmatrix} = \begin{pmatrix} \cos\theta & -\operatorname{sen}\theta \\ \operatorname{sen}\theta & \cos\theta \end{pmatrix}\begin{pmatrix} i' \\ j' \end{pmatrix}$$

$v_{móvel} = V\cdot\cos\theta\, i' - V\operatorname{sen}\theta\, j'$ $\qquad \theta = \omega t$

Raio de curvatura $R = \frac{|v|^3}{|v\times\dot{v}|} =$

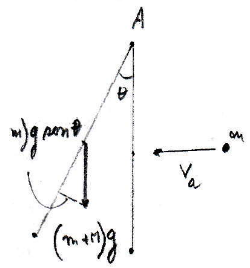
aceleração de Coriolis: $\vec{a}_c = 2\vec{\omega}\times\vec{v} = 2\omega V j =$

$= 2\omega V\operatorname{sen}\theta\, i' + 2\omega V\cos\theta\, j'$

$$\vec{a}_c = 2\begin{vmatrix} i & j & k \\ 0 & 0 & \omega \\ V & 0 & 0 \end{vmatrix} = 2\omega V j$$

16.19 | 16.19

$$I_A = \frac{1}{3}Ml^2 + m\left(\frac{l}{2}\right)^2 = \left(\frac{M}{3}+\frac{m}{4}\right)l^2$$



$I\ddot{\theta} = -(m+M)\,g\,\text{sen}\,\theta\,\frac{l}{2}$ e se θ pequeno vem $I_A\ddot{\theta} + (m+M)\,g\,\frac{l}{2}\,\theta = 0$

$$\ddot{\theta} + \frac{(m+M)g}{\left(\frac{M}{3}+\frac{m}{4}\right)l^2}\,\frac{l}{2}\,\theta = 0\ ;\quad \ddot{\theta} + \frac{(m+M)g}{\left(\frac{M}{3}+\frac{m}{4}\right)l\,2}\,\theta = 0$$

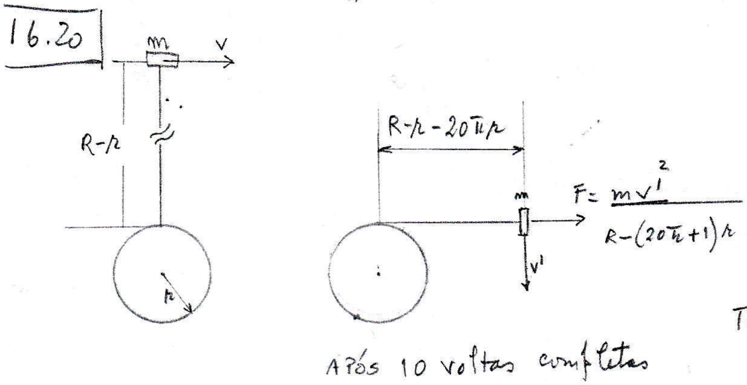
que é a eq. dif. de um oscilador harmónico de pulsação $\omega = \sqrt{\frac{(m+M)g}{\left(\frac{M}{3}+\frac{m}{4}\right)2l}} = \dot{\theta}$

$$v_f = \frac{l}{2}\dot{\theta} = \frac{l}{2}\sqrt{\frac{(m+M)g}{\left(\frac{M}{3}+\frac{m}{4}\right)2l}} = \frac{l}{2}\sqrt{\frac{g\,(m+M)}{l\,2\left(\frac{M}{3}+\frac{m}{4}\right)}}$$

<u>Nota</u>: esta é a solução do livro. Creio que está errada, apesar disso. Não é correcto identificar a pulsação das oscilações, ω, com a velocidade angular instantânea $\dot{\theta}$. Por alguma razão o livro não apresenta solução para as restantes duas alíneas.

Por outro lado a solução não depende da velocidade de impacto v_a da massa m o que, me parece, não faz sentido nenhum.

16.20 | 16.20



Conservação da en. cinética:

$\frac{1}{2}mv^2 = \frac{1}{2}mv'^2$ donde $v' = v$

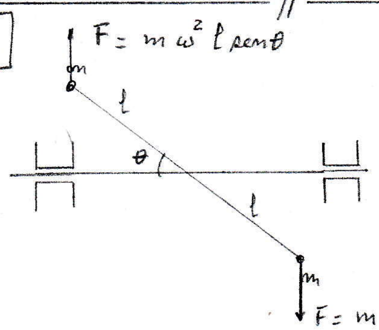
Mas $v = \omega(R-r+r) = \omega_0 R$

Tensão no fio: $T = \frac{mv'^2}{R-(20\pi+1)r} = \frac{m\omega_0^2R^2}{R-(20\pi+1)r}$

O impulso angular J necessário para parar o cilindro: $\tau = I\ddot{\theta}$

$\int \tau\,dt = I\int \ddot{\theta}\,dt$ e então $J = I\dot{\theta} = I\omega_0$

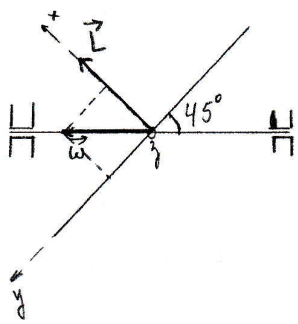
16.21 | 16.21



Torque $= 2\cdot m\omega^2 l\,\text{sen}\,\theta\; l\cos\theta = m\omega^2 l^2\,\text{sen}\,2\theta$

Torque máximo dá-se para $2\theta = 90°$ ou $\theta = 45°$ e

vale $\tau_{max} = m\omega^2 l^2$



Seja o sistema de eixos que coincide com o sistemas de eixos principais de inércia da barra, e então:

$$I_{xx} = I_{zz} = \frac{1}{12} ML^2 \qquad I_{yy} = 0$$

O vector $\vec{\omega}$ pode escrever-se nesse sistema de eixos:

$$\vec{\omega} = \omega_x \hat{i} + \omega_y \hat{j} + \omega_z \hat{k} = \omega \cos 45^\circ \hat{i} + \omega \operatorname{sen} 45 \hat{j} + 0 \hat{k}$$

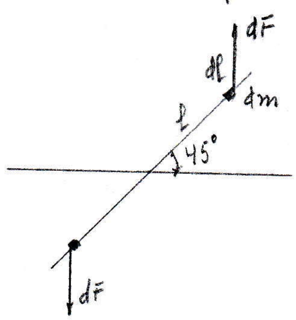
O momento angular $L = I \cdot \omega$ ou, em notação matricial:

$$\vec{L} = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} \frac{1}{12} ML^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1}{12} ML^2 \end{bmatrix} \begin{bmatrix} \omega \cos 45 \\ \omega \operatorname{sen} 45 \\ 0 \end{bmatrix} =$$

$$= \begin{bmatrix} \frac{1}{12} ML^2 \omega \cos 45^\circ \\ 0 \\ 0 \end{bmatrix} = L_x \hat{i} + L_y \hat{j} + L_z \hat{k} \quad \text{e então}$$

o momento angular $\vec{L}$ tem a direcção do eixo dos xx e faz 45° com o eixo de rotação.

b) Cálculo do torque:



$$dF = dm\, \omega^2 l \operatorname{sen}\theta \qquad e \quad dm = \frac{M}{L} dl$$

torque $d\tau = dm\, \omega^2 l \operatorname{sen}\theta\, l \cos\theta \times 2 = dm\, \omega^2 l^2 \operatorname{sen} 2\theta$ e se $\theta = 45°$

$$d\tau = \frac{M}{L} \omega^2 l^2 dl$$

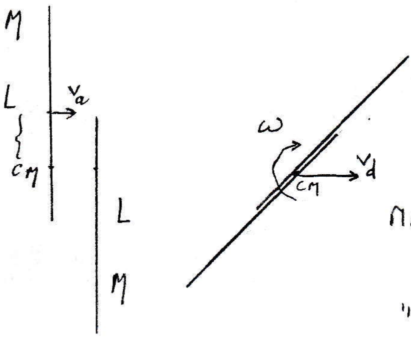
$$\tau = \frac{M}{L} \omega^2 \int_0^{L/2} l^2 dl = \frac{M\omega^2}{L} \left.\frac{l^3}{3}\right|_0^{L/2} = \frac{M\omega^2}{L\,3} \left(\frac{L}{2}\right)^3 = \frac{M\omega^2 L^2}{24}$$

OUTRO MÉTODO: $\vec{\tau} = \vec{\omega} \times \vec{L} = \begin{vmatrix} i & j & k \\ \omega \cos 45 & \omega \operatorname{sen} 45 & 0 \\ L_x & 0 & 0 \end{vmatrix} = -L_x \omega \operatorname{sen} 45\, \hat{k} = -\frac{1}{12} ML^2 \omega \cos 45 \cdot \omega \operatorname{sen} 45\, \hat{k} =$

$$= -\frac{1}{12} ML^2 \omega^2 \frac{\sqrt{2}}{2} \frac{\sqrt{2}}{2} \hat{k} = -\frac{1}{24} ML^2 \omega^2 \hat{k}$$

cujo módulo coincide com o valor anterior.

16.17



$I_{CM_a} = \frac{1}{12}ML^2 + M\left(\frac{L}{4}\right)^2 = \frac{7}{48}ML^2$ Mom. de inércia só da barra móvel

$I_{CM_d} = 2\,I_{CM_a} = \frac{7}{24}ML^2$ " " " do conjunto

Mom. angular antes do impacto: $L_a = MV_a\frac{L}{4}$

" " depois " " : $L_d = I_{CM_d}\cdot\omega$

Conservação do momento angular: $I_{CM_d}\,\omega = MV_a\frac{L}{4}$; $MV_a\frac{L}{4} = \frac{7}{24}ML^2\omega$ donde

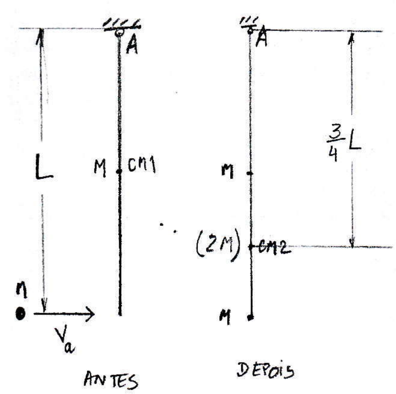
$$\omega = \frac{6}{7}\frac{V_a}{L}$$

Por outro lado: mom. linear antes do impacto: MV_a

" " depois " " $2MV_d$

Conservação do momento linear: $2MV_d = MV_a$ donde $V_d = \frac{1}{2}V_a$

16.18



O CM2, depois do embate, encontra-se a $\frac{3}{4}L$ de A.

Momento de inércia em relação ao eixo que passa por A:

$$I_A = \underbrace{\frac{1}{12}ML^2 + M\left(\frac{L}{2}\right)^2}_{\text{I da barra em relação a A}} + \underbrace{ML^2}_{\text{I da bola em relação a A}} = \left(\frac{1}{12}+\frac{1}{4}+1\right)ML^2 = \frac{1+3+12}{12}ML^2 = \frac{16}{12}ML^2 = \frac{4}{3}ML^2$$

Momento angular antes do embate: MV_aL

" " depois do embate: $I_A\cdot\omega = I_A\cdot\dot{\theta}_0$

Conservação do momento angular: $I_A\dot{\theta}_0 = MV_aL$ ou $\frac{4}{3}ML^2\dot{\theta}_0 = MV_aL$

pelo que $\dot{\theta}_0 = \frac{3}{4}\frac{V_a}{L}$

En. cinética de rotação em torno de A: $\frac{1}{2}I_a\omega^2 = \frac{1}{2}\frac{4}{3}ML^2\left(\frac{3}{4}\right)^2\frac{V_a^2}{L^2} = \frac{1}{2}\frac{3}{4}MV_a^2$

Variação da energia potencial : $2\left(\frac{3}{4}L\right)(2M)g$

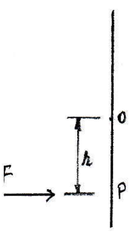
Conservação da energia : $\frac{1}{2}\cancel{\frac{3}{4}}\cancel{M}V_a^2 = 2\cancel{\frac{3}{4}}L\,2\cancel{M}g$; $V_a^2 = 8Lg$ pelo que $V_a = \sqrt{8Lg}$

16.14 contin. — contin. 16.14

Conservação do momento linear: $mv = 6m V_{cm}$ donde: $V_{cm} = \frac{1}{6} v$

" " " angular: $m v a = I\omega$; $\frac{15}{2} \cancel{m} a^{\cancel{2}} \omega = \cancel{m} v \cancel{a}$; $\omega = \frac{2}{15}\frac{v}{a}$

16.15 — 16.15



a) $Ma = F$ $\quad M\int a\,dt = \int F dt$; $M V_{cm} = J$ $\quad V_{cm} = \frac{J}{M}$

b) Momento de inércia em relação ao CM em O: $I = \frac{1}{12} M L^2$

$F \cdot h = I \ddot{\theta}$ $\quad \int F dt \cdot h = \frac{1}{12} M L^2 \int \ddot{\theta} dt \Rightarrow J \cdot h = \frac{1}{12} M L^2 \omega$ $\quad \omega = \frac{12 J h}{M L^2}$

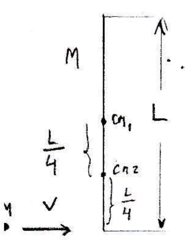
c) $V_A = V_{A/cm} + V_{cm/fixo} = -\frac{L}{2}\omega + \frac{J}{M} = -\frac{L}{2}\frac{12 J h}{M L^2} + \frac{J}{M} = \frac{J}{M}\left(1 - \frac{6}{L} h\right)$

d) Basta fazer $1 - \frac{6}{L} h = 0$ $\quad h = \frac{L}{6}$ $\quad \overline{AP} = \frac{L}{2} + \frac{L}{6} = \frac{2}{3} L$

e) Mesma resposta que em d) $\overline{AP} = \frac{2}{3} L$

16.16 — 16.16

a) $M\vec{V} + M \cdot 0 = 2M \vec{V}_{cm}$ $\quad \vec{V}_{cm} = \frac{1}{2}\vec{V}$



b) o cm do sistema barra-bola fica situado em CM2 a $\frac{L}{4}$ da extremidade

o momento angular antes do impacto vem: $L = M V \frac{L}{4}$

c) $I\omega = \frac{1}{4} M V L$ pela conservação do momento angular, e em que I

é o momento de inércia da barra em relação a CM2 mais o da

bola em relação ao mesmo ponto.

$I = I_{\text{barra em rel. a CM1}} + M\left(\frac{L}{4}\right)^2 + M\left(\frac{L}{4}\right)^2 = \frac{1}{12} M L^2 + \frac{2}{16} M L^2 = \left(\frac{1}{12} + \frac{1}{8}\right) M L^2 = \frac{5}{24} M L^2$

E então $\quad \omega = \dfrac{\frac{1}{4} M V L}{\frac{5}{24} M L^2} = \frac{6}{5}\frac{V}{L}$

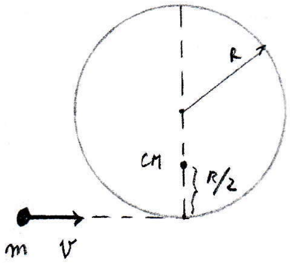
d) En. cinética antes do impacto: $E_{K_a} = \frac{1}{2} M V^2$

En. cinética depois do impacto: $E_{K_d} = \frac{1}{2}(2M)\left(\frac{V}{2}\right)^2 + \frac{1}{2}\left(\frac{5}{24} M L^2\right)\left(\frac{6}{5}\frac{V}{L}\right)^2 = \frac{2}{5} M V^2$

$$\frac{E_{K_a} - E_{Kd}}{E_{K_a}} = \frac{\frac{1}{2} M V^2 - \frac{2}{5} M V^2}{\frac{1}{2} M V^2} = 2\left(\frac{1}{2} - \frac{2}{5}\right) = \frac{2}{10} = 20\%$$ de en. cinética perdida

16.13

16.13



a) Conservação do momento linear: $m v + m \cdot 0 = 2 m V_{CM}$

$V_{CM} = \frac{1}{2} v$

b) Momento angular antes do impacto em relação do CM.

Cálculo do CM: $m 0 + m R = 2 m R'$ $\quad R'_{CM} = \frac{1}{2} R$

$L_a = |r \times \vec{p}| = m v \frac{R}{2}$

c) Momento de inércia do círculo em relação ao CM:

$I = m R^2 + m \left(\frac{R}{2}\right)^2 = \frac{5}{4} m R^2$

Momento de inércia total (com a bala de massa m) em relação ao CM:

$I_t = \frac{5}{4} m R^2 + m \left(\frac{R}{2}\right)^2 = \left(\frac{5}{4} + \frac{1}{4}\right) m R^2 = \frac{3}{2} m R^2$

Momento angular depois do impacto:

$L_d = I_t \omega = \frac{3}{2} m R^2 \omega$

Conservação do momento angular: $\frac{3}{2} m R^2 \omega = m v \frac{R}{2}$; $3 R \omega = v$; $\omega = \frac{v}{3R}$

d) En. cinética do sistema antes do impacto: $E_{K_a} = \frac{1}{2} m v^2$

En. cinética do sistema depois do impacto: $E_{K_d} = \frac{1}{2} 2m V_{CM}^2 + \frac{1}{2} \frac{3}{2} m R^2 \omega^2$

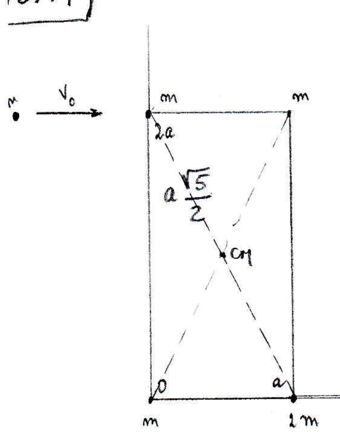
$E_{K_d} = \frac{1}{2} 2m \left(\frac{v}{2}\right)^2 + \frac{1}{2} \frac{3}{2} m R^2 \left(\frac{v}{3R}\right)^2 = \frac{1}{4} m v^2 + \frac{1}{2} \frac{3}{2} \frac{1}{9} m v^2 = \left(\frac{1}{4} + \frac{1}{12}\right) m v^2 = \frac{1}{3} m v^2$

Como se vê, não há pois conservação da energia cinética.

—————— // —————— // ——————

16.14

16.14



CM após colisão: $R_x = \frac{1}{6m}(m a + m 0 + 2m a + m \cdot 0) = \frac{1}{2} a$

$R_y = \frac{1}{6m}(2m\, 2a + m \cdot 2a + m \cdot 0 + 2m\, 0) = a$

Mom. de inércia após a colisão: $I = 2m\left(a \frac{\sqrt{5}}{2}\right)^2 + m\left(a \frac{\sqrt{5}}{2}\right)^2 + m\left(a \frac{\sqrt{5}}{2}\right)^2 + 2m\left(a \frac{\sqrt{5}}{2}\right)^2 = \frac{15}{2} m a^2$

Mom. angular em relação a CM: antes da colisão : $m v a$

depois da colisão : $I\omega$

Mom. linear antes da colisão : $m v$

depois " " : $6 m V_{CM}$

16.32 Contin. Contin. 16.32

O $\sum \vec{M}_A = 0$ e vem: $F.\overline{BP} - F(\overline{BA}-\overline{BP}) + F_R \cdot \overline{BA} = 0$ ou $F_R = \frac{F}{\overline{BA}}(\overline{BA}-\overline{BP}-\overline{BP})$

$F_R = F.\left(1-\frac{2\overline{BP}}{\overline{BA}}\right)$ em que ainda não conhecemos $\overline{BP}$. Para isso podemos escrever:

$\overline{AS} = \frac{2}{3} c$ $\quad \overline{AP} = \overline{AS}-\overline{PS} = \frac{2}{3} c - \overline{PS}$ Mas: $\frac{\overline{PS}}{\overline{TB}} = \frac{\overline{QS}}{a}$; $\overline{PS} = \frac{\overline{QS}}{a}\overline{TB} = \frac{1}{a} \frac{1}{3} a\, \overline{TB} = \frac{1}{3} \overline{TB}$

$\overline{AP} = \frac{2}{3} c - \frac{1}{3}\overline{TB}$ e $\overline{BP} = \overline{AB}-\overline{AP} = c - \frac{2}{3} c + \frac{1}{3} \overline{TB} = \frac{1}{3} c + \frac{1}{3} \overline{TB}$. Mas $\overline{TB} = a\,\text{sen}\,\alpha = \frac{a^2}{c}$

e então vem: $\overline{BP} = \frac{1}{3} c + \frac{1}{3}\frac{a^2}{c} = \frac{c^2+a^2}{3c} = \frac{a^2+b^2+a^2}{3\sqrt{a^2+b^2}} = \frac{1}{3}\frac{2a^2+b^2}{\sqrt{a^2+b^2}}$

Finalmente: $F_R = \frac{M}{2} \omega_0^2 \frac{ab}{3c}\left(1-2\frac{1}{c}\frac{c^2+a^2}{3c}\right) = \frac{M}{2}\omega_0^2\frac{ab}{3c}\frac{3c^2-2(c^2+a^2)}{3c^2} = \frac{M}{2} \omega_0^2\frac{ab}{3c}\frac{c^2-2a^2}{3c^2}$

$F_R = \frac{M}{2}\omega_0^2\frac{ab}{3c}\frac{a^2+b^2-2a^2}{3c^2} = \frac{1}{18}M\omega_0^2\, ab\frac{b^2-a^2}{(a^2+b^2)^{3/2}}$

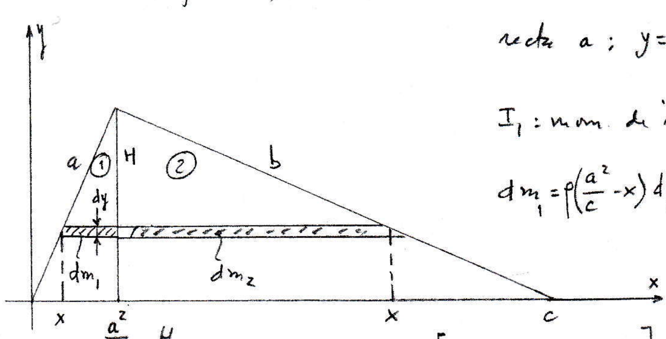
Nota: a solução do livro dá o factor $\frac{1}{12}$ em vez de $\frac{1}{18}$!

e então a força exercida no rolamento é a oposta da reacção do apoio.

b) A energia cinética de rotação é: $E_{K_{rot}} = \frac{1}{2} I\omega_0^2$ em que I é o momento de inércia da placa rectangular de massa M em relação ao eixo diagonal xx.

Das tabelas tira-se: $I = \frac{Ma^2b^2}{6(a^2+b^2)}$ pelo que $E_{K_{rot}} = \frac{1}{2}\frac{Ma^2b^2}{6(a^2+b^2)}\omega_0^2$

Vamos, porém, calcular I.



recta a: $y = \frac{b}{a} x \qquad x = \frac{a}{b} y$

I_1: mom. de inércia do Triângulo 1 em relação ao eixo xx.

$dm_1 = \rho\left(\frac{a^2}{c}-x\right)dy = \frac{2m}{ab}\left(\frac{a^2}{c}-x\right)dy = \frac{2m}{ab}\left(\frac{a^2}{c}-\frac{a}{b} y\right)dy$

$$I_1 = \frac{2m}{ab}\int_0^H \left(\frac{a^2}{c}-\frac{a}{b} y\right)y^2\,dy = \frac{2m}{ab}\left[\frac{a^2}{c}\frac{H^3}{3}-\frac{a}{b}\frac{H^4}{4}\right] = \frac{2m}{ab}\left[\frac{a^2}{c}\frac{a^3b^3}{3c^3}-\frac{a}{b}\frac{a^4b^4}{4c^4}\right] = \frac{2m}{ab}\left[\frac{a^5b^3}{3c^4}-\frac{a^5b^3}{4c^4}\right] =$$

$$= 2m\frac{4a^4b^2-3a^4b^2}{12c^4} = m\frac{a^4b^2}{6c^4}$$

$$I_2 = m\frac{a^2b^4}{6c^4}$$

$$I = I_1+I_2 = m\frac{a^4b^2+a^2b^4}{6c^4} = m\frac{a^2b^2(a^2+b^2)}{6c^4} = m\frac{a^2b^2c^2}{6c^4} = m\frac{a^2b^2}{6(a^2+b^2)}$$

c.q.d.

16.32 Contin. Contin. 16.32

I_2 foi determinado por semelhança com I_1. Mas podia ter sido calculado directamente.

$dm_2 = \frac{2m}{ab}\left(x - \frac{a^2}{c}\right)dy$ a recta é: $y = -\frac{a}{b}x + \frac{ac}{b}$ donde $x = -\frac{b}{a}y + c$

$$dm_2 = \frac{2m}{ab}\left(-\frac{b}{a}y + c - \frac{a^2}{c}\right)dy = 2m\left(-\frac{1}{a^2}y + \frac{c}{ab} - \frac{a}{bc}\right)dy = 2m\left(-\frac{1}{a^2}y + \frac{c^2-a^2}{abc}\right)dy =$$

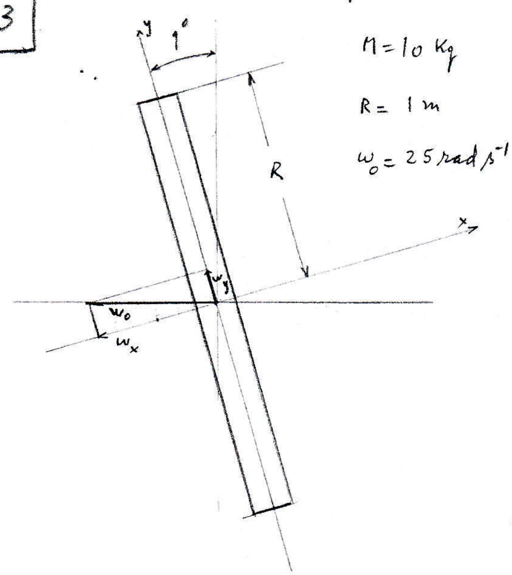
$$= 2m\left(-\frac{1}{a^2}y + \frac{b^2}{abc}\right)dy = 2m\left(-\frac{1}{a^2}y + \frac{b}{ac}\right)dy$$

$$I_2 = 2m\int_0^H \left(-\frac{1}{a^2}y + \frac{b}{ac}\right)y^2\,dy = 2m\left[\left(-\frac{1}{a^2}\right)\int_0^H y^3\,dy + \frac{b}{ac}\int_0^H y^2\,dy\right] = 2m\left[-\frac{1}{a^2}\frac{H^4}{4} + \frac{b}{ac}\frac{H^3}{3}\right]$$

$$= 2m\left[-\frac{1}{a^2 4}\frac{a^4b^4}{c^4} + \frac{b}{ac3}\frac{a^3b^3}{c^3}\right] = 2m\left[-\frac{a^2b^4}{4c^4} + \frac{a^2b^4}{3c^4}\right] = 2m\frac{-3a^2b^4+4a^2b^4}{12c^4} =$$

$$= 2m\frac{a^2b^4}{12c^4} = \frac{ma^2b^4}{6c^4}$$ que confirma o valor indicado antes.

16.33 16.33



$M = 10$ Kg

$R = 1$ m

$\omega_0 = 25\ rad\,s^{-1}$

$$L = \begin{bmatrix} L_x \\ L_y \\ L_z \end{bmatrix} = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix}\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} I_{xx}\,\omega_x \\ I_{yy}\,\omega_y \\ I_{zz}\,\omega_z \end{bmatrix} = mR^2\begin{bmatrix} \frac{1}{2}\omega \\ \frac{1}{4}\cdot 0{,}01745\,\omega \\ 0 \end{bmatrix}$$

$I_{xx} = \frac{1}{2}mR^2 \qquad I_{yy} = I_{zz} = \frac{1}{4}mR^2$

$\omega_x = \omega_0 \cos 1° \simeq \omega_0$

$\omega_y = \omega_0 \,\text{sen}\, 1° = 0{,}01745\,\omega_0$

$\omega_z = 0$

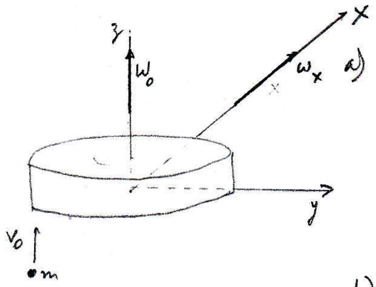
$$\vec{L} = mR^2\frac{1}{2}\omega_0\,\hat{i} + mR^2\frac{1}{4}0{,}01745\,\omega_0\,\hat{j} = mR^2\omega_0\left(\frac{1}{2}\hat{i} + 0{,}0044\,\hat{j}\right)$$

Torque: $$\vec{\tau} = \vec{\omega}\times\vec{L} = \begin{vmatrix} i & j & k \\ \omega_x & \omega_y & 0 \\ \frac{1}{2}mR^2\omega_0 & \frac{1}{4}mR^2 0{,}01745\,\omega_0 & 0 \end{vmatrix} = \frac{1}{4}mR^2 0{,}01745\,\omega^2 - \frac{1}{2}mR^2\omega^2 0{,}01745$$

$$= -mR^2\omega^2\frac{1}{4}0{,}01745 = -10\cdot 1^2\cdot 25^2\frac{1}{4}0{,}01745 =$$

$$= 27{,}26\ Nm$$

16,34 | 16,34



a)

$$L_z = \frac{1}{2} M R^2 \omega_0$$

$$L_x = 2 m V_0 R$$

$$\frac{L_x}{L_z} = \frac{4 m V_0 R}{M R^2 \omega_0} = \frac{4 m V_0}{M R \omega_0}$$

b) $2 m V_0 R = I_{xx} \cdot \omega_x \qquad I_{xx} = \frac{1}{4} M R^2 \qquad \omega_x = \frac{8 m V_0 R}{M R^2}$

$$\frac{\omega_x}{\omega_0} = \frac{8 m V_0}{M R^2 \omega_0}$$

Nota: o momento transmitido ao disco pela massa m é duas vezes $m V_0$ pois ressalta depois do impacto