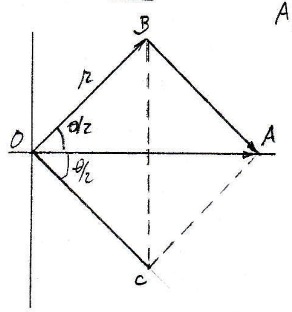
a) Analiticamente: $A = re^{i\theta/2} + re^{-i\theta/2} = r\cos\frac{\theta}{2} + ir\,\mathrm{sen}\,\frac{\theta}{2} + r\cos\frac{\theta}{2} - ir\,\mathrm{sen}\,\frac{\theta}{2} = 2r\cos(\theta/2) = |A|$

Geometricamente:



A soma dos dois vectores $\vec{OB}$ e $\vec{OC}$ dá o vector $\vec{OA}$ cujo módulo é $|A| = 2r\cos\frac{\theta}{2}$

b) Analiticamente: $A = \sum_{n=0}^{N} re^{in\theta} = r + re^{i\theta} + re^{i2\theta} + \dots = r\left(1 + e^{i\theta} + e^{i2\theta} + \dots\right)$. A soma entre parêntesis é uma progressão aritmética de razão $e^{i\theta}$. Esta soma vale: $\dfrac{1-e^{i(N+1)\theta}}{1-e^{i\theta}}$

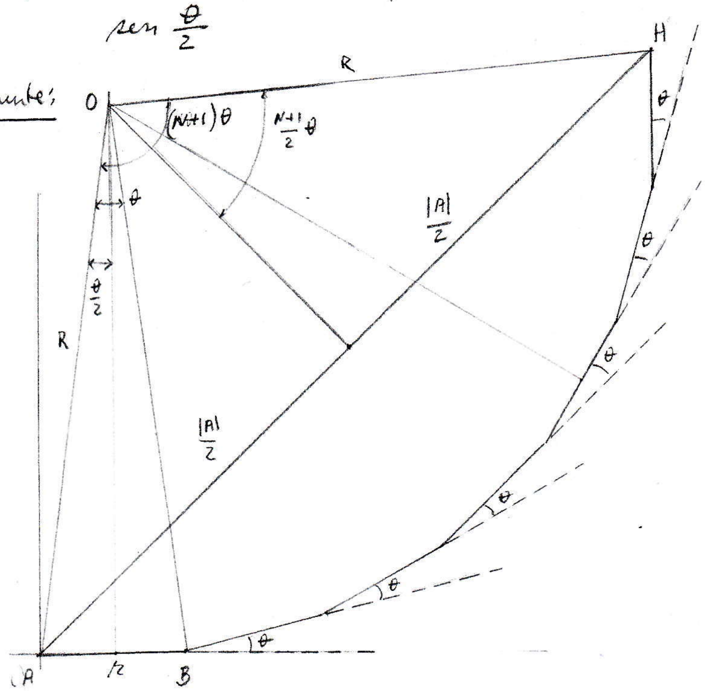
Então $A = r\dfrac{1-e^{i(N+1)\theta}}{1-e^{i\theta}}$ e $|A|^2 = r\dfrac{1-e^{i(N+1)\theta}}{1-e^{i\theta}}\cdot r\dfrac{1-e^{-i(N+1)\theta}}{1-e^{-i\theta}} = r^2\dfrac{\left(1-e^{i(N+1)\theta}\right)\left(1-e^{-i(N+1)\theta}\right)}{\left(1-e^{i\theta}\right)\left(1-e^{-i\theta}\right)} =$

$$|A|^2 = r^2\frac{1+1-\left(e^{i(N+1)\theta}+e^{-i(N+1)\theta}\right)}{1+1-\left(e^{i\theta}+e^{-i\theta}\right)} = r^2\frac{2-2\cos(N+1)\theta}{2-2\cos\theta} = r^2\frac{1-\cos(N+1)\theta}{1-\cos\theta} =$$

$$= r^2\frac{\cancel{\cos^2\frac{N+1}{2}\theta} + \mathrm{sen}^2\frac{N+1}{2}\theta - \cancel{\cos^2\frac{N+1}{2}\theta} + \mathrm{sen}^2\frac{N+1}{2}\theta}{\cancel{\cos^2\frac{\theta}{2}} + \mathrm{sen}^2\frac{\theta}{2} - \cancel{\cos^2\frac{\theta}{2}} + \mathrm{sen}^2\frac{\theta}{2}} = r^2\frac{\cancel{2}\,\mathrm{sen}^2\frac{N+1}{2}\theta}{\cancel{2}\,\mathrm{sen}^2\frac{\theta}{2}}$$ e então:

$$|A| = r\frac{\mathrm{sen}\frac{N+1}{2}\theta}{\mathrm{sen}\frac{\theta}{2}}$$ c.q.d.

Geometricamente:



Na figura N=6

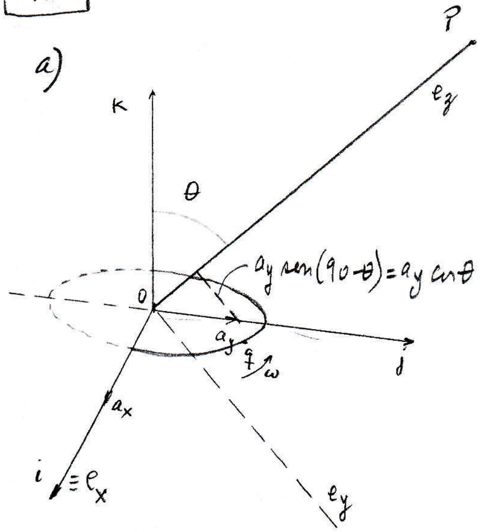
$r = 2R\,\mathrm{sen}\,\frac{\theta}{2}$ donde $R = \frac{1}{2}r\frac{1}{\mathrm{sen}\frac{\theta}{2}}$

$\frac{|A|}{2} = R\,\mathrm{sen}\,\frac{N+1}{2}\theta = \frac{1}{2}r\frac{1}{\mathrm{sen}\frac{\theta}{2}}\,\mathrm{sen}\,\frac{N+1}{2}\theta$

$|A| = r\dfrac{\mathrm{sen}\frac{N+1}{2}\theta}{\mathrm{sen}\frac{\theta}{2}}$ c.q.d.

21.2

a)



Cálculo da aceleração da carga q:

$x = a \cos \omega t \; ; \; \dot{x} = -a\omega \, \text{sen}\, \omega t \; ; \; \ddot{x} = -a\omega^2 \cos \omega t = a_x$

$y = a \, \text{sen}\, \omega t \; ; \; \dot{y} = a\omega \cos \omega t \; ; \; \ddot{y} = -a\omega^2 \, \text{sen}\, \omega t = a_y$

Seja $\overline{OP}$ no plano (y,z) o que não é limitativo pois podemos sempre rodar o sistema de eixos i, j, k até $\overline{OP}$ ficar no plano (y,z)

Só interessam no cálculo do campo em P as componentes perpendiculares a $\overrightarrow{OP}$.

A componente a_x é já $\perp$ a $\overline{OP}$. A componente a_y não é $\perp$. Mas $a_y \cos\theta$ dirigido segundo o eixo e_y (no novo sistema de eixos em que $\hat{i} \equiv \hat{e}_x$, $\hat{e}_y$, e $\hat{e}_z$ que tem a direcção $\overrightarrow{OP}$). A aceleração $\perp$ a $\overline{OP}$ neste novo sistema de eixos é: $a_x \hat{e}_x + a_y \cos\theta \, \hat{e}_y = -a\omega^2 \cos\omega t \, \hat{e}_x - a\omega^2 \cos\theta \, \text{sen}\, \omega t \, \hat{e}_y$

O campo é dado por: $E = -\dfrac{q}{4\pi\varepsilon_0 c^2 R} \, a\left(t - \dfrac{R}{c}\right)$ e vem:

$$E = \frac{q a \omega^2}{4\pi\varepsilon_0 c^2 R}\left[\cos\omega t' \hat{e}_x + \cos\theta \, \text{sen}\, \omega t' \, \hat{e}_y\right] \quad \text{com } t' = t - \frac{R}{c}$$

b) $I(\theta = 0^\circ) = \left(\dfrac{q a \omega^2}{4\pi\varepsilon_0 c^2 R}\right)^2 \left(\cos^2\omega t' + \text{sen}^2 \omega t'\right) = \left(\dfrac{q a \omega^2}{4\pi\varepsilon_0 c^2 R}\right)^2 =$

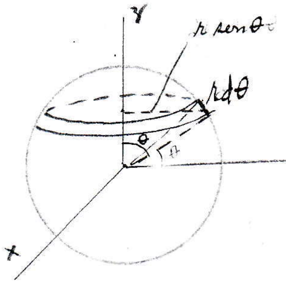
$I\left(\theta = \dfrac{\pi}{2}\right) = \left(\dfrac{q a \omega^2}{4\pi\varepsilon_0 c^2 R}\right)^2 \cdot \dfrac{1}{T}\displaystyle\int_0^T \cos^2 \omega\left(t - \dfrac{R}{c}\right) dt = \dfrac{1}{2} I(\theta = 0)$. Notar que, neste caso, só contribui para o campo a componente segundo e_x da aceleração

21.3 **21.3**

a)

$$E = -\frac{q}{4\pi\varepsilon_0 c^2 R}\cdot \sin\theta \cdot a_0 \cos\omega\left(t-\frac{R}{c}\right) \quad ; \quad P(\theta) = \left(\frac{q\,a_0}{4\pi\varepsilon_0 c^2 R}\right)^2 \sin^2\theta \cdot \cos^2\omega\left(t-\frac{R}{c}\right)$$

Pot. média por unidade de área: $\overline{P}(\theta) = \left(\frac{q a_0}{4\pi\varepsilon_0 c^2 R}\right)^2 \sin^2\theta \cdot \underbrace{\frac{1}{T}\int_0^T \cos^2\omega\left(t-\frac{R}{c}\right)dt}_{\frac{1}{2}} = \frac{1}{2}\left(\frac{q a_0}{4\pi\varepsilon_0 c^2 R}\right)^2 \sin^2\theta$



elemento de área assinalado: $2\pi R\,\mathrm{sen}\,\theta \cdot R\,d\theta = 2\pi R^2 \mathrm{sen}\,\theta\, d\theta$

Potência total radiada: $\overline{P}_{total} = \left(\frac{q a_0}{4\pi\varepsilon_0 c^2 R}\right)^2 2\pi R^2 \underbrace{\int_0^\pi \mathrm{sen}^3\theta\, d\theta}_{A}\ \underbrace{\frac{1}{T}\int_0^T \cos^2\omega\left(t-\frac{R}{c}\right)dt}_{\frac{1}{2}}$

$$A = \int_0^\pi \mathrm{sen}^3\theta\, d\theta = \frac{1}{12}\left[\cos 3\theta - 9\cos\theta\right]_0^\pi = \frac{1}{12}\left[-1+9-(1-9)\right] = \frac{1}{12}\left[-2+18\right] = \frac{16}{12} = \frac{4}{3}$$ e vem:

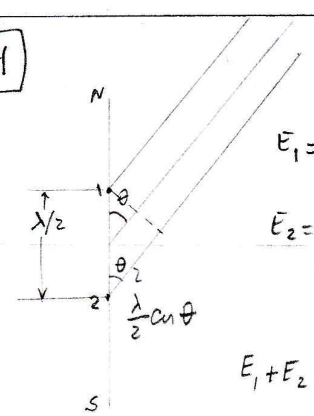
$$\overline{P}_{total} = \left(\frac{q a_0}{4\pi\varepsilon_0 c^2}\right)^2 2\pi\,\frac{4}{3}\cdot\frac{1}{2} = \frac{4\pi}{3}\left(\frac{q a_0}{4\pi\varepsilon_0 c^2}\right)^2$$

$$\frac{\overline{P}}{\overline{P}_{total}} = \frac{\frac{1}{2}\left(\frac{q a_0}{4\pi\varepsilon_0 c^2}\right)^2 \frac{1}{R^2}\sin^2\theta}{\frac{4\pi}{3}\left(\frac{q a_0}{4\pi\varepsilon_0 c^2}\right)^2} = \frac{3}{8\pi}\,\frac{\sin^2\theta}{R^2}$$

b) $R = 25000\cdot\sqrt{2}$; $\sin\theta = \frac{1}{\sqrt{2}}$; $\overline{P}_{total} = 0{,}5\ \mathrm{W}$

$$P = \frac{3}{8\pi}\,\frac{\sin^2\theta}{R^2}\,\overline{P}_{total} = \frac{3}{8\pi}\cdot\frac{\frac{1}{2}}{25000^2\cdot 2}\cdot\frac{1}{2} = \frac{3}{8\pi}\,\frac{1}{8\cdot 25000^2} = 2{,}39\cdot 10^{-11}\ \mathrm{W/m^2}$$

21.4 **21.4**



Situação em que as antenas radiam isoladamente:

$E_1 = K_1 \cos\left(\omega\left(t-\frac{R}{c}\right)\right)$ com $I_1 = K_1^2 \frac{1}{T}\int \cos^2\omega\left(t-\frac{R}{c}\right)dt = \frac{1}{2}K_1^2 = I_0$ donde $K_1 = \sqrt{2 I_0}$

$E_2 = K_2 \cos\left(\omega\left(t-\frac{R}{c}-\frac{\lambda/2}{c}\cos\theta\right)\right)$ com $I_2 = K_2^2 \frac{1}{T}\int \cos^2\left(\omega\left(t-\frac{R}{c}-\frac{\lambda/2}{c}\right)\right)dt = \frac{1}{2}K_2^2 = 2 I_0 = 2\sqrt{I_0}$

Situação em que as antenas radiam em simultâneo:

$$E_1 + E_2 = \sqrt{2 I_0}\cos\left(\omega\left(t-\frac{R}{c}\right)\right) + 2 I_0 \cos\left(\omega\left(t-\frac{R}{c}-\frac{\lambda/2}{c}\cos\theta\right)\right) \quad \text{eq. } 1$$

1° caso: $\theta = 0°$ $\quad \vec{E}_1 + \vec{E}_2 = \sqrt{2 I_0}\left[\cos\left(\omega\left(t-\frac{R}{c}\right)\right) + \sqrt{2}\cos\left(\omega\left(t-\frac{R}{c}-\frac{\lambda/2}{c}\right)\right)\right]$ e se $\alpha = \omega\left(t-\frac{R}{c}\right)$ e $\beta = -\omega\frac{\lambda/2}{c} = -\pi$

Mas $\cos\alpha + \sqrt{2}\cos(\alpha+\beta) = \cos\alpha + \sqrt{2}\left[\cos\alpha\cos\beta - \mathrm{sen}\,\alpha\,\mathrm{sen}\,\beta\right] = (1+\sqrt{2}\cos\beta)\cos\alpha - \mathrm{sen}\,\alpha\,\mathrm{sen}\,\beta$ e vem:

21.4 Contin. Contin. 21.4

$$\vec{E}_1+\vec{E}_2=\sqrt{2}\sqrt{I_0}\left[\left(1+\sqrt{2}\underbrace{\cos\beta}_{\cos(-\pi)=-1}\right)\cos\alpha-\sqrt{2}\underbrace{\operatorname{sen}\beta}_{\operatorname{sen}(-\pi)=0}\operatorname{sen}\alpha\right]=\sqrt{2}\sqrt{I_0}\left(1-\sqrt{2}\right)\cos\omega\left(t-\frac{R}{c}\right) \quad \text{e}$$

a intensidade será de: $I=2I_0\cdot(1-\sqrt{2})^2\cdot\frac{1}{2}=(1-\sqrt{2})^2 I_0=0{,}17\,I_0$

2° caso: $\theta=90^\circ$

Da eq. 1 e $\theta=90^\circ$ vem: $\vec{E}_1+\vec{E}_2=\sqrt{2}\sqrt{I_0}\left[\cos\omega\left(t-\frac{R}{c}\right)+\sqrt{2}\cos\omega\left(t-\frac{R}{c}\right)\right]$

$\vec{E}_1+\vec{E}_2=\sqrt{2I_0}\left(1+\sqrt{2}\right)\cos\omega\left(t-\frac{R}{c}\right)$ e a intensidade será de:

$$I=2I_0\left(1+\sqrt{2}\right)^2\frac{1}{2}=\left(1+\sqrt{2}\right)^2 I_0=5{,}8\,I_0$$

3° caso: $\theta=60^\circ$

Da eq. 1 e com $\theta=60^\circ$ vem: $\vec{E}_1+\vec{E}_2=\sqrt{2I_0}\left[\cos\omega\left(t-\frac{R}{c}\right)+\sqrt{2}\cos\left[\omega\left(t-\frac{R}{c}\right)-\underbrace{\frac{\omega\lambda}{2c}\cos 60^\circ}_{\frac{\pi}{2}}\right]\right]$

e fazendo $\alpha=\omega\left(t-\frac{R}{c}\right)$ e $\beta=\frac{-\pi}{2}$ vem:

$$\vec{E}_1+\vec{E}_2=\sqrt{2I_0}\left[\cos\alpha+\sqrt{2}\cos(\alpha+\beta)\right]=\sqrt{2I_0}\left[\cos\alpha+\sqrt{2}\cos\left(\alpha-\frac{\pi}{2}\right)\right]=\sqrt{2I_0}\left[\cos\alpha-\sqrt{2}\operatorname{sen}\alpha\right]=$$

$$=\sqrt{2I_0}\left[\cos\alpha-\operatorname{tg}\gamma\operatorname{sen}\alpha\right]=\sqrt{2I_0}\,\frac{1}{\cos\gamma}\left[\cos\alpha\cos\gamma-\operatorname{sen}\alpha\operatorname{sen}\gamma\right]=\sqrt{2I_0}\,\frac{1}{\cos\gamma}\cos(\alpha+\gamma)$$

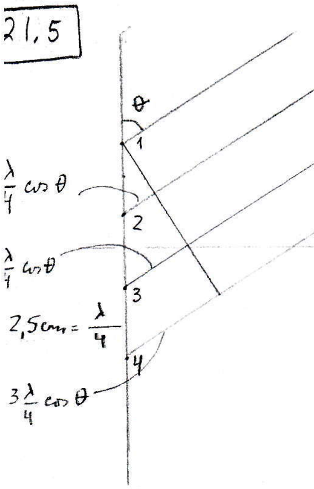
Com $\operatorname{tg}\gamma=\sqrt{2}$ e $\cos\gamma=\dfrac{1}{\sqrt{1+\operatorname{tg}^2\gamma}}=\dfrac{1}{\sqrt{1+2}}=\dfrac{1}{\sqrt{3}}$ pelo que:

$\therefore$

$\vec{E}_1+\vec{E}_2=\sqrt{2I_0}\sqrt{3}\cos\left[\omega\left(t-\frac{R}{c}\right)+\operatorname{arctg}\sqrt{2}\right]$ e vem para a intensidade média:

$$I=2I_0\cdot 3\cdot\frac{1}{2}=3\,I_0$$

21.5 21.5



$f=3\cdot10^9\,Hz$ o que dá $\lambda=\dfrac{c}{f}=\dfrac{3\cdot10^{10}}{3\cdot10^9}=10\,cm$

Campos de radiação:

$E_1=k\cos\left(\omega\left(t-\frac{R}{c}\right)\right)$ $\quad E_1=k\,e^{i\omega\left(t-\frac{R}{c}\right)}$

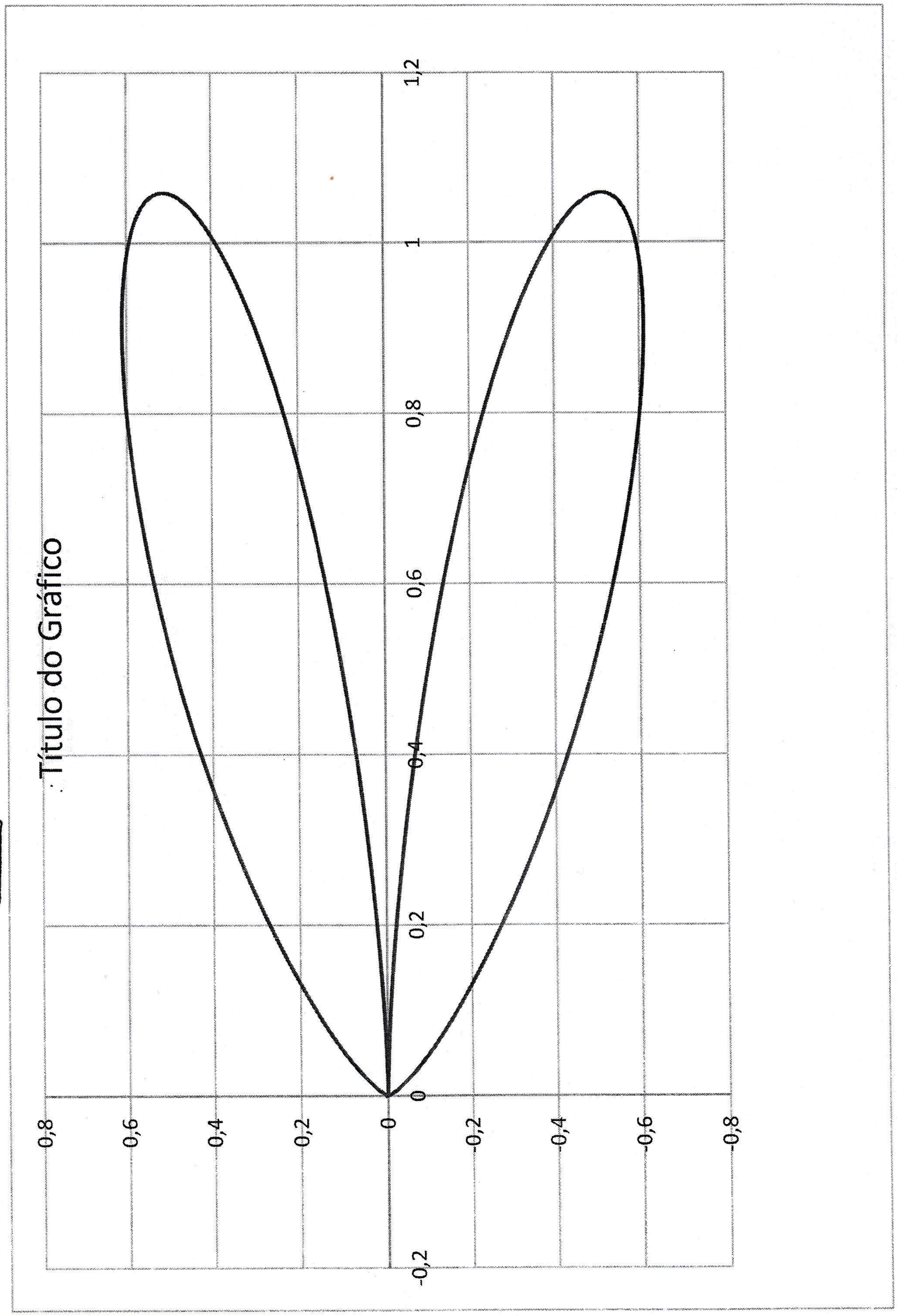
$E_2=k\cos\left(\omega\left(t-\frac{R}{c}-\frac{\lambda/4}{c}\cos\theta\right)-\frac{\pi}{2}\right)$ $\quad E_2=k\,e^{i\omega\left(t-\frac{R}{c}-\frac{\lambda/4}{c}\cos\theta\right)-\frac{\pi}{2}}$

$E_3=k\cos\left(\omega\left(t-\frac{R}{c}-\frac{2\lambda/4}{c}\cos\theta\right)-\pi\right)$ $\quad E_3=k\,e^{i\omega\left(t-\frac{R}{c}-\frac{2\lambda/4}{c}\cos\theta\right)-\pi}$

$E_4=k\cos\left(\omega\left(t-\frac{R}{c}-\frac{3\lambda/4}{c}\cos\theta\right)-3\pi\right)$ $\quad E_4=k\,e^{i\omega\left(t-\frac{R}{c}-\frac{3\lambda/4}{c}\cos\theta\right)-\frac{3\pi}{2}}$

e fazendo $\alpha=\omega\left(t-\frac{R}{c}\right)$ e $\beta=\omega\frac{\lambda/4}{c}\cos\theta+\frac{\pi}{2}=\frac{\omega\lambda}{4c}\cos\theta+\frac{\pi}{2}=\frac{\pi}{2}\cos\theta+\frac{\pi}{2}$

21.5



21,5 Contin. Contin. 21,5

Somando os campos complexos, tendo em conta que a solução é a parte real da solução, vem:

$$E_1+E_2+E_3+E_4 = K\left[e^{i\alpha}+e^{i(\alpha-\beta)}+e^{i(\alpha-2\beta)}+e^{i(\alpha-3\beta)}\right] = K e^{i\alpha}\left[1+e^{-i\beta}+e^{-i2\beta}+e^{-i3\beta}\right]$$

$$= K e^{i\alpha}\frac{e^{-i4\beta}-1}{e^{-i\beta}-1}$$

A intensidade é o quadrado do campo que pode ser obtida multiplicando a expressão anterior pelo seu conjugado e vem:

$$I = K^2 e^{i\alpha}\cdot e^{-i\alpha}\frac{e^{-i4\beta}-1}{e^{-i\beta}-1}\,\frac{e^{i4\beta}-1}{e^{i\beta}-1} = K^2\frac{1+1-(e^{-i4\beta}+e^{i4\beta})}{1+1-(e^{-i\beta}+e^{i\beta})} = K^2\frac{2-2\cos 4\beta}{2-2\cos\beta} =$$

$$= K^2\frac{1-\cos 4\beta}{1-\cos\beta} = K^2\frac{\cos^2 2\beta+\operatorname{sen}^2 2\beta-\cos^2 2\beta+\operatorname{sen}^2 2\beta}{\cos^2\frac{\beta}{2}+\operatorname{sen}^2\frac{\beta}{2}-\cos^2\frac{\beta}{2}+\operatorname{sen}^2\frac{\beta}{2}} = K^2\frac{2\operatorname{sen}^2 2\beta}{2\operatorname{sen}^2\frac{\beta}{2}} =$$

$$= K^2\frac{\operatorname{sen}^2 2\cdot\left(\frac{\pi}{2}\cos\theta+\frac{\pi}{2}\right)}{\operatorname{sen}^2\frac{1}{2}\left(\frac{\pi}{2}\cos\theta+\frac{\pi}{2}\right)} = K^2\cdot\frac{\operatorname{sen}^2\pi(1+\cos\theta)}{\operatorname{sen}^2\frac{\pi}{4}(1+\cos\theta)}$$

em que K^2 é a intensidade I_0 de um único emissor e vale $K^2\frac{1}{T}\int_0^T \cos\omega(t-\frac{R}{c})\,dt = \frac{1}{2}K^2$

pelo que $\frac{1}{2}K^2 = I_0$ donde $K=\sqrt{2I_0}$, e então $I = 2I_0\dfrac{\operatorname{sen}^2\pi(1+\cos\theta)}{\operatorname{sen}^2\frac{\pi}{4}(1+\cos\theta)}$

Nota: na solução em vez de θ está o complementar para $90°$ pelo que

$$I = 2I_0\frac{\operatorname{sen}^2\pi(1+\operatorname{sen}\theta')}{\operatorname{sen}^2\frac{\pi}{4}(1+\operatorname{sen}\theta')} \quad \text{com } \theta' = 90°-\theta$$

É interessante notar que para $\theta'=0°$ e $\theta'=90°$ a intensidade é nula.

A radiação máxima dá-se para um ângulo intermédio entre $0°$ e $90°$.

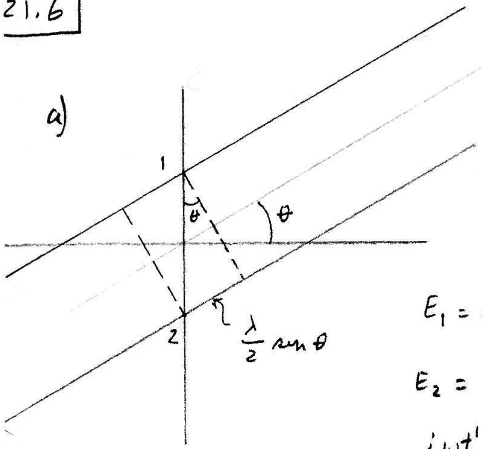
A análise com uso de computador permite verificar que o máximo de I se dá para $\theta=28°$ e vale $I = 2I_0\cdot 1.18396$.

Se $\theta<0$ então $I = 2I_0\dfrac{\operatorname{sen}^2\pi(-1+\operatorname{sen}\theta)}{\operatorname{sen}^2\frac{\pi}{4}(-1+\operatorname{sen}\theta)}$

21.6

21.6

a)



$$E_1 = k\cos\omega t'$$

$$E_2 = k\cos\omega\left(t' + \frac{\lambda/2\,\text{sen}\,\theta}{c}\right) = k\cos\left[\omega t' + \frac{\omega\lambda}{2c}\,\text{sen}\,\theta\right] = k\cos\left[\omega t' + \pi\,\text{sen}\,\theta\right]$$

$$E_1 = k e^{i\omega t'}$$

$$E_2 = k e^{i\omega t'} e^{+i\pi\,\text{sen}\,\theta} \qquad E_1 + E_2 = k e^{i\omega t'}\left[1 + e^{+i\pi\,\text{sen}\,\theta}\right]$$

$$I = k^2 e^{i\omega t'} e^{-i\omega t'}\left[\left(1 + e^{+i\pi\,\text{sen}\,\theta}\right)\left(1 + e^{-i\pi\,\text{sen}\,\theta}\right)\right] = k^2\left[2 + \underbrace{e^{-i\pi\,\text{sen}\,\theta} + e^{i\pi\,\text{sen}\,\theta}}_{\cos(\pi\,\text{sen}\,\theta)}\right]$$

$$I = k^2 \cdot 2\left[1 + \cos(\pi\,\text{sen}\,\theta)\right] = 2I_0\left[1 + \cos(\pi\,\text{sen}\,\theta)\right]$$

b) $E_1 = k\cos\omega t'$

$E_2 = k\cos\left[\omega t' + \pi\,\text{sen}\,\theta + \phi\right]$ e comparando com a alínea anterior vem:

$$I = 2I_0\left[1 + \cos\left(\phi + \pi\,\text{sen}\,\theta\right)\right]$$

Mas se $\theta = 210°$ então pretende-se que I seja máximo. Qual é o valor de ϕ que torna máximo I quando $\theta = 210°$? Derivando:

$$\frac{dI}{d\phi} = -2I_0\,\text{sen}(\phi + \pi\,\text{sen}\,\theta) = 0 \Rightarrow \phi + \pi\,\text{sen}\,210 = 0$$ o que dá: $\phi = -\pi\,\text{sen}\,210 = 1{,}57\,\text{rad} = \frac{\pi}{2}$

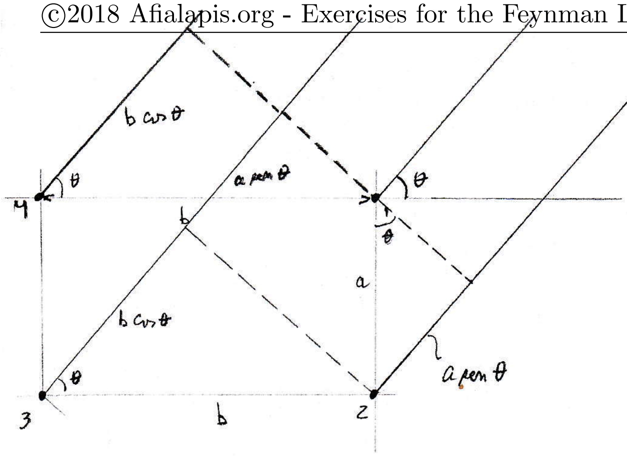
pelo que $I = 2I_0\left[1 + \cos\left(\frac{\pi}{2} + \pi\,\text{sen}\,\theta\right)\right]$

Não haverá sinal se $I = 0$ o que implica: $1 + \cos\left(\frac{\pi}{2} + \pi\,\text{sen}\,\theta\right) = 0$

$\cos\left(\frac{\pi}{2} + \pi\,\text{sen}\,\theta\right) = -1$; $\frac{\pi}{2} + \pi\,\text{sen}\,\theta = +\pi$; $\pi\,\text{sen}\,\theta = \frac{\pi}{2}$; $\text{sen}\,\theta = \frac{1}{2}$; $\theta = 30°$

Isto é na direcção $\theta = 30°$ o sinal que parte do emissor 2 está desfazado do sinal que parte de 1 de π. Há pois oposição de fase e a interferência é destrutiva e então não há sinal a grande distância nessa direcção,

A fase do emissor 2 é: $\frac{\omega\lambda}{2c}\,\text{sen}\,30 + \frac{\pi}{2} = \pi$ pois $\omega\lambda = 2\pi c$ e $\text{sen}\,30° = \frac{1}{2}$



$$\frac{\omega a}{c} = \frac{2\pi f a}{c} = \frac{2\pi}{\lambda} a$$

$$E_1 = k \cos \omega\left(t - \frac{R}{c}\right)$$

$$E_2 = k \cos\left(\omega\left(t - \frac{R}{c} - \frac{a \,\text{sen}\,\theta}{c}\right)\right) = k \cos\left(\omega\left(t - \frac{R}{c}\right) - \frac{\omega a \,\text{sen}\,\theta}{c}\right) = k \cos\left(\omega\left(t - \frac{R}{c}\right) - \frac{2\pi}{\lambda} a \,\text{sen}\,\theta\right)$$

$$E_3 = k \cos\left(\omega\left(t - \frac{R}{c} - \frac{b\cos\theta + a \,\text{sen}\,\theta}{c}\right)\right) = k \cos\left(\omega\left(t - \frac{R}{c}\right) - \frac{2\pi}{\lambda}\left(b \cos\theta + a \,\text{sen}\,\theta\right)\right)$$

$$E_4 = k \cos\left(\omega\left(t - \frac{R}{c} - \frac{b\cos\theta}{c}\right)\right) = k \cos\left(\omega\left(t - \frac{R}{c}\right) - \frac{2\pi}{\lambda} b \cos\theta\right)$$

No campo complexo e fazendo $\alpha = \omega\left(t - \frac{R}{c}\right)$; $\beta = \frac{2\pi}{\lambda} a \,\text{sen}\,\theta$ e $\gamma = \frac{2\pi}{\lambda} b \cos\theta$, vem:

$$E_1 = k e^{i\alpha}$$
$$E_2 = k e^{i\alpha} \cdot e^{-i\beta}$$
$$E_3 = k e^{i\alpha} \cdot e^{-i\gamma}$$
$$E_4 = k e^{i\alpha} \cdot e^{-i(\beta+\gamma)}$$

$$E_1 + E_2 + E_3 + E_4 = k e^{i\alpha}\left[1 + e^{-i\beta} + e^{-i\gamma} + e^{-i\beta} \cdot e^{-i\gamma}\right]$$

$$I = k^2 e^{i\alpha} e^{-i\alpha}\left[1 + e^{-i\beta} + e^{-i\gamma} + e^{-i\beta} \cdot e^{-i\gamma}\right]\left[1 + e^{i\beta} + e^{i\gamma} + e^{i\beta} e^{i\gamma}\right] =$$

$$= k^2\left[1 + e^{i\beta} + e^{i\gamma} + e^{i\beta} \cdot e^{i\gamma} + e^{-i\beta} + 1 + e^{-i\beta} \cdot e^{i\gamma} + e^{i\gamma} + e^{-i\gamma} + e^{-i\gamma} \cdot e^{i\beta} + 1 + e^{i\beta} + e^{-i\beta} e^{-i\gamma} + e^{-i\gamma} + e^{-i\beta} + 1\right] =$$

$$= k^2\left[4 + 2\left(e^{i\beta} + e^{-i\beta}\right) + 2\left(e^{i\gamma} + e^{-i\gamma}\right) + e^{i(\beta+\gamma)} + e^{-i(\beta+\gamma)} + e^{-i(\beta-\gamma)} + e^{i(\beta-\gamma)}\right] =$$

$$= k^2\left[4 + 4\cos\beta + 4\cos\gamma + 2\cos(\beta+\gamma) + 2\cos(\beta-\gamma)\right]$$

$\theta = 30^\circ$ pelo que $\beta = \frac{2\pi}{\lambda} a \,\text{sen}\,\theta = \frac{\pi}{\lambda} a$ $\quad \gamma = \frac{2\pi}{\lambda} b \cos\theta = \frac{2\pi}{\lambda} b \frac{\sqrt{3}}{2} = \frac{\pi\sqrt{3}}{\lambda} b$

$$I = 2k^2\left[2 + 2\cos\frac{\pi}{\lambda} a + 2\cos\frac{\pi\sqrt{3}}{\lambda} b + \cos\left(\frac{\pi}{\lambda}\left(a + \sqrt{3} b\right)\right) + \cos\left(\frac{\pi}{\lambda}\left(a - \sqrt{3} b\right)\right)\right]$$

Para I ser máximo então $\frac{\pi}{\lambda} a = 2\pi$ ou $a = 2\lambda$ e $\frac{\pi\sqrt{3} b}{\lambda} = 2\pi$ ou $b = \frac{2\lambda}{\sqrt{3}}$

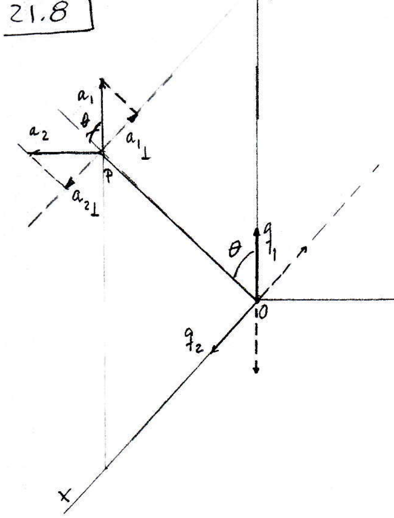
$\frac{\pi}{\lambda}\left(a + \sqrt{3} b\right) = \frac{\pi}{\lambda}\left(2\lambda + \frac{\sqrt{3} 2\lambda}{\sqrt{3}}\right) = \frac{\pi 2\lambda}{\lambda}\left(1 + \frac{\sqrt{3}}{\sqrt{3}}\right) = 2\pi(1+1) = 4\pi$ que dá $\cos\left(\frac{\pi}{\lambda}\left(a + \sqrt{3} b\right)\right) = 4$

$\frac{\pi}{\lambda}\left(a - \sqrt{3} b\right) = \ldots\ldots$ $\quad = 0$ " " $\cos 0 = 1$

assim: $I_{max}(\theta = 30^\circ) = 2k^2\left[2+2+2+1+1\right] = 16 \cdot k^2$ e $a = 2\lambda$

$b = \frac{2\lambda}{\sqrt{3}}$

21.8 | 21.8



a) $z = d\sin\omega t$; $\dot z = \omega d\cos\omega t$; $\ddot z = -\omega^2 d\,\text{sen}\,\omega t$

$x = d\sin\omega t$; $\dot x = \omega d\cos\omega t$; $\ddot x = -\omega^2 d\,\text{sen}\,\omega t$

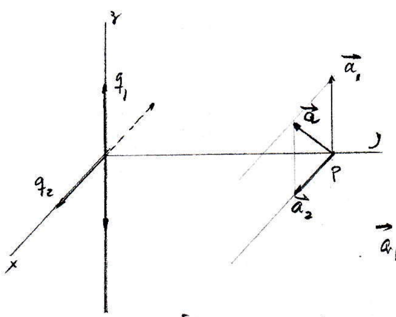
$a_{1\perp} = -\omega^2 d\,\text{sen}\,\omega t\,\text{sen}\,\theta$

$a_{2\perp} = -\omega^2 d\,\text{sen}\,\omega t\cos\theta$ e se $\theta = 45^\circ \Rightarrow |a_{\perp 1}| = |a_{\perp 2}|$

e têm sentidos opostos. Assim a componente perpendicular à linha de vista $\overline{OP}$ é nula e então o campo é nulo.

$\vec E = 0$

b)



$P = (0, R, 0)$

$Q_1 = (0, 0, d\sin\omega t)$

$P - Q_1 = (0, R, -d\sin\omega t)$

$\vec a_1 = \dfrac{d^2(P-Q_1)}{dt^2} = (0, 0, +\omega^2 d\sin\omega t)$

$P = (0, R, 0)$

$Q_2 = (d\sin\omega t, 0, 0)$

$P - Q_2 = (-d\sin\omega t, 0, 0)$

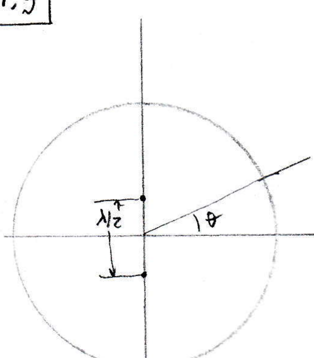
$\vec a_2 = \dfrac{d^2(P-Q_2)}{dt^2} = (\omega^2 d\sin\omega t, 0, 0)$

cuja soma é $\vec a = \vec a_1 + \vec a_2 = (\omega^2 d\sin\omega t, 0, \omega^2 d\sin\omega t) = \omega^2 d\sin\omega t\,(\hat i + \hat k)$

E o campo eléctrico vem: $\vec E = -\dfrac{q\omega^2 d}{4\pi\varepsilon_0 c^2 R}\sin\omega t'(\hat i + \hat k)$ ou, em módulo:

$$E = \frac{\sqrt{2}\,q\,\omega^2 d}{4\pi\varepsilon_0 c^2 R}\sin\omega\left(t - \frac{R}{c}\right)$$

21.9 | 21.9



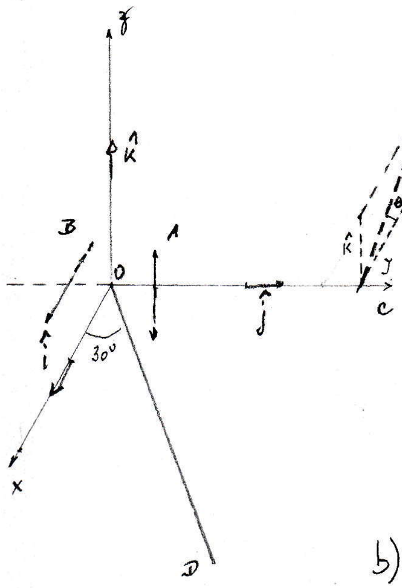
No problema 21.6 viu-se que a intensidade radiada na direcção θ em que os emissores emitem com diferença de fase ϕ, é dada por: $I = 2I_0[1 + \cos(\phi + \pi\,\text{sen}\,\theta)]$. O que se pretende é actuar em ϕ de forma a que $\phi + \pi\,\text{sen}\,\theta = c^{te}$, isto é, tal que $\phi = c^{te} - \pi\,\text{sen}\,\theta$. A variação de ϕ no tempo vem: $\dfrac{d\phi}{dt} = -\pi\cos\theta\dfrac{d\theta}{dt}$ rad s^{-1}

Qual o valor de θ? Perímetro por volta do helicóptero: $2\pi \cdot 2 = 4\pi$ mi

$\dfrac{d\theta}{dt} = \dfrac{120\ \text{mi}\ h^{-1}}{4\pi\ \text{mi}} \cdot 2\pi\ \text{rad} \cdot \dfrac{1}{3600\ s} = \dfrac{1}{60}\ \text{rad}\ s^{-1}$ mas 1 Hertz = 2π rad s^{-1} ou 1 rad s$^{-1} = \dfrac{1}{2\pi}$ Hz

$\dfrac{d\theta}{dt} = \dfrac{1}{60}\cdot\dfrac{1}{2\pi}$ Hz e $\dfrac{d\phi}{dt} = -\pi\dfrac{1}{120\pi}\cos\theta$ Hz $= -\dfrac{1}{120}\cos\theta$ [Hz]

21,10



a) em E:

Componente $\perp$ à direcção do eixo dos xx devido ao dipolo B: 0

" " " " " " " " " A:

$$E_{A_\perp} = K \cos \omega\left(t - \frac{R}{c}\right)$$

A intensidade média é pois: $I = K^2 \frac{1}{T}\int_0^T \cos^2 \omega\left(t - \frac{R}{c}\right) dt = \frac{1}{2}K^2 = I_A$

e a direcção do campo é $\perp$ ao plano xy

b) Em C:

componente $\perp$ a Oy devido ao dipolo A: $K \cos \omega\left(t - \frac{R}{c}\right)\hat{k} = \vec{E}_A$

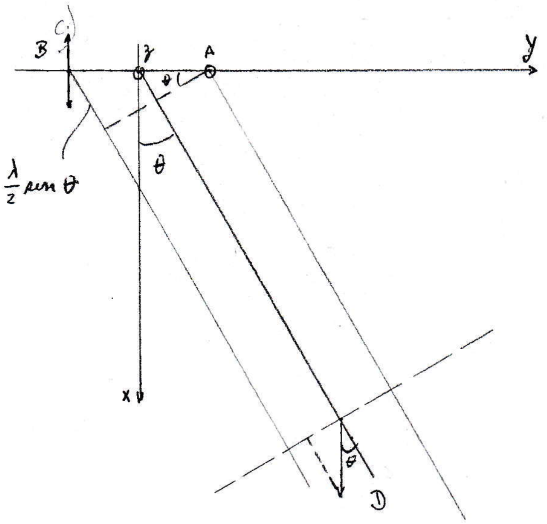
Componente $\perp$ a Oy devido ao dipolo B: $\vec{E}_B = 2K \cos \omega\left(t - \frac{R}{c} - \frac{\lambda}{2c}\right)\hat{i} = 2K\left[\cos \omega\left(t - \frac{R}{c}\right) - \pi\right]\hat{i}$

$$\vec{E}_B = -2K \cos \omega\left(t - \frac{R}{c}\right)\hat{i}$$

Campo total $\vec{E}_A + \vec{E}_B = K \cos \omega\left(t - \frac{R}{c}\right)\left[-2\hat{i} + \hat{k}\right]$

Intensidade média $= K^2\left(2^2 + 1^2\right)\frac{1}{T}\int_0^T \cos^2 \omega\left(t - \frac{R}{c}\right)dt = \frac{5K^2}{2} = 5\, I_A$

Direcção: $\operatorname{tg}\theta = \frac{1}{2}$ donde $\theta = 26{,}56^0$ e é paralela ao plano xz e faz este ângulo com o plano xy



$\vec{E}_A = K \cos \omega t' \hat{k}$ com $t' = t - \frac{R}{c}$

$E_B = 2K\cos\left(\omega\left(t' - \frac{\lambda}{2c}\operatorname{sen}\theta\right)\right) = 2K\cos\left(\omega t' - \pi\frac{1}{2}\right) = 2K\cos\left(\omega t' - \frac{\pi}{2}\right)$

$= -2K \operatorname{sen} \omega t'$

Componente $\perp$ de E_B à direcção da recta D:

$E_B = -2K \operatorname{sen} \omega t' \operatorname{sen}\theta$

Vector do campo: Versor da direcção de D: $\vec{n} = \cos\theta\,\hat{i} + \operatorname{sen}\theta\,\hat{j}$

Versor da direcção $\perp$ a D: $\hat{p} = -\operatorname{sen}\theta\,\hat{i} + \cos\theta\,\hat{j}$

$\vec{E}_B = -2K \operatorname{sen}\omega t' \operatorname{sen}\theta\left[-\operatorname{sen}\theta\,\hat{i} + \cos\theta\,\hat{j}\right]$

Campo total: $\vec{E}_A + \vec{E}_B = +2K\operatorname{sen}\omega t' \operatorname{sen}^2\theta\,\hat{i} - 2K\operatorname{sen}\omega t'\operatorname{sen}\theta\cos\theta\,\hat{j} + K\cos\omega t'\,\hat{k}$ e, substituindo os senos e cosenos de θ, vem:

$\vec{E}_A + \vec{E}_B = \frac{K}{2}\operatorname{sen}\omega t'\,\hat{i} - \frac{K\sqrt{3}}{2}\operatorname{sen}\omega t'\,\hat{j} + K\cos\omega t'\,\hat{k}$ e a intensidade é o quadrado do módulo deste vector vindo:

$$I = \frac{K^2}{4}\,\text{sen}^2\omega t' + \frac{K^2 3}{4}\,\text{sen}^2\omega t' + K^2\cos^2\omega t' = \left(\frac{K^2}{4}+\frac{K^2\cdot 3}{4}\right)\text{sen}^2\omega t' + K^2\cos^2\omega t' =$$

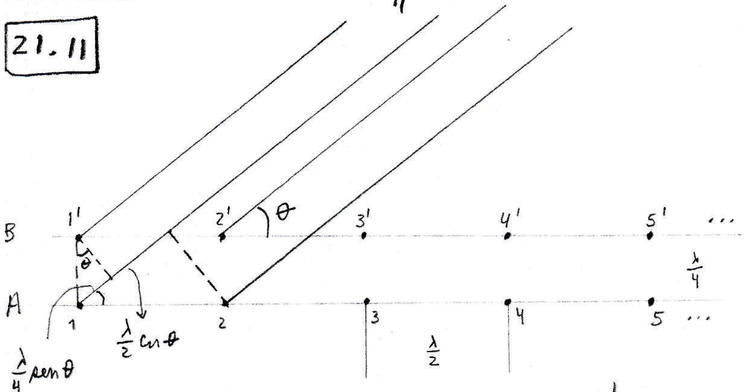
$$= K^2\,\text{sen}^2\omega t' + K^2\cos^2\omega t' = K^2 = 2\,I_A$$

Qual a direcção do campo?

$\vec{E}_A+\vec{E}_B = K\,\text{sen}\,\omega t'\left[\frac{1}{2}\hat{i} - \frac{\sqrt{3}}{2}\hat{j}\right] + K\cos\omega t'\,\hat{k}$ e vê-se que o campo descreve um círculo no plano ⊥ a xy e que o intersecta segundo o vector $\frac{1}{2}i - \frac{\sqrt{3}}{2}$. No plano definido por este vector e o versor $\hat{k}$, o campo descreve uma circunferência de raio K, como se pode ver calculando o módulo de $\vec{E}_A+\vec{E}_B$. A polarização é pois circular.

21.11 21.11

Ver!



$E_1 = K_1\cos\omega t' = K_1\cos\omega t'$; $E_{1'} = K_1\cos\left(\omega\left(t+\frac{\lambda}{4c}\,\text{sen}\,\theta\right)-\frac{\pi}{2}\right) = K_1\cos\left(\omega t' + \frac{\pi}{2}\,\text{sen}\,\theta - \frac{\pi}{2}\right)$

$E_2 = K_1\cos\omega\left(t'-\frac{\lambda}{2c}\cos\theta\right) = K_1\cos\left(\omega t' - \pi\cos\theta\right)$; $E_{2'} = K_1\cos\left(\omega\left(t'-\frac{\lambda}{4c}\,\text{sen}\,\theta\right) - \frac{\pi}{2} - \pi\cos\theta\right)$

$E_3 = K_1\cos\omega\left(t' - 2\frac{\lambda}{2c}\cos\theta\right) = K_1\cos\left(\omega t' - 2\pi\cos\theta\right)$; $E_{3'} = K_1\cos\left(\omega\left(t'-\frac{\lambda}{4c}\,\text{sen}\,\theta\right) - \frac{\pi}{2} - 2\pi\cos\theta\right)$

$\vdots$

$E_N = K_1\cos\omega\left(t'-(N-1)\frac{\lambda}{2c}\cos\theta\right) = K_1\cos\left(\omega t' - (N-1)\pi\cos\theta\right)$; $E_{N'} = K_1\cos\left(\omega\left(t'-\frac{\lambda}{4c}\,\text{sen}\,\theta\right) - \frac{\pi}{2} - (N-1)\pi\cos\theta\right)$

$E_1+E_{1'} = K_1e^{i\alpha} + K_1e^{i\alpha}e^{-i\left(\frac{\pi}{2}+\frac{\pi}{2}\text{sen}\,\theta\right)} = K_1e^{i\alpha}\left[1+e^{-i\left[\frac{\pi}{2}+\frac{\pi}{2}\text{sen}\,\theta\right]}\right]$

$E_2+E_{2'} = K_1e^{i\alpha}e^{-i\pi\cos\theta} + K_1e^{i\alpha}e^{-i\left(\frac{\pi}{2}+\frac{\pi}{2}\text{sen}\,\theta\right)}e^{-i\pi\cos\theta} = K_1e^{i\alpha}e^{-i\pi\cos\theta}\left[1+e^{-i\left(\frac{\pi}{2}+\frac{\pi}{2}\text{sen}\,\theta\right)}\right]$

$E_N+E_{N'} = K_1e^{i\alpha}e^{-i(N-1)\pi\cos\theta} + K_1e^{i\alpha}e^{-i(N-1)\pi\cos\theta}e^{-i\left(\frac{\pi}{2}+\frac{\pi}{2}\text{sen}\,\theta\right)} = K_1e^{i\alpha}e^{-i(N-1)\pi\cos\theta}\left[1+e^{-i\left(\frac{\pi}{2}+\frac{\pi}{2}\text{sen}\,\theta\right)}\right]$

Campo total: $E+E = K_1e^{i\alpha}\left[1+e^{-i\left(\frac{\pi}{2}+\frac{\pi}{2}\text{sen}\,\theta\right)}\right]\sum_{N=1}^{N} e^{-i(n-1)\pi\cos\theta} =$

$$= K_1e^{i\alpha}\left[\left[1+e^{-i\left(\frac{\pi}{2}+\frac{\pi}{2}\text{sen}\,\theta\right)}\right]\frac{1-e^{iN\pi\cos\theta}}{1-e^{i\pi\cos\theta}}\right]$$

A intensidade é:

$$I = K_1^2\left[1+e^{-i\left(\frac{\pi}{2}+\frac{\pi}{2}\text{sen}\,\theta\right)}\right]\left[1+e^{i\left(\frac{\pi}{2}+\frac{\pi}{2}\text{sen}\,\theta\right)}\right]\frac{1-e^{iN\pi\cos\theta}}{1-e^{i\pi\cos\theta}}\,\frac{1-e^{-iN\pi\cos\theta}}{1-e^{-i\pi\cos\theta}} =$$

21.11 Contin. | Contin. 21.11

$$I = k_1^2\left[2 + e^{-i\left(\frac{\pi}{2}+\frac{\pi}{2}\operatorname{sen}\theta\right)} + e^{i\left(\frac{\pi}{2}+\frac{\pi}{2}\operatorname{sen}\theta\right)}\right]\frac{2-\left(e^{iN\pi\cos\theta}+e^{-iN\pi\cos\theta}\right)}{2-\left(e^{i\pi\cos\theta}+e^{-i\pi\cos\theta}\right)} =$$

$$= k_1^2\left[2+2\cos\left(\frac{\pi}{2}+\frac{\pi}{2}\operatorname{sen}\theta\right)\right]\frac{2-2\cos N\pi\cos\theta}{2-2\cos\pi\cos\theta} = k_1^2\cdot 2\left(1+\cos\left(\frac{\pi}{2}+\frac{\pi}{2}\operatorname{sen}\theta\right)\right)\frac{1-\cos(N\pi\cos\theta)}{1-\cos(\pi\cos\theta)} =$$

$$= k_1^2\cdot 2\cdot\left(1+\cos\left(\frac{\pi}{2}(1+\operatorname{sen}\theta)\right)\right)\frac{\cos^2\frac{N\pi\cos\theta}{2}+\operatorname{sen}^2\frac{N\pi\cos\theta}{2}-\cos^2\frac{N\pi\cos\theta}{2}+\operatorname{sen}^2\frac{N\pi\cos\theta}{2}}{\cos^2\frac{\pi\cos\theta}{2}+\operatorname{sen}^2\frac{\pi\cos\theta}{2}-\cos^2\frac{\pi\cos\theta}{2}+\operatorname{sen}^2\frac{\pi\cos\theta}{2}} =$$

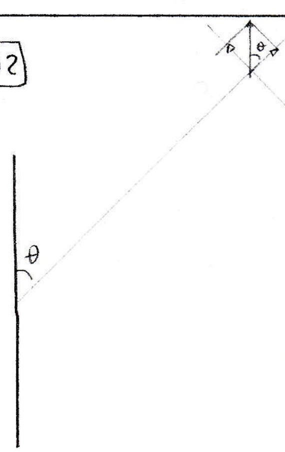
$$= 2k_1^2\left(1+\cos\left(\frac{\pi}{2}(1+\operatorname{sen}\theta)\right)\right)\frac{\operatorname{sen}^2\frac{N\pi\cos\theta}{2}}{\operatorname{sen}^2\frac{\pi\cos\theta}{2}}$$

Na solução do livro foi tomado o ângulo complementar para 90° e vem assim

$$I(\theta) = 2k_1^2\left(1+\cos\left(\frac{\pi}{2}(1+\cos\theta')\right)\right)\frac{\operatorname{sen}^2\frac{N\pi\operatorname{sen}\theta'}{2}}{\operatorname{sen}^2\frac{\pi\cos\theta'}{2}}$$ pois que $\cos\theta = \cos(90-\theta') = \operatorname{sen}\theta'$

$\cos\left(-\frac{\pi}{2}+\frac{\pi}{2}\cos\theta\right) = \cos\left(-\frac{\pi}{2}(1-\cos\theta)\right) = \cos\left(\frac{\pi}{2}(1-\cos\theta)\right)$. Se o dipólo de B estiver em avanço de fase de 90° em relação ao dipólo de A então na expressão aparece $1+\cos\left(\frac{\pi}{2}(1-\cos\theta)\right)$ em vez de $1+\cos\left(\frac{\pi}{2}(1+\cos\theta)\right)$

21.12 | 21.12



$z = a\cos\omega t$; $\dot{z} = -a\omega\operatorname{sen}\omega t$; $\ddot{z} = -a\omega^2\cos\omega t$

$\ddot{z}_\perp = -a\omega^2\cos\omega\left(t-\frac{R}{c}\right)\operatorname{sen}\theta$

$$E = \frac{q a\omega^2}{4\pi\varepsilon_0 c^2 R}\cos\omega\left(t-\frac{R}{c}\right)\operatorname{sen}\theta$$