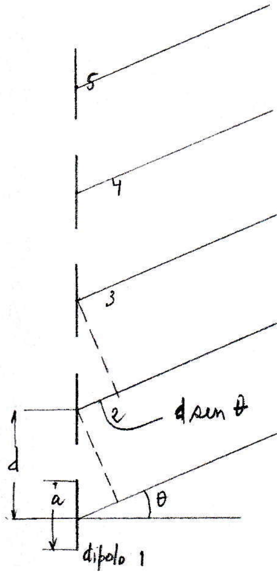


Campo criado a grande distância pelo dipolo atómico situado à distância x do centro O

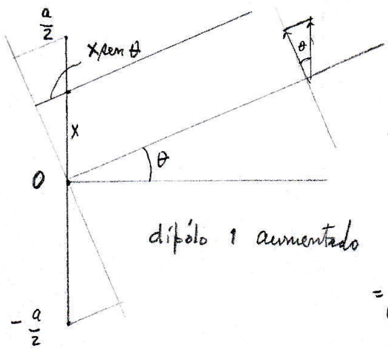
$$E_x = + \frac{qA\omega^2}{4\pi\varepsilon_0 c^2 R}\cos\theta \cdot \cos\left(\omega\left(t-\frac{R}{c}\right) - \frac{x\,\mathrm{sen}\,\theta}{c}\right) \qquad \begin{array}{l} x = A\cos\omega t \\ \dot{x} = -A\omega\,\mathrm{sen}\,\omega t \\ \ddot{x} = -A\omega^2\cos\omega t \end{array}$$

e, no campo complexo, vem:

$$E_x = \frac{qA\omega^2}{4\pi\varepsilon_0 c^2 R}\cdot\cos\theta\cdot e^{i\omega\left(t-\frac{R}{c}\right)}\cdot e^{-i\frac{2\pi}{\lambda}x\,\mathrm{sen}\,\theta} \qquad \text{pois } \frac{\omega}{c}x\,\mathrm{sen}\,\theta = \frac{2\pi}{\lambda}x\,\mathrm{sen}\,\theta$$

Campo total criado pelo dipolo 1:

$$E_1 = \frac{qA\omega^2}{4\pi\varepsilon_0 c^2 R}\cdot\cos\theta\, e^{i\left(\omega\left(t-\frac{R}{c}\right)\right)}\int_{-\frac{a}{2}}^{\frac{a}{2}} e^{-i\frac{2\pi}{\lambda}x\,\mathrm{sen}\,\theta}\,dx =$$



$$= \frac{qA\omega^2}{4\pi\varepsilon_0 c^2 R}\cos\theta\, e^{i\left(\omega\left(t-\frac{R}{c}\right)\right)}\left[\frac{1}{-i\frac{2\pi}{\lambda}\mathrm{sen}\,\theta}\, e^{-i\frac{2\pi}{\lambda}x\,\mathrm{sen}\,\theta}\Big|_{-\frac{a}{2}}^{\frac{a}{2}}\right] =$$

$$= \frac{qA\omega^2}{4\pi\varepsilon_0 c^2 R}\cos\theta\cdot e^{i\omega\left(t-\frac{R}{c}\right)}\left[\frac{1}{-i\frac{2\pi}{\lambda}\mathrm{sen}\,\theta}\underbrace{\left(e^{-i\frac{2\pi}{\lambda}\frac{a}{2}\mathrm{sen}\,\theta} - e^{i\frac{2\pi}{\lambda}\frac{a}{2}\mathrm{sen}\,\theta}\right)}_{-2i\,\mathrm{sen}\,\frac{2\pi}{\lambda}\frac{a}{2}\mathrm{sen}\,\theta}\right] =$$

$$= \frac{qA\omega^2}{4\pi\varepsilon_0 c^2 R}\cos\theta\, e^{i\omega\left(t-\frac{R}{c}\right)}\frac{1}{-i\frac{2\pi}{\lambda}\mathrm{sen}\,\theta}\cdot\left(-2i\,\mathrm{sen}\,\frac{2\pi}{\lambda}\frac{a}{2}\mathrm{sen}\,\theta\right) =$$

$$= \frac{qA\omega^2}{4\pi\varepsilon_0 c^2 R}\cos\theta\, e^{i\omega\left(t-\frac{R}{c}\right)}\frac{\frac{a}{2}}{\frac{2\pi}{\lambda}\mathrm{sen}\,\theta\cdot\frac{a}{2}}\,2\,\mathrm{sen}\,\frac{2\pi}{\lambda}\frac{a}{2}\mathrm{sen}\,\theta = \frac{qA\omega^2}{4\pi\varepsilon_0 c^2 R}\cos\theta\cdot e^{i\omega\left(t-\frac{R}{c}\right)}\cdot a\,\frac{\mathrm{sen}\,\frac{\beta}{2}}{\frac{\beta}{2}} \qquad \text{com } \beta = \frac{2\pi}{\lambda}a\,\mathrm{sin}\,\theta$$

O campo criado pelo dipólo 2 é igual ao criado pelo dipólo 1 com uma diferença na fase $\phi = \frac{\omega}{c}d\,\mathrm{sen}\,\theta + \alpha = \frac{2\pi}{\lambda}d\,\mathrm{sen}\,\theta + \alpha$, pelo que deve multiplicar-se a expressão anterior por $e^{i\phi}$. O dipólo 3 cria um campo igual ao dipólo 1 multiplicado por $e^{i2\phi}$ e assim sucessivamente até ao dipolo n cujo novo factor é $e^{i(n-1)\phi}$.

O campo total é pois a soma: $E = \sum_{i=1}^{n} E_i$ e vem:

$$\sum_{i=1}^{n} E_i = \frac{qA\omega^2}{4\pi\varepsilon_0 c^2 R}\cos\theta\, e^{i\left(\omega\left(t-\frac{R}{c}\right)\right)} a\cdot\frac{\mathrm{sen}\,\frac{\beta}{2}}{\frac{\beta}{2}}\underbrace{\left[1 + e^{i\phi} + e^{i2\phi} + \dots + e^{i(n-1)\phi}\right]}_{S} =$$

$$= \frac{qA\omega^2}{4\pi\varepsilon_0 c^2 R}\cdot\cos\theta\cdot e^{i\omega\left(t-\frac{R}{c}\right)}\cdot a\cdot\frac{\mathrm{sen}\,\frac{\beta}{2}}{\frac{\beta}{2}}\,\frac{1-e^{+in\phi}}{1-e^{+i\phi}} = K\cdot\cos\theta\cdot\frac{\mathrm{sen}\,\frac{\beta}{2}}{\frac{\beta}{2}}\, e^{i\omega\left(t-\frac{R}{c}\right)}\,\frac{1-e^{in\phi}}{1-e^{i\phi}}$$

A intensidade é o quadrado do campo e pode ser obtida multiplicando o campo pelo seu conjugado vindo assim:

22.1 Contin. Contin. 22.1

$$I = K^2 \cos^2\theta \left(\frac{\operatorname{sen}\frac{\beta}{2}}{\frac{\beta}{2}}\right)^2 \frac{1-e^{in\phi}}{1-e^{i\phi}}\,\frac{1-e^{-in\phi}}{1-e^{-i\phi}} = K^2\cos^2\theta\left(\frac{\operatorname{sen}\frac{\beta}{2}}{\frac{\beta}{2}}\right)^2 \frac{2-(e^{in\phi}+e^{-in\phi})}{2-(e^{i\phi}+e^{-i\phi})} =$$

$$= K^2\cos^2\theta\left(\frac{\operatorname{sen}\frac{\beta}{2}}{\frac{\beta}{2}}\right)^2 \frac{1-\cos n\phi}{1-\cos\phi} = K^2\cos^2\theta\left(\frac{\operatorname{sen}\frac{\beta}{2}}{\frac{\beta}{2}}\right)^2 \frac{2\operatorname{sen}^2\frac{n\phi}{2}}{2\operatorname{sen}^2\frac{\phi}{2}}$$

e assim: $I = K^2\cos^2\theta\left(\frac{\operatorname{sen}\frac{\beta}{2}}{\frac{\beta}{2}}\right)^2 \frac{\operatorname{sen}^2\frac{n\phi}{2}}{\operatorname{sen}^2\frac{\phi}{2}}$ em que:

$\beta = \frac{2\pi}{\lambda} a \operatorname{sen}\theta$ e $\phi = \alpha + \frac{2\pi}{\lambda} d \operatorname{sen}\theta$

O factor $\cos^2\theta$ resulta de se considerar a componente perpendicular à linha de vista do campo criado a grande distância. Ele não figura na solução mas deveria figurar.

22.2 22.2

$$Q = \frac{\omega}{\gamma} = \frac{5500}{10} = 5,5\cdot 10^2$$

Nota: na solução o expoente é 6.

22.3 22.3

22.4 22.4

$$\theta = 1,22\,\frac{\lambda}{L}$$ com $\lambda = 5500\,\text{Å} = 5500\cdot10^{-10}\,\text{m} = 5,5\cdot10^{-7}\,\text{m}$

$L = 5\cdot10^{-3}\,\text{m}$

$$\theta_{min} = 1,22\,\frac{5,5\cdot10^{-7}}{5\cdot10^{-3}}\text{ radianos} = 1,342\cdot10^{-4}\text{ radianos}$$

$$\operatorname{tg}\theta = \frac{d}{D} \qquad D = \frac{d}{\operatorname{tg}\theta} \simeq \frac{d}{\theta} = \frac{1,2}{1,342\cdot10^{-4}} = 8,94\text{ Km}$$

22.5 **22.5**

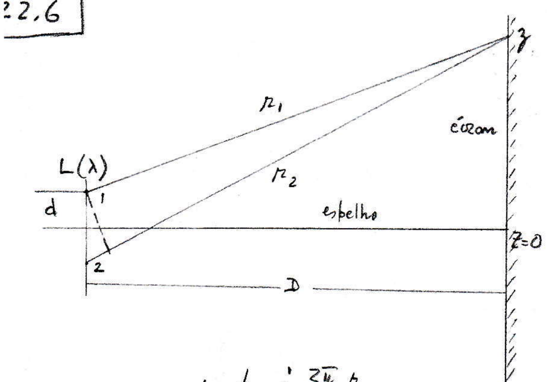
$\lambda_1 = 5889{,}95\ \mathring{A}$ e $5895{,}92\ \mathring{A}$

Sabe-se (pág. 30-6) que $\dfrac{\Delta\lambda}{\lambda} = \dfrac{1}{mn}$ m: ordem da risca; n: nº de linhas da rede

Então $\dfrac{5895{,}92 - 5889{,}95}{5895{,}92} = \dfrac{1}{n}$ donde $n = 988$ linhas

Se a rede tem 600 linhas por mm então a rede tem de ter $\dfrac{988}{600}\,\text{mm} = 1{,}64\ \text{mm}$

22.6 **22.6**



$r_1 = \sqrt{(z-d)^2 + D^2}$ $r_2 = \sqrt{(z+d)^2 + D^2}$

$E = -\dfrac{q}{4\pi\varepsilon_0 c^2}\dfrac{1}{R}\, a_\perp\!\left(t - \dfrac{R}{c}\right)$ em geral. Neste caso:

$E_1 = \dfrac{K}{r_1}\cos\omega\left(t - \dfrac{r_1}{c}\right) \rightsquigarrow \bar{E}_1 = \dfrac{K}{r_1} e^{i\omega\left(t - \frac{r_1}{c}\right)}$

$E_2 = \dfrac{K}{r_2}\cos\left[\omega\left(t - \dfrac{r_2}{c}\right) - \pi\right] \rightsquigarrow \bar{E}_2 = \dfrac{K}{r_2} e^{i\left[\omega\left(t - \frac{r_2}{c}\right) - \pi\right]}$

$\bar{E}_1 = \dfrac{K}{r_1} e^{i\omega t} e^{-i\frac{2\pi}{\lambda}r_1}$ $\bar{E}_2 = \dfrac{K}{r_2} e^{i\omega t} e^{-i\frac{2\pi}{\lambda}r_2} e^{-i\pi}$

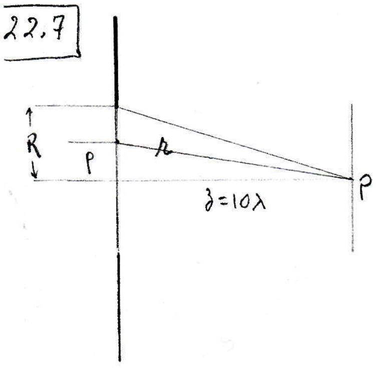
$\bar{E}_1 + \bar{E}_2 = K e^{i\omega t}\left[\dfrac{1}{r_1} e^{-i\frac{2\pi}{\lambda}r_1} - \dfrac{1}{r_2} e^{-i\frac{2\pi}{\lambda}r_2}\right]$ e então vem para a intensidade:

$$I(z) = K^2 e^{i\omega t} e^{-i\omega t}\left[\frac{1}{r_1} e^{-i\frac{2\pi}{\lambda}r_1} - \frac{1}{r_2} e^{-i\frac{2\pi}{\lambda}r_2}\right]\left[\frac{1}{r_1} e^{+i\frac{2\pi}{\lambda}r_1} - \frac{1}{r_2} e^{+i\frac{2\pi}{\lambda}r_2}\right] =$$

$$= K^2\left[\frac{1}{r_1^2} + \frac{1}{r_2^2} - \frac{1}{r_1 r_2} e^{-i\frac{2\pi}{\lambda}(r_1 - r_2)} - \frac{1}{r_1 r_2} e^{+i\frac{2\pi}{\lambda}(r_1 - r_2)}\right] = K^2\left[\frac{1}{r_1^2} + \frac{1}{r_2^2} - \frac{1}{r_1 r_2}\left(e^{-i\frac{2\pi}{\lambda}(r_1 - r_2)} + e^{i\frac{2\pi}{\lambda}(r_1 - r_2)}\right)\right] =$$

$$= K^2\left[\frac{1}{r_1^2} + \frac{1}{r_2^2} - \frac{1}{r_1 r_2}\cdot 2\cdot\cos\frac{2\pi}{\lambda}(r_1 - r_2)\right]$$

22.7 **22.7**



Campo total em P (pág. 30.10, FLP):

$$E_P = \int_0^R \frac{q}{4\pi\varepsilon_0 c^2}\,\frac{\omega^2 x_0\, e^{i\omega\left(t - \frac{r}{c}\right)}}{r}\,\eta\, 2\pi\rho\, d\rho =$$

$$= \frac{q\,\omega^2 x_0\, \eta\, \cancel{2}\cancel{\pi}}{2\cancel{4}\cancel{\pi}\varepsilon_0 c^2}\int_z^{\sqrt{R^2+z^2}} e^{i\omega\left(t - \frac{r}{c}\right)}\, dr$$ pois $\rho^2 + z^2 = r^2$; $2\rho\, d\rho = 2r\, dr$

22.7 Contin. | Contin 22.7

$$E_p = \underbrace{\frac{q\omega_0^2 x_0 \eta}{2\varepsilon_0 c^2}}_{A} e^{i\omega t}\int_z^{\sqrt{R^2+z^2}} e^{-i\frac{2\pi}{\lambda}r}\,dr = A\cdot e^{i\omega t}\frac{1}{-i\frac{2\pi}{\lambda}}\left(e^{-i\frac{2\pi}{\lambda}r}\Big|_z^{\sqrt{R^2+z^2}}\right) =$$

$$= Ae^{i\omega t}\cdot i\frac{\lambda}{2\pi}\left[e^{-i\frac{2\pi}{\lambda}\sqrt{R^2+z^2}} - e^{-i\frac{2\pi}{\lambda}z}\right]$$ pelo que a intensidade virá:

$$I = A^2\left(\frac{\lambda}{2\pi}\right)^2\left[e^{-i\frac{2\pi}{\lambda}\sqrt{R^2+z^2}} - e^{-i\frac{2\pi}{\lambda}z}\right]\left[e^{i\frac{2\pi}{\lambda}\sqrt{R^2+z^2}} - e^{i\frac{2\pi}{\lambda}z}\right] =$$

$$= A^2\left(\frac{\lambda}{2\pi}\right)^2\left[1+1-e^{-i\frac{2\pi}{\lambda}\left[\sqrt{R^2+z^2}-z\right]} - e^{i\frac{2\pi}{\lambda}\left[\sqrt{R^2+z^2}-z\right]}\right] =$$

$$= A^2\left(\frac{\lambda}{2\pi}\right)^2\left[2-2\cos\left[\frac{2\pi}{\lambda}\left(\sqrt{R^2+z^2}-z\right)\right]\right] = 2A^2\left(\frac{\lambda}{2\pi}\right)^2\left[1-\cos\left[\frac{2\pi}{\lambda}\left(\sqrt{R^2+z^2}-z\right)\right]\right]$$

O I_{MAX} dá-se se $\frac{2\pi}{\lambda}\left(\sqrt{R^2+z^2}-z\right) = \pi$; $\sqrt{R^2+z^2}-z = \frac{\lambda}{2}$; $\sqrt{R^2+z^2} = z+\frac{\lambda}{2}$

$R^2+z^2 = z^2+\lambda z+\left(\frac{\lambda}{2}\right)^2$; $R^2 = \lambda z + \left(\frac{\lambda}{2}\right)^2$; $R^2 = \lambda\cdot 10\lambda + \frac{\lambda^2}{4} = 10.25\,\lambda^2$;

pelo que $R = \sqrt{10.25}\,\lambda$ para se obter uma intensidade máxima em P.

b) Na expressão do campo E_p, se $R\to\infty$ que corresponde à remoção do écran então E_p vem igual a: $E_p = Ae^{i\omega t}\,i\frac{\lambda}{2\pi}\left[-e^{-i\frac{2\pi}{\lambda}z}\right]$ e a intensidade $I_{R=\infty} = A^2\left(\frac{\lambda}{2\pi}\right)^2\left[\left(-e^{-i\frac{2\pi}{\lambda}z}\right)\left(-e^{i\frac{2\pi}{\lambda}z}\right)\right] = A^2\left(\frac{\lambda}{2\pi}\right)^2\cdot 1$

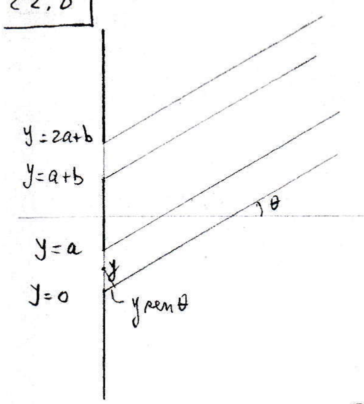
Por outro lado a intensidade máxima ocorre para $R = \sqrt{10.25}\,\lambda$ e vale:

$$I_{MAX} = 2A^2\left(\frac{\lambda}{2\pi}\right)^2\cdot\left[1-(-1)\right] = 4A^2\left(\frac{\lambda}{2\pi}\right)^2$$ e então:

$$I_{R=\infty} = \frac{1}{4}I_{MAX}$$

22.8

22.8



$$E=\int_0^a K\, e^{i\left(\omega\left(t-\frac{R}{c}+\frac{y\,\mathrm{sen}\,\theta}{c}\right)\right)}dy+\int_{a+b}^{2a+b} K\, e^{i\left(\omega\left(t-\frac{R}{c}\right)+\frac{y\,\mathrm{sen}\,\theta}{c}\right)}dy$$

$$E=K e^{i\alpha}\int_0^a e^{i\frac{2\pi}{\lambda}\mathrm{sen}\,\theta\, y}\,dy+K e^{i\alpha}\int_{a+b}^{2a+b} e^{i\frac{2\pi}{\lambda}\mathrm{sen}\,\theta\, y}\,dy=$$

$$=K e^{i\alpha}\left[\frac{1}{i\frac{2\pi}{\lambda}\mathrm{sen}\,\theta}\left(e^{i\frac{2\pi}{\lambda}\mathrm{sen}\,\theta\, y}\Big|_0^a\right)+\frac{1}{i\frac{2\pi}{\lambda}\mathrm{sen}\,\theta}\left(e^{iy\frac{2\pi}{\lambda}\mathrm{sen}\,\theta}\Big|_{a+b}^{2a+b}\right)\right]=\qquad \text{com } \alpha=\omega\left(t-\frac{R}{c}\right)$$

$$=K e^{i\alpha}\frac{1}{i\frac{2\pi}{\lambda}\mathrm{sen}\,\theta}\left[e^{i\frac{2\pi}{\lambda}a\,\mathrm{sen}\,\theta}-1+e^{i\frac{2\pi}{\lambda}\mathrm{sen}\,\theta\,(2a+b)}-e^{i\frac{2\pi}{\lambda}\mathrm{sen}\,\theta\,(a+b)}\right]=$$

$$=K e^{i\alpha}\frac{-\lambda i}{2\pi\,\mathrm{sen}\,\theta}\left[e^{i\frac{2\pi}{\lambda}a\,\mathrm{sen}\,\theta}-1+e^{i\frac{2\pi}{\lambda}a\,\mathrm{sen}\,\theta}\cdot e^{i\frac{2\pi}{\lambda}(a+b)\,\mathrm{sen}\,\theta}-e^{i\frac{2\pi}{\lambda}\mathrm{sen}\,\theta\, a}\cdot e^{i\frac{2\pi}{\lambda}\mathrm{sen}\,\theta\, b}\right]=$$

$$=K e^{i\alpha}\frac{-i\lambda}{2\pi\,\mathrm{sen}\,\theta}\left[e^{i\frac{2\pi}{\lambda}a\,\mathrm{sen}\,\theta}-1+e^{i\frac{2\pi}{\lambda}(a+b)\,\mathrm{sen}\,\theta}\left(e^{i\frac{2\pi}{\lambda}a\,\mathrm{sen}\,\theta}-1\right)\right]=$$

$$=K e^{i\alpha}\frac{-i\lambda}{2\pi\,\mathrm{sen}\,\theta}\left(e^{i\frac{2\pi}{\lambda}a\,\mathrm{sen}\,\theta}-1\right)\left(e^{i\frac{2\pi}{\lambda}(a+b)\,\mathrm{sen}\,\theta}+1\right)$$

A intensidade é pois:

$$I=K^2\frac{-i\lambda}{2\pi\,\mathrm{sen}\,\theta}\frac{i\lambda}{2\pi\,\mathrm{sen}\,\theta}\left(e^{i\frac{2\pi}{\lambda}a\,\mathrm{sen}\,\theta}-1\right)\left(e^{-i\frac{2\pi}{\lambda}a\,\mathrm{sen}\,\theta}-1\right)\left(e^{i\frac{2\pi}{\lambda}(a+b)\,\mathrm{sen}\,\theta}+1\right)\left(e^{-i\frac{2\pi}{\lambda}(a+b)\,\mathrm{sen}\,\theta}+1\right)=$$

$$=K^2\frac{\lambda^2}{(2\pi\,\mathrm{sen}\,\theta)^2}\left(2-2\cos\frac{2\pi}{\lambda}a\,\mathrm{sen}\,\theta\right)\left(2+2\cos\frac{2\pi}{\lambda}(a+b)\,\mathrm{sen}\,\theta\right)=\qquad \text{e se } \beta=\frac{2\pi}{\lambda}\mathrm{sen}\,\theta$$

$$=4K^2\frac{1}{\beta^2}\left(1-\cos\beta a\right)\left(1+\cos\beta(a+b)\right)=4K^2\frac{1}{\beta^2}\,2\,\mathrm{sen}^2\frac{\beta a}{2}\cdot\left(1+\cos\beta(a+b)\right)$$

Visto que $1-\cos\beta a=\cos^2\frac{\beta a}{2}+\mathrm{sen}^2\frac{\beta a}{2}-\cos^2\frac{\beta a}{2}+\mathrm{sen}^2\frac{\beta a}{2}=2\,\mathrm{sen}^2\frac{\beta a}{2}$

$$I=8K^2\frac{\lambda^2}{(2\pi\,\mathrm{sen}\,\theta)^2}\mathrm{sen}^2\left(\frac{2\pi}{\lambda}\frac{a}{2}\mathrm{sen}\,\theta\right)\left(1+\cos\frac{2\pi}{\lambda}\mathrm{sen}\,\theta\,(a+b)\right),$$ e se $a=6$ $b=6$ $\lambda=3$ vem:

$$I=8K^2\frac{9}{4\pi^2\,\mathrm{sen}^2\theta}\cdot\mathrm{sen}^2\frac{2\pi}{3}\frac{6}{2}\mathrm{sen}\,\theta\left(1+\cos\frac{2\pi}{3}(6+6)\,\mathrm{sen}\,\theta\right)=8\cdot 9\cdot K^2\frac{1}{4\pi^2\,\mathrm{sen}^2\theta}\mathrm{sen}^2(2\pi\,\mathrm{sen}\,\theta)\left(1+\cos(8\pi\,\mathrm{sen}\,\theta)\right)$$

$\boxed{22.8}$ Contin. Contin. $\boxed{22.8}$

$$I = 8 \cdot 9 \cdot k^2 \left(\frac{\operatorname{sen}(2\pi \operatorname{sen}\theta)}{2\pi \operatorname{sen}\theta} \right)^2 \left(1 + \cos(8\pi \operatorname{sen}\theta)\right) = \frac{I_{MAX}}{2} \frac{1}{4\pi^2 \operatorname{sen}^2\theta} \operatorname{sen}^2(2\pi \operatorname{sen}\theta)\left(1+\cos(8\pi \operatorname{sen}\theta)\right)$$

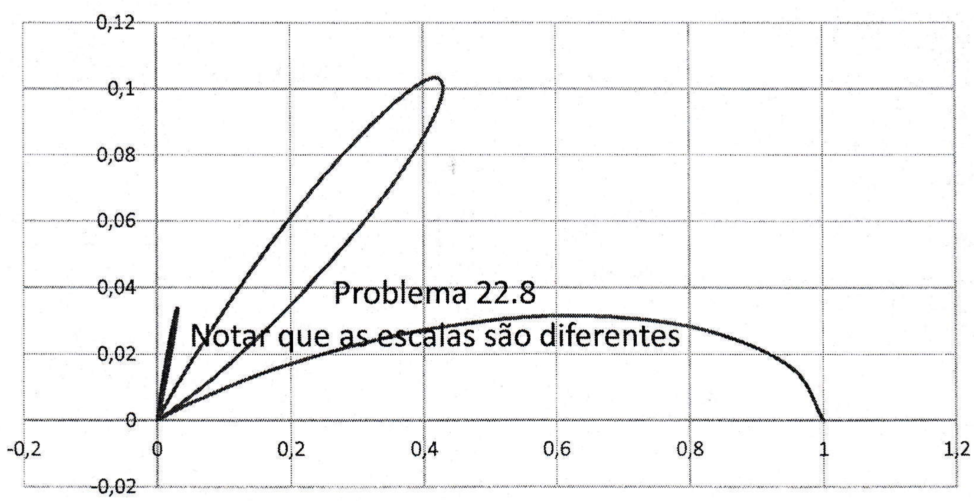
Para $\theta = 0°$ $I = I_{MAX}$ e vale $I_{MAX} = 8 \cdot 9 \cdot k^2 \cdot 1 \cdot 2 \Rightarrow 8 \cdot 9 \cdot k^2 = \frac{I_{MAX}}{2}$

Finalmente:
$$\frac{I}{I_{MAX}} = \frac{1}{8 \cdot \pi^2 \operatorname{sen}^2\theta} \operatorname{sen}^2(2\pi \operatorname{sen}\theta)\left(1 + \cos(8\pi \operatorname{sen}\theta)\right)$$

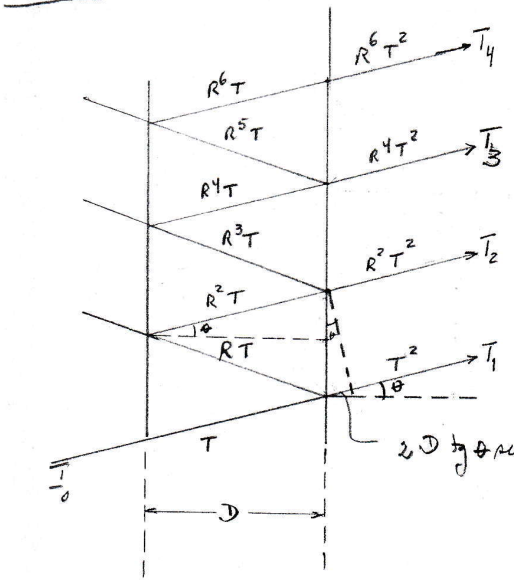
$$= \frac{1}{2} \frac{1}{(2\pi \operatorname{sen}\theta)^2} \operatorname{sen}^2(2\pi \operatorname{sen}\theta)\left(1 + \cos(4 \cdot 2\pi \operatorname{sen}\theta)\right)$$

Valores de máximos obtidos com o Excel:

θ(radian) θ (°)	0 0	0,23 13	0,44 25	0,59 34	0,84 48	1,2 69
$\frac{I}{I_{MAX}}$	1	0,44	0,0098	0,0057	0,045	0,002



22.10 | 22.10



$2D\,\mathrm{tg}\,\theta\,\mathrm{sen}\,\theta = 2D\dfrac{\mathrm{sen}^2\theta}{\cos\theta}$

A fase de T_2 está atrasada de $\frac{\omega}{c}\cdot\frac{2D}{\cos\theta}=\frac{2\pi}{\lambda}\frac{2D}{\cos\theta}$ em relação à fase de T_1. Mas, por outro lado, a fase de T_2 está adiantada em relação à fase de T_1 de $\frac{\omega}{c}2D\,\mathrm{tg}\,\theta\cdot\mathrm{sen}\,\theta = \frac{2\pi}{\lambda}2D\frac{\mathrm{sen}^2\theta}{\cos\theta}$. Então, no total, a fase de T_2 está, em relação à fase de T_1, atrasada de:

$$-\frac{2\pi}{\lambda}\frac{2D}{\cos\theta}+\frac{2\pi}{\lambda}2D\frac{\mathrm{sen}^2\theta}{\cos\theta} = -\frac{2\pi}{\lambda}2D\left(\frac{1}{\cos\theta}-\frac{\mathrm{sen}^2\theta}{\cos\theta}\right) = -\frac{2\pi}{\lambda}2D\frac{\cos^2\theta}{\cos\theta} = -\frac{2\pi}{\lambda}2D\cos\theta$$

A fase de T_3 em relação a T_2 é igual à fase de T_2 em relação a T_1 e assim de seguida. Então podemos escrever:

$$E_1=\sqrt{I_0}\,T^2\cos\omega\left(t-\frac{R}{c}\right) \longrightarrow \bar{E}_1=\sqrt{I_0}\,T^2 e^{i\alpha} \quad \text{com } \alpha=\omega\left(t-\frac{R}{c}\right)$$

$$E_2=\sqrt{I_0}\,T^2R^2\cos\left[\omega\left(t-\frac{R}{c}\right)-\frac{2\pi}{\lambda}2D\cos\theta\right] \longrightarrow \bar{E}_2=\sqrt{I_0}\,T^2R^2e^{i\alpha}e^{-i\frac{2\pi}{\lambda}2D\cos\theta}$$

$$E_3=\sqrt{I_0}\,T^2R^4\cos\left[\omega\left(t-\frac{R}{c}\right)-2\frac{2\pi}{\lambda}2D\cos\theta\right] \longrightarrow E_3=\sqrt{I_0}\,T^2R^4e^{i\alpha}e^{-i2\frac{2\pi}{\lambda}2D\cos\theta}$$

$$\vdots$$

$$E_N=\sqrt{I_0}\,T^2R^{2(N-1)}\cos\left[\omega\left(t-\frac{R}{c}\right)-(N-1)\frac{2\pi}{\lambda}2D\cos\theta\right] \rightarrow E_N=\sqrt{I_0}\,T^2R^{2(N-1)}e^{i\alpha}e^{-i(N-1)\frac{2\pi}{\lambda}2D\cos\theta}$$

cuja soma dá: $\displaystyle\sum_{i=1}^{N}\bar{E}_i=\sqrt{I_0}\,T^2e^{i\alpha}\left[1+R^2e^{-i\frac{2\pi}{\lambda}2D\cos\theta}+R^4e^{-i2\frac{2\pi}{\lambda}2D\cos\theta}+R^6e^{-i3\frac{2\pi}{\lambda}2D\cos\theta}+\dots\right]=$

$=\sqrt{I_0}\,T^2e^{i\alpha}\dfrac{1-\left(R^2e^{-i\frac{2\pi}{\lambda}2D\cos\theta}\right)^N}{1-R^2e^{-i\frac{2\pi}{\lambda}2D\cos\theta}}$ e, quando $N\to\infty$, vem: $\displaystyle\sum_{i=1}^{\infty}E_i=\sqrt{I_0}\,T^2e^{i\alpha}\frac{1}{1-R^2e^{-i\frac{2\pi}{\lambda}2D\cos\theta}}$

A intensidade é o quadrado desta função e vem:

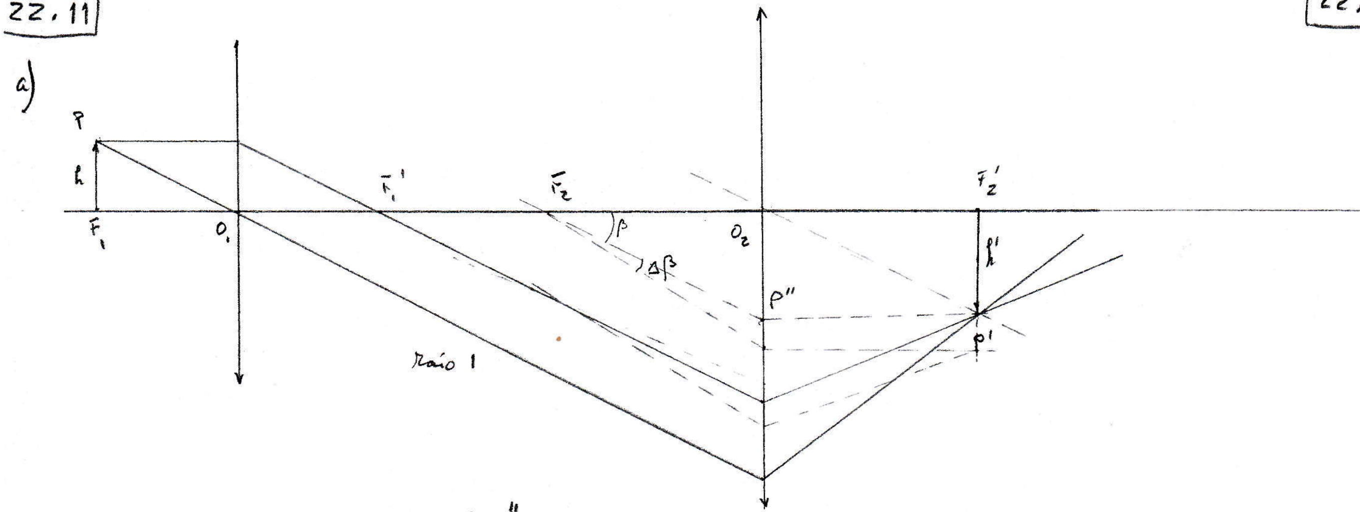
$$I=I_0T^4e^{i\alpha}e^{-i\alpha}\frac{1}{1-R^2e^{-i\frac{2\pi}{\lambda}2D\cos\theta}}\cdot\frac{1}{1-R^2e^{-i\frac{2\pi}{\lambda}2D\cos\theta}}=I_0T^4\frac{1}{1+R^4-R^2\left(e^{-i\frac{2\pi}{\lambda}2D\cos\theta}+e^{i\frac{2\pi}{\lambda}2D\cos\theta}\right)}=$$

$=I_0T^4\dfrac{1}{1+R^4-2R^2\cos\left(\frac{2\pi}{\lambda}2D\cos\theta\right)}$ e se considerarmos θ muito pequeno e tal que $\cos\theta\simeq 1$ vem, finalmente: $I=I_0\dfrac{T^4}{1+R^4-2R^2\cos\left(\frac{4\pi D}{\lambda}\right)}$ e como $T^2+R^2=1$ pode ainda escrever-se:

$$I=I_0\frac{T^4}{1+(1-T^2)^2-2R^2\cos\left(\frac{4\pi D}{\lambda}\right)}=I_0\frac{T^4}{2-2T^2+T^4-2R^2\cos\left(\frac{4\pi D}{\lambda}\right)}=I_0\frac{T^4}{2(1-T^2)-2R^2\cos\left(\frac{4\pi D}{\lambda}\right)+T^4}=\frac{I_0T^4}{2R^2\left(1-\cos\frac{4\pi D}{\lambda}\right)+T^4}$$

22.11 | 22.11

a)

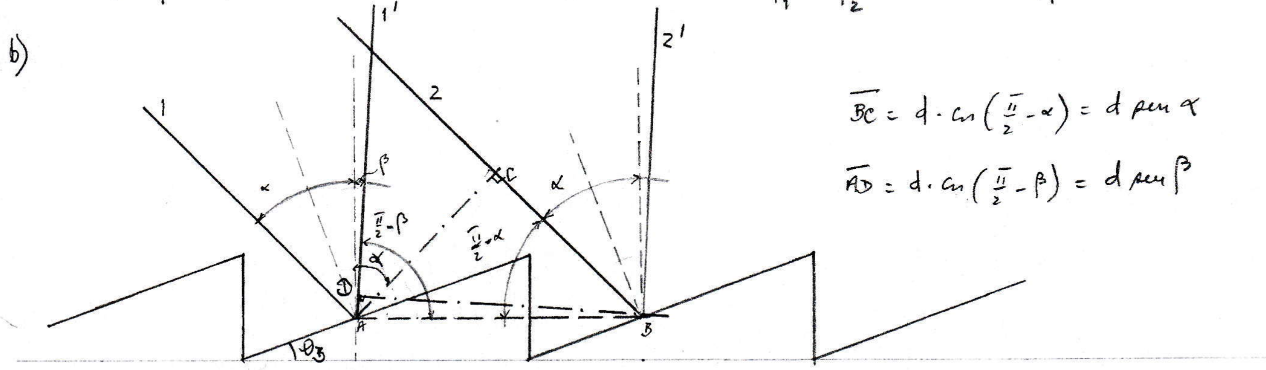


$\Delta\, O_2 F_2' P'$ é igual ao $\Delta\, O_2 F_2 P''$

$\overline{F_2 P''} \parallel \overline{O_2 P'}$ $\quad$ $\overline{O_2 P'} \parallel$ raio 1 por construção!

$\Delta F_1 P O_1$ é semelhante ao $\Delta\, O_2 F_2' P'$ e então $\dfrac{h}{F_1} = \dfrac{h'}{F_2}$ ou $h' = \dfrac{F_2}{F_1} h$

b)



$\overline{BC} = d \cdot \cos\left(\frac{\pi}{2} - \alpha\right) = d \operatorname{sen} \alpha$

$\overline{AD} = d \cdot \cos\left(\frac{\pi}{2} - \beta\right) = d \operatorname{sen} \beta$

O raio incidente 2 percorre um caminho superior ao do raio incidente 1, em $\overline{BC} = d \operatorname{sen}\alpha$

O raio reflectido 2' percorre um caminho inferior ao do raio reflectido 1, em $\overline{AD} = d \operatorname{sen}\beta$.

Assim o raio reflectido 2' percorre um caminho que difere do raio 1' em:
$\overline{BC} - \overline{AD} = d \operatorname{sen}\alpha - d \operatorname{sen}\beta$. A diferença de fase entre 1' e 2' é pois:

$\dfrac{2\pi}{\lambda}(d \operatorname{sen}\alpha - d \operatorname{sen}\beta)$ que se for igual a $2\pi m$ dá interferência construtiva

ou $d \operatorname{sen}\alpha - d \operatorname{sen}\beta = m \cdot \lambda_m$ donde $\lambda_m = \dfrac{d}{m}(\operatorname{sen}\alpha - d \operatorname{sen}\beta)$ com $d = \dfrac{1\,mm}{N} = \dfrac{10^7\,\mathring{A}}{N}$

e então $\lambda_m = \dfrac{10^7}{m \cdot N}(\operatorname{sen}\alpha - \operatorname{sen}\beta)$ em $\mathring{A}$

22.11 Contin. Contin. 22.11

c) $d(\sin\alpha - \sin\beta) = m\lambda_m$. Se o ângulo do raio incidência α se mantiver constante, então, para uma certa ordem fixa, isto é $m = c^{te}$, se λ variar um pouco, para que a relação se mantenha, β variará também.

Na figura da alínea a) se o raio se tornar mais inclinado de $\Delta\beta$ então haverá um desvio de D no plano focal.

Podemos agora escrever: $\sin\beta = -\frac{m\lambda_m}{d} + \sin\alpha$; $\cos\beta\, d\beta = -\frac{m}{d} d\lambda$ ou

$$d\beta = -\frac{m}{d}\frac{1}{\cos\beta} d\lambda$$

Por outro lado $P'' = F_2 \operatorname{tg}\beta$ pelo que $dP'' = F_2 \frac{d}{d\beta}\operatorname{tg}\beta = F_2 \frac{1}{\cos^2\beta} d\beta = -F_2 \frac{m}{d}\frac{1}{\cos^3\beta} d\lambda$

Mas $dP'' = dP' = D$ e portanto $D = -F_2 \frac{m}{d}\frac{1}{\cos^3\beta} d\lambda = -F_2 \frac{m}{\frac{1}{N}}\frac{1}{\cos^3\beta} \cdot 10^{-7}$ mm

$$D = -10^{-7}\frac{F_2\, m\, N}{\cos^3\beta} [mm]$$

Nota: na solução em vez de $\cos^3\beta$ está só $\cos\beta$!

<u>outra abordagem:</u>

$$m\lambda_m = d(\sin\alpha - \sin\beta)$$

$$m\lambda_m + m\Delta\lambda_m = d(\sin\alpha - \sin(\beta + \Delta\beta))$$

$$-m\Delta\lambda_m = -d\sin\beta + d\sin(\beta + \Delta\beta)$$

$$m\Delta\lambda_m = d(\sin\beta - \sin(\beta+\Delta\beta))$$

$m\Delta\lambda_m = d(\sin\beta - \sin\beta\cos\Delta\beta - \sin\Delta\beta\cos\beta)$ mas $\cos\Delta\beta \simeq 1$ e vem

$$m\Delta\lambda_m = -d\sin\Delta\beta\cos\beta \quad (1)$$

Por outro lado, a variação $\Delta\beta$ provoca uma variação D tal que $\operatorname{tg}\Delta\beta = \frac{D}{F_2}$ (ver figura a))

pelo que $\operatorname{sen}\Delta\beta = \frac{\operatorname{tg}\Delta\beta}{\sqrt{1+\operatorname{tg}^2\Delta\beta}} = \frac{\frac{D}{F_2}}{\sqrt{1+\frac{D^2}{F_2^2}}} = \frac{D}{\sqrt{F_2^2 + D^2}}$ e como $F_2 >> D$ vem

$\operatorname{sen}\Delta\beta \simeq \frac{D}{F_2}$ que, substituindo em eq. (1), vem:

$m\Delta\lambda_m = -d \cdot \frac{D}{F_2}\cos\beta$ ou $D = -\frac{F_2 \cdot m}{d}\frac{\Delta\lambda}{\cos\beta} = -\frac{F_2 \cdot m}{\frac{1}{N}}\frac{\Delta\lambda}{\cos\beta} = -\frac{F_2 \cdot m \cdot N}{\cos\beta} \cdot \Delta\lambda$ e

se $\Delta\lambda = 1\,\text{Å} = 10^{-7}$ mm vem $D[mm] = -\frac{F_2 \cdot m \cdot N}{\cos\beta} 10^{-7} [mm]$ que é a solução do livro.

22.12 22.12

a) Se na expressão obtida na alínea a) do problema anterior se fizer $\alpha = -\beta$, caso em que a incidência é normal ao flanco ascendente da rampa, vem: $d(\sin\alpha - \sin\beta) = m\lambda_m$ ou $2d\sin\theta_B = m\lambda_m$ pois, nesse caso $\alpha = -\beta = \theta_B$ que é o ângulo de inclinação da rampa.

assim $\sin\theta_B = \dfrac{m\lambda_m}{2d} = \dfrac{5 \cdot 525{,}0218}{2 \cdot \frac{10^6}{600}} = 0{,}7875$ pelo que $\theta_B = 52°$

b) Se $\theta_B = 52°$, e é um valor fixo, então $m\lambda = 2d\sin\theta_B$ e $\lambda_m = \dfrac{2d\sin\theta_B}{m}$

Assim, a cada m corresponde um valor para λ.

$m=4 \quad \lambda = \dfrac{2\,\frac{10^6}{600} \cdot 0{,}7875}{4} = 656{,}26$ nm

$m=3 \quad \lambda = \quad = 875$ nm > 700 nm fora do intervalo

$m=6 \quad \lambda = \quad = 437$ nm

$m=7 \quad \lambda = \quad = 375$ nm

$m=8 \quad \lambda = \quad = 328$ nm < 360 nm fora do intervalo

assim; $\lambda = 375$ nm, 437 nm, 656 nm são outros comprimentos de onda que irão coincidir com o correspondente a $m=5$

c)

d) À semelhança do que se fez na alínea c) do problema anterior:

$m\lambda_m = 2d\sin\theta_B$ em que $\theta_B = \theta_d$

$m\lambda_m + m\Delta\lambda_m = 2d\sin(\theta_d + \Delta\theta_d)$ e subtraindo vem: $m\Delta\lambda = 2d\sin\Delta\theta_d\cos\theta_d$

Pelas mesmas razões da alínea c) do problema anterior vem: $\sin\Delta\theta_d = \dfrac{D}{F_2}$

e então: $m\Delta\lambda_m = 2d\,\dfrac{D}{F_2}\cos\theta_d$ donde $D = \dfrac{mF_2\Delta\lambda}{2d\cos\theta_d} = \dfrac{mF_2\Delta\lambda}{2\frac{1}{N}\cos\theta_d} = \dfrac{mNF_2}{2\cos\beta}\Delta\lambda$

$D = \dfrac{mNF_2}{2\cos\theta_d}10^{-7}$ mm Å.

Com os valores numéricos $m=5$; $N=600$; $F_2 = 23 \cdot 10^3$ mm; $\theta_d = 52°$ vem $D = 5{,}6$ mm Å^{-1}

e)