36.1

a) $y(t) = \text{const.}$

Embora $y(t)$ não seja uma função periódica típica cumpre a condição

$$y(t) = y(t+T) \quad \text{para todo o } t \text{ e}$$

qualquer período arbitrário T.

1.º $a_0 = \frac{1}{T}\int_0^T y(t)\,dt \Rightarrow a_0 = \frac{\text{Const.}}{T}\int_0^T dt$

$$\boxed{a_0 = \text{Const.}}$$

2.º $a_n = \frac{2}{T}\int_0^T \text{const.}\cos(n\omega t)\,dt \qquad c/\ \omega = \frac{2\pi}{T}$

$$= \frac{2\,\text{Const}}{T}\underbrace{\int_0^T \cos(n\omega t)\,dt}_{=0}$$

$$\boxed{a_n = 0}$$

3.º $b_n = \frac{2}{T}\int_0^T \text{const.}\,\text{sen}(n\omega t)\,dt \qquad c/\ \omega = \frac{2\pi}{T}$

$$= \frac{2\,\text{const.}}{T}\underbrace{\int_0^T \sin(n\omega t)\,dt}_{=0}$$

$$\boxed{b_n = 0}$$

$$y(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega t + b_n \,\text{sen}\, n\omega t$$

$$y(t) = \text{Const.}$$

36.2

a) $f\left(\frac{\pi}{2}\right) = 1$

$$f\left(\frac{\pi}{2}\right) = \frac{4}{\pi}\left(\sin\frac{\pi}{2} + \frac{1}{3}\sin\frac{3\pi}{2} + \frac{1}{5}\sin\frac{5\pi}{2} + \cdots\right)$$

$$= \frac{4}{\pi}\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots\right)$$

$$1 = \frac{4}{\pi}\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots\right)$$

$$\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots\right) = \frac{\pi}{4} \quad \text{c.q.d.}$$

---

b) Recorrendo à Identidade de Parseval

$$\frac{1}{L}\cdot\int_0^{2L}[f(x)]^2\,dx = \frac{a_0^2}{2} + \sum_{n=1}^{\infty}(a_n^2 + b_n^2)$$

Neste caso $L = \pi$; $a_0 = 0$; $a_n = 0$

$$\frac{1}{\pi}\int_0^{2\pi}dx = \frac{4^2}{\pi^2}\left(1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots\right)$$

$$2 = \frac{16}{\pi^2}\left(1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots\right)$$

$$\left(1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots\right) = \frac{\pi^2}{8} \quad \text{c.q.d}$$

---

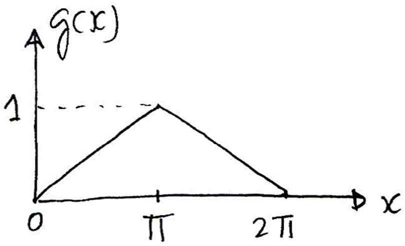
c) $$\sum_{n=1}^{\infty}\frac{1}{n^2} = \left(1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots\right) + \left(\frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{6^2} + \cdots\right)$$

$$= \frac{\pi^2}{8} + \frac{1}{2^2}\underbrace{\left(1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots\right)}_{\sum_{n=1}^{\infty}\frac{1}{n^2}}$$

Então:

$$\sum_{n=1}^{\infty}\frac{1}{n^2} = \frac{\pi^2}{8} + \frac{1}{4}\sum_{n=1}^{\infty}\frac{1}{n^2} \Rightarrow \sum_{n=1}^{\infty}\frac{1}{n^2} = \frac{4}{3}\frac{\pi^2}{8} = \frac{\pi^2}{6} \quad \text{c.q.}$$

36.3



$$g(x) = \begin{cases} \dfrac{x}{\pi} & \text{se} \quad 0 \le x \le \pi \\ 2 - \dfrac{x}{\pi} & \text{se} \quad \pi \le x \le 2\pi \end{cases}$$

a) $$a_n = \frac{1}{L}\int_0^{2L} g(x) \cos\frac{n\pi}{L} x \, dx \Rightarrow a_n = \frac{1}{\pi}\int_0^{2\pi} g(x) \cos nx \, dx$$

$$a_n = \frac{1}{\pi}\int_0^{\pi} \frac{x}{\pi} \cos nx \, dx + \frac{1}{\pi}\int_{\pi}^{2\pi} \left(2 - \frac{x}{\pi}\right) \cos nx \, dx$$

$$= \frac{1}{\pi^2}\int_0^{\pi} x \cos nx \, dx + \frac{2}{\pi}\int_{\pi}^{2\pi} \cos nx \, dx - \frac{1}{\pi^2}\int_{\pi}^{2\pi} x \cos nx \, dx \quad (1)$$

1. Cálculo de $\int_0^{\pi} x \cos nx \, dx$

Façamos: $g = x \Rightarrow g' = 1$

$f' = \cos nx \Rightarrow f = \dfrac{\operatorname{sen} nx}{n}$

$$\int_0^{\pi} x \cos nx \, dx = \underbrace{x \frac{\operatorname{sen} nx}{n} \Big|_0^{\pi}}_{=0} - \int_0^{\pi} \frac{\operatorname{sen} nx}{n} \, dx$$

$$= \frac{\cos nx}{n^2} \Big|_0^{\pi} = \boxed{\frac{\cos n\pi}{n^2} - \frac{1}{n^2}} \quad (2)$$

2. Cálculo de $\int_{\pi}^{2\pi} \cos nx \, dx$

$$\boxed{\int_{\pi}^{2\pi} \cos nx \, dx = \frac{\operatorname{sen} nx}{n} \Big|_{\pi}^{2\pi} = 0} \quad (3)$$

3. Cálculo de $\int_{\pi}^{2\pi} x\cos nx\, dx$

$$\int_{\pi}^{2\pi} x\cos nx\, dx = \frac{\cos nx}{n^2}\Big|_{\pi}^{2\pi} = \frac{\cos n2\pi}{n^2} - \frac{\cos n\pi}{n^2}$$

$$= \boxed{\frac{1}{n^2} - \frac{\cos n\pi}{n^2}} \quad (4)$$

Substituindo (2), (3) e (4) em (1):

$$a_n = \frac{1}{\pi^2}\left(\frac{\cos n\pi}{n^2} - \frac{1}{n^2}\right) - \frac{1}{\pi^2}\left(\frac{1}{n^2} - \frac{\cos n\pi}{n^2}\right)$$

$$= \boxed{\frac{2\cos n\pi}{\pi^2 n^2} - \frac{2}{\pi^2 n^2}}$$

$a_0 = ?$

$$a_n = \frac{1}{\pi^2}\int_0^{\pi} x\cos nx\, dx + \frac{2}{\pi}\int_{\pi}^{2\pi}\cos nx\, dx - \frac{1}{\pi^2}\int_{\pi}^{2\pi} x\cos x\, dx$$

$$a_0 = \frac{1}{\pi^2}\int_0^{\pi} x\, dx + \frac{2}{\pi}\int_{\pi}^{2\pi} dx - \frac{1}{\pi^2}\int_{\pi}^{2\pi} x\, dx$$

$$= \frac{1}{\pi^2}\frac{x^2}{2}\Big|_0^{\pi} + \frac{2}{\pi}x\Big|_{\pi}^{2\pi} - \frac{1}{\pi^2}\frac{x^2}{2}\Big|_{\pi}^{2\pi}$$

$$= \boxed{1}$$

$$b_n = \frac{1}{\pi^2}\int_0^{\pi} x\,\mathrm{sen}\, nx\, dx + \frac{2}{\pi}\int_{\pi}^{2\pi}\mathrm{sen}\, nx\, dx - \frac{1}{\pi^2}\int_{\pi}^{2\pi} x\,\mathrm{sen}\, nx\, dx \quad (5)$$

4. Cálculo de $\int_0^{\pi} x \operatorname{sen} mx \, dx$

$$g = x \Rightarrow g' = 1$$
$$f' = \operatorname{sen} mx \Rightarrow f = -\frac{\cos mx}{m}$$

$$\boxed{\int_0^{\pi} x \operatorname{sen} mx \, dx} = -x \frac{\cos mx}{m}\Big|_0^{\pi} + \frac{1}{m}\int_0^{\pi} \cos mx \, dx$$

$$= -\frac{\pi}{m}\cos m\pi + \underbrace{\frac{1}{m}\frac{\operatorname{sen} mx}{m}\Big|_0^{\pi}}_{=0}$$

$$= \boxed{-\frac{\pi}{m}\cos m\pi} \quad (6)$$

5. Cálculo de $\int_{\pi}^{2\pi} \operatorname{sen} mx \, dx$

$$\boxed{\int_{\pi}^{2\pi} \operatorname{sen} mx \, dx} = -\frac{\cos mx}{m}\Big|_{\pi}^{2\pi}$$

$$= \boxed{-\frac{\cos m2\pi}{m} + \frac{\cos m\pi}{m}} \quad (7)$$

6. Cálculo de $\int_{\pi}^{2\pi} x \operatorname{sen} mx \, dx$

$$g = x \Rightarrow g' = 1$$
$$f' = \operatorname{sen} mx \Rightarrow f = -\frac{\cos mx}{m}$$

$$\int_{\pi}^{2\pi} x \operatorname{sen} mx \, dx = -x \frac{\cos mx}{m}\Big|_{\pi}^{2\pi} + \frac{1}{m}\int_{\pi}^{2\pi} \cos mx \, dx$$

$$= -\frac{2\pi}{m}\cos m2\pi + \frac{\pi}{m}\cos m\pi + \underbrace{\frac{1}{m^2}\operatorname{sen} mx\Big|_{\pi}^{2\pi}}_{=0}$$

$$\int_{\pi}^{2\pi} x \operatorname{sen} mx \, dx = -\frac{2\pi}{m} + \frac{\pi}{m} \cos m\pi \qquad (8)$$

Sustituindo (6), (7) e (8) em (5):

$$\boxed{b_m} = \frac{1}{\pi^2}\left(-\frac{\pi}{m}\cos m\pi\right) + \frac{2}{\pi}\left(\frac{\cos m\pi}{m} - \frac{\cos m2\pi}{m}\right) - \frac{1}{\pi^2}\left(-\frac{2\pi}{m} + \frac{\pi\cos m\pi}{m}\right)$$

$$= -\cancel{\frac{1}{\pi m}\cos m\pi} + \cancel{\frac{2}{\pi m}\cos m\pi} - \frac{2}{\pi m}\cos m2\pi + \frac{2}{\pi m} - \cancel{\frac{1}{\pi m}\cos m\pi}$$

$$= \frac{2}{\pi m} - \frac{2}{\pi m}\underbrace{\cos m2\pi}_{=1}$$

$$\boxed{= 0}$$

Então:

$$g(x) = \frac{a_0}{2} + \sum_{m=1}^{\infty}\left(a_m \cos mx + b_m \operatorname{sen} mx\right)$$

$$= \frac{1}{2} + \sum_{m=1}^{\infty}\left(\frac{2}{\pi^2 m^2}\cos m\pi - \frac{2}{\pi^2 m^2}\right)\cos mx$$

$$= \frac{1}{2} + \sum_{m=1}^{\infty}\frac{2}{\pi^2 m^2}\left(\cos m\pi - 1\right)\cos mx$$

$$= \frac{1}{2} + \sum_{m=1}^{\infty}\frac{2}{\pi^2 m^2}\left[(-1)^m - 1\right]\cos mx$$

$$(-1)^m - 1 = \begin{cases} 0 & \text{se } m \text{ par} \\ -2 & \text{se } m \text{ ímpar} \end{cases}$$

$$g(x) = \frac{1}{2} - \frac{4}{\pi^2}\cos x - \frac{4}{\pi^2 9}\cos 3x - \frac{4}{\pi^2 25}\cos 5x - \cdots$$

$$= \frac{1}{2} - \frac{4}{\pi^2}\left(\cos x + \frac{\cos 3x}{9} + \frac{\cos 5x}{25} + \cdots\right)$$

b) Recorrendo à Identidade de Parseval:

$$\frac{1}{\pi}\int_0^{2\pi}[f(x)]^2dx = \frac{a_0^2}{2}+\sum_{n=1}^{\infty}(a_n^2+b_n^2) \quad (1)$$

$a_0 = 1$

$a_n = \frac{2}{\pi^2 n^2}(\cos n\pi - 1)$ se n é par: $(\cos n\pi - 1) = 0$

$$a_1 = \frac{2}{\pi^2 1^2}(-2) \Rightarrow a_1^2 = \frac{16}{\pi^4 1^4}$$

$$a_3 = \frac{2}{\pi^2 3^2}(-2) \Rightarrow a_3^2 = \frac{16}{\pi^4 3^4}$$

$$a_5 = \frac{2}{\pi^2 5^2}(-2) \Rightarrow a_5^2 = \frac{16}{\pi^4 5^4}$$

$$\frac{a_0^2}{2}+\sum_{n=1}^{\infty} a_n^2 = \frac{1}{2}+\frac{16}{\pi^4}\left(\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+\cdots\right) \quad (2)$$

$$\frac{1}{\pi}\int_0^{2\pi}[f(x)]^2dx = \frac{1}{\pi}\int_0^{\pi}\left(\frac{x}{\pi}\right)^2dx + \frac{1}{\pi}\int_{\pi}^{2\pi}\left(2-\frac{x}{\pi}\right)^2dx$$

$$= \frac{1}{\pi^3}\frac{x^3}{3}\Big|_0^{\pi} + \frac{4}{\pi}x\Big|_{\pi}^{2\pi} + \frac{1}{\pi^3}\frac{x^3}{3}\Big|_{\pi}^{2\pi} - \frac{4}{\pi^2}\frac{x^2}{2}\Big|_{\pi}^{2\pi}$$

$$= \frac{1}{\pi^3}\frac{\pi^3}{3} + \frac{4}{\pi}(2\pi-\pi) + \frac{1}{\pi^3}\left(\frac{8\pi^3}{3}-\frac{\pi^3}{3}\right) - \frac{4}{\pi^2}\left(\frac{4\pi^2}{2}-\frac{\pi^2}{2}\right)$$

$$= \frac{1}{3}+4+\frac{7}{3}-6$$

$$= \frac{2}{3} \quad (3)$$

Aplicando (2) e (3) em (1):

$$\frac{2}{3} = \frac{1}{2}+\frac{16}{\pi^4}\left(\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+\cdots\right)$$

$$\frac{1}{6} = \frac{16}{\pi^4}\left(\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \cdots\right)$$

$$\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \cdots = \frac{1}{6} \cdot \frac{\pi^4}{16}$$

$$= \frac{\pi^4}{96} \quad \underline{\text{c.q.d}}$$

$$\sum_{n=1}^{\infty} \frac{1}{n^4}$$

$$\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \cdots = \left(\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \cdots\right) + \left(\frac{1}{2^4} + \frac{1}{4^4} + \frac{1}{6^4} + \cdots\right)$$

$$= \left(\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \cdots\right) + \frac{1}{2^4}\left(\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \cdots\right)$$

Então:

$$\sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{96} + \frac{1}{16}\sum_{n=1}^{\infty} \frac{1}{n^4}$$

$$\frac{15}{16}\sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{96} \Rightarrow \sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{16}{15} \cdot \frac{\pi^4}{96}$$

$$\sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90} \quad \underline{\text{c.q.d.}}$$

36.4

$$\int_0^\infty \frac{x^3}{e^x-1}\,dx \Rightarrow \int_0^\infty \frac{x^3 e^{-x}}{1-e^{-x}}\,dx$$

mas $\dfrac{1}{1-e^{-x}} = 1 + e^{-x} + e^{-2x} + e^{-3x} + \cdots$

Então:

$$\int_0^\infty \frac{x^3}{e^x-1}\,dx = \int_0^\infty x^3 e^{-x}\left(1+e^{-x}+e^{-2x}+e^{-3x}+\cdots\right)dx$$

$$= \int_0^\infty x^3 e^{-x}\,dx + \int_0^\infty x^3 e^{-2x}\,dx + \int_0^\infty x^3 e^{-3x}\,dx + \cdots$$

Nota: $\displaystyle\int_0^\infty x^3 e^{-nx}\,dx$

fazemos $u = nx \Rightarrow du = n\,dx$

$$\int_0^\infty \left(\frac{u}{n}\right)^3 e^{-u}\,\frac{du}{n} \Rightarrow \int_0^\infty \frac{u^3}{n^4} e^{-u}\,du$$

Então:

$$\int_0^\infty x^3 e^{-nx}\,dx = \frac{1}{n^4}\int_0^\infty u^3 e^{-u}\,du$$

$$\int_0^\infty \frac{x^3}{e^x-1}\,dx = \frac{1}{1^4}\int_0^\infty u^3 e^{-u}\,du + \frac{1}{2^4}\int_0^\infty u^3 e^{-u}\,du + \frac{1}{3^4}\int_0^\infty u^3 e^{-u}\,du + \cdots$$

$$= \int_0^\infty u^3 e^{-u}\,du \left(1 + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \cdots\right)$$

$$\int_0^\infty \frac{x^3}{e^x-1}\,dx = \int_0^\infty u^3 e^{-u}\,du \cdot \sum_{n=1}^{\infty} \frac{1}{n^4}$$

$$= 6 \cdot \frac{\pi^4}{90}$$

$$= \frac{\pi^4}{15} \,//$$

NOTA:

$$\int_0^\infty u^3 e^{-u}\,du = A \; ; \; \left\{ \begin{array}{l} g = u^3 \Rightarrow g' = 3u^2 \\ f' = e^{-u} \Rightarrow f = -e^{-u} \end{array} \right. \Rightarrow A = -u^3 e^{-u}\Big|_0^\infty + \underbrace{\int_0^\infty 3u^2 e^{-u}\,du}_{B}$$

Cálculo de B:

$$\left. \begin{array}{l} g = 3u^2 \Rightarrow g' = 6u \\ f' = e^{-u} \Rightarrow f = -e^{-u} \end{array} \right\} \Rightarrow B = -3u^2 e^{-u} + \underbrace{\int_0^\infty 6u\, e^{-u}\,du}_{C}$$

Cálculo de C:

$$\left. \begin{array}{l} g = 6u \Rightarrow g' = 6 \\ f' = e^{-u} \Rightarrow f = -e^{-u} \end{array} \right\} \Rightarrow C = -6u e^{-u}\Big|_0^\infty + \underbrace{\int_0^\infty 6\, e^{-u}\,du}_{D}$$

Cálculo de D:

$$6\int_0^\infty e^{-u}\,du = -6e^{-u}\Big|_0^\infty = -\frac{6}{e^u}\Big|_0^\infty = 0-(-6) = 6 \,//$$

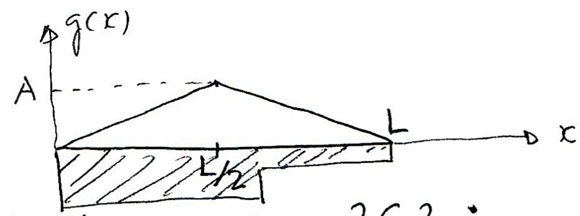
Então:

$$\int_0^\infty u^3 e^{-u}\,du = -\frac{u^3}{e^u}\Big|_0^\infty - \frac{3u^2}{e^u}\Big|_0^\infty - \frac{6u}{e^u}\Big|_0^\infty + 6$$

Aplicando a regra de L'Hospital ao 1.º, 2.º e 3.º termos do membro direito da equação anterior resulta:

$$\int_0^\infty u^3 e^{-u}\,du = -0-0-0+6 = 6 \,//$$

36.5



Usando o resultado do exercício 36.3:

$$g(x) = \frac{A}{2} - \frac{4A}{\pi^2}\left[\frac{\cos\left(\frac{2\pi}{L}x\right)}{1^2} + \frac{\cos\left(\frac{6\pi}{L}x\right)}{3^2} + \frac{\cos\left(\frac{10\pi}{L}x\right)}{5^2} + \cdots\right]$$

$$g\left(\frac{L}{2}\right) = \frac{A}{2} - \frac{4A}{\pi^2}\left[\frac{\cos\left(\frac{2\pi}{L}\frac{L}{2}\right)}{1} + \frac{\cos\left(\frac{6\pi}{L}\frac{L}{2}\right)}{9} + \frac{\cos\left(\frac{10\pi}{L}\frac{L}{2}\right)}{25} + \cdots\right]$$

$$g\left(\frac{L}{2}\right) = \frac{A}{2} - \frac{4A}{\pi^2}\left[\cos\pi + \frac{\cos 3\pi}{9} + \frac{\cos 5\pi}{25} + \cdots\right]$$

$$g\left(\frac{L}{2}\right) = \frac{A}{2} + \frac{4A}{\pi^2}\left[1 + \frac{1}{9} + \frac{1}{25} + \cdots\right]$$

A amplitude da fundamental é A_0. A fundamental também se chama primeiro harmónico A_1.

Então:

$$A_0 = A_1 = \frac{4A}{\pi^2}$$

$$A_2 = 0$$

$$A_3 = \frac{1}{9}\frac{4A}{\pi^2}$$

Então:

$$\frac{A_1}{A_0} = 1 \; ; \; \frac{A_2}{A_0} = 0 \; ; \; \frac{A_3}{A_0} = \frac{1}{9}$$

36.6

$$h(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi}{L}x\right) + b_n \operatorname{sen}\left(\frac{n\pi}{L}x\right)$$

sendo:

$$1.\quad a_n = \frac{1}{L}\int_0^{2L} f(x)\cos\left(\frac{n\pi}{L}x\right)dx$$

$$= \frac{1}{\pi}\int_0^{2\pi}\left(\frac{x}{2\pi}\right)\cos(nx)\,dx$$

$$a_n = \frac{1}{2\pi^2}\int_0^{2\pi} x\cos(nx)\,dx \qquad \text{Façamos:}\quad g = x \Rightarrow g' = 1,\quad f' = \cos nx \Rightarrow f = \frac{\operatorname{sen} nx}{n}$$

$$\frac{1}{2\pi^2}\int_0^{2\pi} x\cos(nx)\,dx = \underbrace{x\frac{\operatorname{sen} nx}{n}\Big|_0^{2\pi}}_{=0} - \int_0^{2\pi}\frac{\operatorname{sen} nx}{n}\,dx$$

$$\frac{1}{2\pi^2}\int_0^{2\pi} x\cos(nx)\,dx = \frac{\cos nx}{n^2}\Big|_0^{2\pi} = \frac{\cos n2\pi}{n^2} - \frac{1}{n^2}$$

Para $\forall n \neq 0 \Rightarrow \boxed{a_n = 0}$

2. $a_0 = ?$

$$a_n = \frac{1}{2\pi^2}\int_0^{2\pi} x\cos(nx)\,dx \Rightarrow a_0 = \frac{1}{2\pi^2}\int_0^{2\pi} x\cos(0)\,dx$$

$$a_0 = \frac{1}{2\pi^2}\frac{x^2}{2}\Big|_0^{2\pi} \Rightarrow \boxed{a_0 = \frac{1}{2\pi^2}\frac{4\pi^2}{2} = 1}$$

$$3.\quad b_n = \frac{1}{L}\int_0^{2L} f(x)\operatorname{sen}\left(\frac{n\pi}{L}x\right)dx$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi}\left(\frac{x}{2\pi}\right)\operatorname{sen}(nx)\,dx$$

$$b_n = \frac{1}{2\pi^2} \int_0^{2\pi} x \operatorname{sen} nx \, dx$$

Façamos: $g = x \Rightarrow g' = 1$

$f' = \operatorname{sen} nx \Rightarrow f = -\frac{\cos nx}{n}$

$$\int_0^{2\pi} x \operatorname{sen} nx \, dx = -x \frac{\cos nx}{n} \Big|_0^{2\pi} + \int_0^{2\pi} \frac{\cos nx}{n} dx$$

$$= -\frac{2\pi \cos n 2\pi}{n} + \underbrace{\frac{\operatorname{sen} nx}{n^2} \Big|_0^{2\pi}}_{=0}$$

Então: $b_n = \frac{1}{2\pi^2} \left(-\frac{2\pi}{n} \cos n 2\pi \right)$

Para qualquer $n \Rightarrow b_n = -\frac{1}{\pi n}$

Então:

$$h(x) = \frac{1}{2} + \sum_{n=1}^{\infty} \left(-\frac{1}{\pi n} \operatorname{sen} nx \right)$$

$$h(x) = \frac{1}{2} - \frac{1}{\pi} \left(\operatorname{sen} x + \frac{\operatorname{sen} 2x}{2} + \frac{\operatorname{sen} 3x}{3} + \cdots \right)$$

36.7

$$y(x) = 3 \sin \frac{\pi}{L} x + \sin 3 \frac{\pi}{L} x$$

a) $y(x) = 3 \sin kx + \sin 3 kx$

Então: $k = \frac{\pi}{L}$

Como $\omega = kv \Rightarrow \frac{2\pi}{T} = kv$

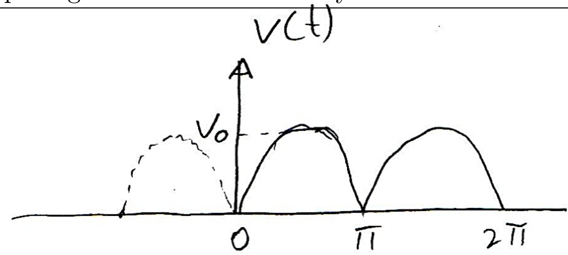
numa corda a velocidade é dada por:

$v = \sqrt{\frac{S}{\sigma}}$ sendo S a tensão da corda e

σ a densidade linear de massa da corda.

$$\frac{2\pi}{T} = \frac{\pi}{L} \cdot \sqrt{\frac{S}{\sigma}} \Rightarrow T = 2L \sqrt{\frac{\sigma}{S}} \quad \text{c.q.d.}$$

36.8



V(t) é uma função par pelo que o coeficiente de Fourier b_m é nulo; $\underline{b_m = 0}$

$$a_m = \frac{1}{L}\int_0^{2L} V(t)\cos\frac{m\pi}{L}x\,dx$$

$$a_m = \frac{1}{\pi}\int_0^{2\pi} V(t)\cos mx\,dx \Rightarrow a_m = \frac{2V_0}{\pi}\int_0^{\pi} \operatorname{sen} x\cos mx\,dx$$

$$a_0 = \frac{2V_0}{\pi}\int_0^{\pi}\operatorname{sen} x\,dx \Rightarrow a_0 = -\frac{2V_0}{\pi}\cos x\Big|_0^{\pi}$$

$$a_0 = -\frac{2V_0}{\pi}(-1-1) \Rightarrow \boxed{a_0 = \frac{4V_0}{\pi}}$$

Nota: $\operatorname{sen} x\cos mx = \frac{1}{2}\left\{\operatorname{sen}[(m+1)x] + \operatorname{sen}[-(m-1)x]\right\}$

$$a_m = \frac{V_0}{\pi}\int_0^{\pi}\left\{\operatorname{sen}[(m+1)x] - \operatorname{sen}[(m-1)x]\right\}dx$$

$$= \frac{V_0}{\pi}\left\{-\frac{\cos[(m+1)x]}{m+1} + \frac{\cos[(m-1)x]}{m-1}\right\}\Bigg|_0^{\pi}$$

$$= \frac{V_0}{\pi}\left\{-\frac{\cos[(m+1)\pi]}{m+1} + \frac{1}{m+1} + \frac{\cos[(m-1)\pi]}{m-1} - \frac{1}{m-1}\right\}$$

se m ímpar:

$$a_m = \frac{V_0}{\pi}\left\{-\frac{1}{m+1} + \frac{1}{m+1} + \frac{1}{m-1} - \frac{1}{m-1}\right\} = 0$$

se n for par:

$$a_n = \frac{V_0}{\pi}\left\{\frac{2}{n+1} - \frac{2}{n-1}\right\} = \frac{V_0}{\pi}\left(\frac{-4}{n^2-1}\right)$$

$$V(t) = \frac{a_0}{2} + \sum_{\substack{n=2 \\ n \text{ par}}}^{\infty} \frac{V_0}{\pi}\left(\frac{-4}{n^2-1}\right)\cos nx$$

$$= \frac{2V_0}{\pi} - \frac{4V_0}{\pi}\left(\frac{1}{n^2-1}\cos nx\right)$$

$$= \frac{2V_0}{\pi} - \frac{4V_0}{\pi}\left(\frac{1}{3}\cos 2x + \frac{1}{15}\cos 4x + \frac{1}{35}\cos 6x + \dots\right.$$

O valor médio de $V(t)$ é:

$$\langle V(t)\rangle = \frac{a_0}{2} = \frac{2V_0}{\pi}$$

A amplitude da segunda harmónica de $V(t)$ é:

$$A_2 = \frac{4V_0}{15\pi}$$

$\boxed{36.9}$

$$V_{out} = V_{in} + \varepsilon(V_{in})^3 \Rightarrow V_{out} = \sin x + \varepsilon \sin^3 x \Rightarrow V_{out} = \sin x(1 + \varepsilon \sin^2 x)$$

Como $\sin^2 x = \frac{1}{2} - \frac{1}{2}\cos 2x \Rightarrow V_{out} = \sin x\left(1 + \frac{\varepsilon}{2} - \frac{\varepsilon}{2}\cos 2x\right)$

$V_{out} = \sin x + \frac{\varepsilon}{2}\sin x - \frac{\varepsilon}{2}\cos 2x \sin x$

Como $\cos 2x \sin x = \frac{1}{2}[\sin(3x) + \sin(x)]$, então:

$$V_{out} = \sin x + \frac{\varepsilon}{2}\sin x - \frac{\varepsilon}{4}\sin(3x) - \frac{\varepsilon}{4}\sin x$$

$$= \sin x + \frac{3}{4}\varepsilon \sin x - \frac{\varepsilon}{4}\sin(3x)$$

$$V_{out} = \left(1 + \frac{3}{4}\varepsilon\right)\sin x - \frac{\varepsilon}{4}\sin(3x)$$