Lift & Drag Derived from Pressure Coefficients

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Objective of this lab is to find lift and drag coefficients from pressure distributions on thin airfoils. NACA 0012 and NACA 4412 were placed in a wind tunnel where a scannivalve recorded pressure at different pressure taps on the airfoil. This data was recorded along with dynamic pressure and fluid velocity. It was found that NACA 0012 achieved maximum lift at ten degrees angle of attack while NACA 4412 did as well. Results of this lab indicate that the Thin Airfoil Theory has validity for small cambered airfoils.

Nomenclature

α	=	angle of attack
$\hat{n}$	=	unit vector in normal direction
$\hat{t}$	=	unit vector in tangential direction
C_p	=	pressure coefficient
C_x	=	force coefficient in the *x* direction
C_z	=	force coefficient in the *z* direction
c	=	chord
s	=	distance on airfoil
F_z	=	*Z* component of the resultant pressure force acting on the airfoil
F_x	=	*X* component of the resultant pressure force acting on the airfoil
ds	=	incremental distance on airfoil
p	=	pressure
p_∞	=	pressure in the free stream
τ_w	=	shear stress
θ	=	angle between x-component and tangential component at a point, counter-clockwise
dx	=	change in x direction
dz	=	change in z direction
l	=	lift per unit span
d	=	drag per unit span
v	=	velocity of fluid
v_∞	=	velocity in the free stream
V	=	fluid viscosity
ρ	=	density
q_∞	=	dynamic pressure
C_f	=	friction coefficient
C_l	=	lift coefficient
C_d	=	drag coefficient
R	=	Reynolds Number

I. Introduction

THE pressure coefficient in fluid dynamics is a dimensionless, quantitative value that offers a basis of relativity when comparing the pressure in a flow of fluid. When a fluid moves through a body, pressure is exerted

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perpendicular to the surface of that body. To describe the distribution of pressure over a body, every point on the body is given a unique pressure coefficient, C_p.

Lift & drag are two aerodynamic forces that affect how aircraft move through air. Lift is a force that acts perpendicular to the flow of a fluid and is what enables aircraft to fly. An airfoil is a shape that creates uneven pressure on its top & bottom surfaces yielding lift. Airfoil shapes are most commonly known to be used for aircraft wings but they can also be found on birds, Formula 1, turbines, and submarines to name a few. The *lift coefficient* is a dimensionless number describing the behavior of a body moving through a fluid with respect to the amount, or lack of, lift it will have. Drag is a friction force that acts against an object moving through a fluid. The *drag coefficient* is a dimensionless number much like the pressure coefficient where it quantifies the behavior of drag for a body moving through a fluid.

Airfoils can be either symmetrical or cambered. A symmetrical airfoil is symmetrical about the chord from the leading edge to the trailing edge. Cambered airfoils are typically thicker above the chord line and thinner below. Symmetrical and cambered airfoils can be described as a four-digit series such as NACA 0012 and NACA 4412, which will be the two airfoils studied in this paper. NACA was formally known as National Advisory Committee for Aeronautics until 1958 when President Eisenhower renamed the agency to NASA. The first number is the series describes max camber thickness as a percentage of the chord length. In 4412, the max camber thickness is 4% of the chord length. The second number describes where along the chord the max camber thickness occurs as a percentage of chord length. In 4412, the max camber thickness occurs at 40% of the chord length from the leading edge. The last two digits describe the maximum thickness of the airfoil as a percentage of chord length.

Every airfoil has a *lift curve* and *drag curve* where the lift and drag coefficients are plotted against the angle of attack. The lift and drag curves offer useful information such as the *stall angle* and *zero lift angle.* The stall angle is the angle of attack where the airfoil reaches a maximum lift coefficient and begins to decrease. The air moving over the top of the airfoil begins to separate from the surface which causes the plane a sudden loss of lift. The zero lift angle is the angle of attack that yields a lift coefficient of zero. Airfoils also have a lift to drag ratio curve. This curve can show at which angle an airfoil can achieve the most lift with respect to minimizing the amount of drag induced.

II. Procedure

The following experiment consists of measuring pressure distributions across two different airfoils at various angles of attack. The initial step in the procedure is to measure the ambient pressure and temperature. Then take both airfoils and measure the chord length and the length from the leading edge to each pressure tap on the airfoil. Place airfoil NACA 0012 in the wind tunnel test section at an angle of attack of zero degrees. Make a connection from each pressure tap on the airfoil to the scannivalve, you may only be able to do one surface at a time. Use one scannivalve port to measure the gauge pressure in the test section by connecting a scannivalve tube to a pressure port on the bottom of the test section. Measure the dynamic pressure of the test section by connecting the pressure transducer across the pitot-static tube. For NACA 0012, use both positive and negative values of 0, 4, 8, 10, and 12 degrees for the angle of attack. Measure the top surface of NACA 0012 and use the negative angles of attack and the airfoils symmetry to derive the pressure coefficients for the bottom surface. Set the wind tunnel to a setting of 40 Hz and obtain data for the test section static pressure and velocity along with the data from each pressure tap for each angle of attack. Record all data in LabView. Repeat for NACA 4412 airfoil. For NACA 4412, record both the top and bottom surfaces and use 0, 4, 8, 10, and 14 degrees for angle of attack. Record all uncertainties throughout the procedure.

When air flows over an airfoil and pressure is exerted at a specific point on the surface, there is a force on the surface that points outwards and denoted by the unit vector, $\hat{n}$. Also at that same point is a tangential force that points clockwise to the surface from trailing edge to trailing edge and denoted by the unit vector, $\hat{t}$. Let *s* be an arbitrary distance along the surface and *ds* an incremental distance along the surface. On the surface of the airfoil, there is a distribution of pressure, *p*, and shear stress, τ_w. The pressure on the surface will always act *normal* to the surface and can be expressed as,

$$-p\hat{n}ds \tag{1}$$

and the shear stress can be expressed as,

$$\tau_w \hat{t} ds \tag{2}$$

where both expressions are in terms of force per unit length. On a two-dimensional airfoil with the fluid flowing in the direction of the x-axis and perpendicular to the z-axis, the force in both the x & z directions can be found. Let the force in the z-direction be equal to,

$$f_z = \oint -p\hat{n}\cdot\hat{k}ds + \oint \tau_w \hat{t}\cdot\hat{k}ds \tag{3}$$

where $\hat{k}$ is a vector in the z-direction. If we define $\hat{n}\cdot\hat{k} = |\hat{n}| \times |\hat{k}| \cdot \cos(\theta) = \cos(\theta)$ and $\hat{t}\cdot\hat{k} = |\hat{t}| \times |\hat{k}| \cdot \cos\left(\frac{\pi}{2} - \theta\right) = \sin(\theta)$, then Eq. (3) can be rewritten as,

$$f_z = \oint (-pcos(\theta) + \tau_w \sin(\theta))ds \tag{4}$$

where θ is the angle from the $\hat{\imath}$ direction counter-clockwise to the tangential vector, $\hat{t}$. Similarly, the force in the x-direction can be written as,

$$f_x = \oint (psin(\theta) + \tau_w \cos(\theta))ds \tag{5}$$

where the pressure term is now positive because $\hat{n}\cdot\hat{n} = \sin\left(\frac{\pi}{2} + \theta\right) = -\sin(\theta)$. If we then take a point along a curved surface and let a tangential line to that surface be *ds* and the x & z components of *ds* be *dx* & *dz* respectively, then $\cos(\theta) = \frac{dx}{ds}$ and $\sin(\theta) = \frac{dz}{ds} = \frac{dz}{dx}\frac{dx}{ds}$, then Eq. (4) & Eq. (5) can be rewritten as,

$$f_z = \oint (-pdx + \tau_w dz) \tag{6}$$

$$f_x = \oint (pdz + \tau_w dx) \tag{7}$$

Using Eq. (6) & Eq. (7), an expression for both lift & drag can be written as,

$$l = f_z \cos(\alpha) - f_x \sin(\alpha) \tag{8}$$

$$d = f_z \sin(\alpha) + f_x \cos(\alpha) \tag{9}$$

where α is the angle of attack. Now let the pressure coefficient be defined as,

$$C_p = \frac{p - p_\infty}{\frac{1}{2}\rho v_\infty^2} = \frac{p - p_\infty}{q_\infty} \tag{10}$$

where $q_\infty = \frac{1}{2}\rho v_\infty^2$, where ρ is the air density and v_∞ is the velocity in the free stream. Similarly, the friction coefficient can be defined as,

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho v_\infty^2} = \frac{\tau_w}{q_\infty} \tag{11}$$

To obtain a dimensionless number for lift & drag in Eq. (8) & Eq. (9), which has units of Newton per meter (lift/drag per unit span), we will need to divide lift and drag by a term with the same units. The units for $q_\infty c$ are,

$$\frac{kg}{m^3}\cdot\frac{m^2}{s^2}\cdot m = \frac{kg}{s^2}$$

Then if we divide *l or d* by $q_\infty c$ we obtain,

$$\frac{\frac{N}{m}}{\frac{kg}{s^2}} = \frac{N}{m}\cdot\frac{s^2}{kg} = \frac{\frac{kg\cdot m}{s^2}}{m}\cdot\frac{s^2}{kg} = \frac{kg\cdot m}{s^2}\cdot\frac{1}{m}\cdot\frac{s^2}{kg} = no\ dimensions$$

Therefore, the lift and drag coefficient can be expressed as,

$$C_l = \frac{l}{q_\infty c} \tag{12}$$

$$C_d = \frac{d}{q_\infty c} \tag{13}$$

Now using Eq. (8), Eq. (9), Eq. (12), and Eq. (13) we can express the lift and drag coefficients as,

$$C_l = C_z \cos(\alpha) - C_x \sin(\alpha)$$
$$C_d = C_z \sin(\alpha) + C_x \cos(\alpha) \tag{14}$$

where C_z & C_x are force coefficients in the x & z-direction. Then by using $C_z = \frac{f_z}{q_\infty c}$, we can obtain,

$$C_z = \oint \frac{-p}{q_\infty}\frac{dx}{c} + \oint \frac{\tau_w}{q_\infty}\frac{dz}{c} = \oint -\frac{p - p_\infty}{q_\infty} d\bar{x} - \oint \frac{p_\infty}{q_\infty} d\bar{x} + \oint C_f d\bar{z} \tag{15}$$

Simplifying further,

$$C_z = \oint -C_p d\bar{x} + \oint C_f d\bar{z}$$

but since the pressure is much bigger than the shear stress, we can neglect the friction coefficient terms.

$$C_z = \oint -C_p d\bar{x} \tag{16}$$

For the force coefficient in the x-direction, we have a similar derivation as above where we will obtain,

$$C_x = \oint C_p d\bar{z} \tag{17}$$

Uncertainties will be calculated in this lab by using $\delta C_l = (C_{l\,max} + C_{l\,min})/2$ and then $C_l \pm \delta C_l$. Same will be used for the drag coefficient with C_d replaced for C_l.

Results

A. Plotting –Cp Versus x/c for all Alpha

The following plots demonstrate the pressure distributions on NACA 0012 & NACA 4412 for all alpha values. For numerical C_p values vs α, please check table A-1 in the appendix.

i. NACA 0012

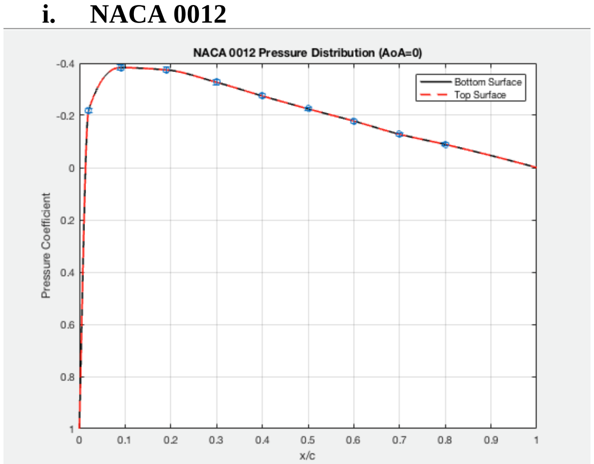

Figure 1 a). NACA 0012 pressure distribution at zero angle of attack.

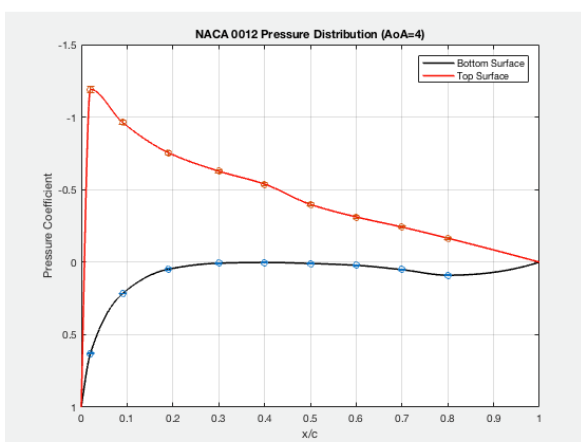

Figure 1 b). NACA 0012 pressure distribution at four degrees angle of attack.

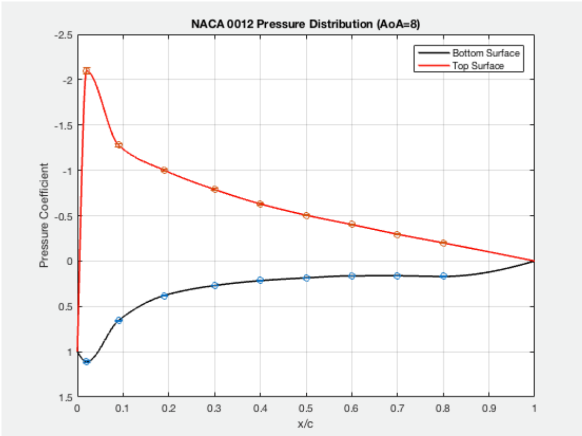

Figure 1 c). Eight degrees angle of attack.

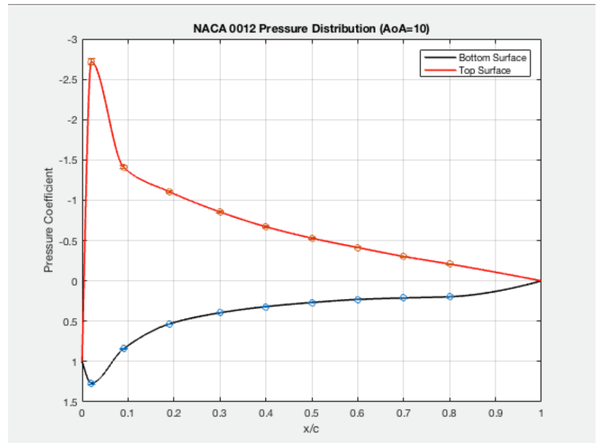

Figure 1 d). Ten degrees angle of attack.

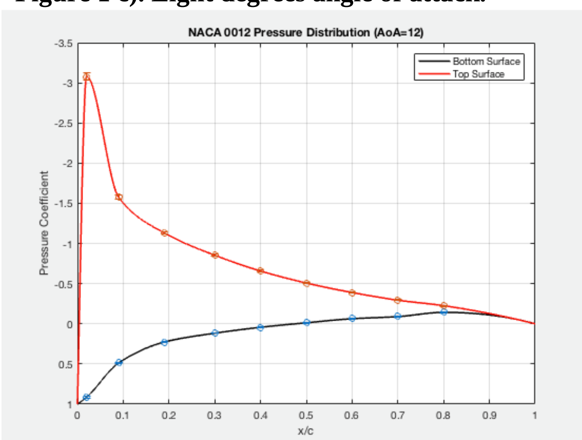

Figure 1 e). Twelve degrees angle of attack

Because of the symmetry of NACA 0012, it is expected that the lift coefficient for $\alpha = 0°$ is approximately zero. From the plot, since there is no enclosed area, the lift coefficient is approximately zero. Which makes an angle of attack of zero degrees the zero lift angle for NACA 0012. The lift coefficients for these plots are not dependent on the shape, but on the enclosed area between the top and bottom surfaces. Knowing this, without doing any actual integrating, it appears that the maximum lift coefficient for NACA 0012 is at an angle of attack of 10 degrees. Further more, the enclosed area for when the angle of attack is 12 degrees is smaller than 10 degrees. This shows that somewhere between 10 & 12 degrees lies the stall angle.

ii. NACA 4412

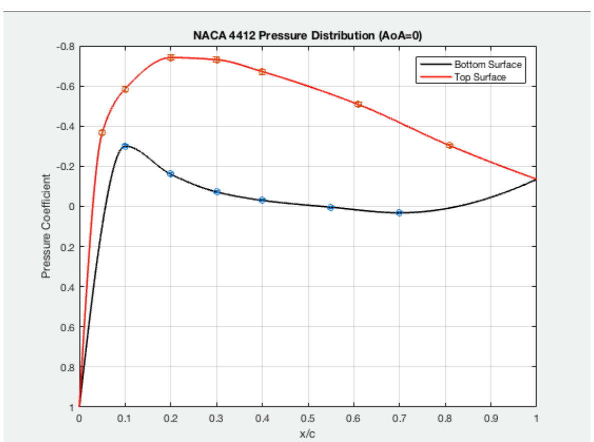

Figure 2 a). Zero degrees angle of attack.

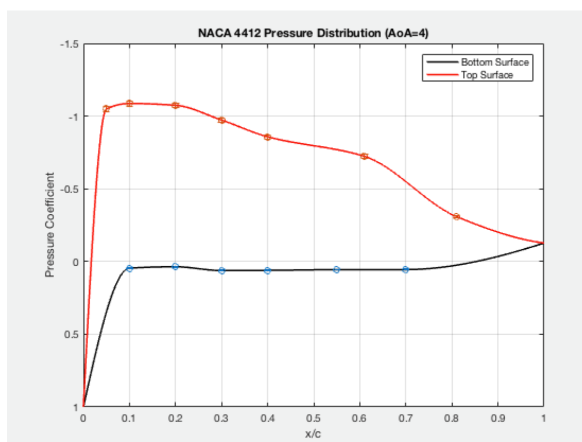

Figure 2 b). Four degrees angle of attack.

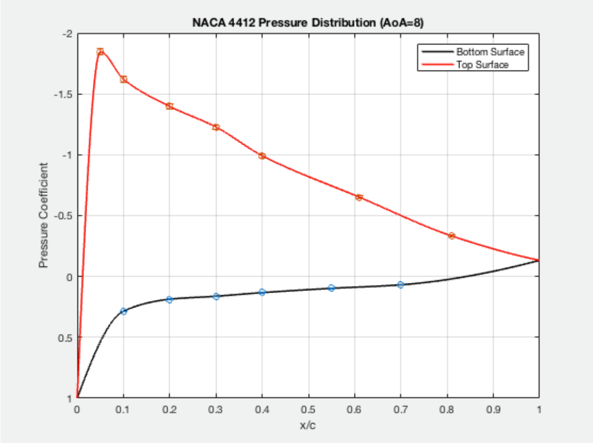

Figure 2 c). Eight degrees angle of attack.

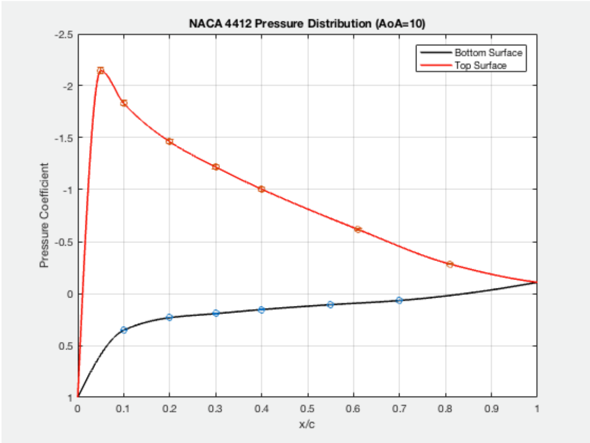

Figure 2 d). Ten degrees angle of attack.

Looking at the plots for NACA 4412, it is not immediately clear where the zero lift angle is, but we can speculate that it is most likely in the negative range for angle of attack, given that the enclosed areas are decreasing along with alpha. The largest enclosed area appears to occur at an attack of attack of 10 degrees with a drop off at 14 degrees. According to the data, the max lift coefficient should occur at an angle of attack of 10 degrees, but if more angles had been tested between 10 and 14 degrees, it would be reasonable to expect the max lift coefficient to occur at an angle higher than 10 degrees.

Comparing NACA 0012 and NACA 4412, these two airfoils should not have maximum lift coefficients at the same angle. Given that NACA 4412 is cambered, the max lift coefficient should be at a higher angle of attack. Similarly, the zero lift angle should occur at a lower angle of attack as stated earlier.

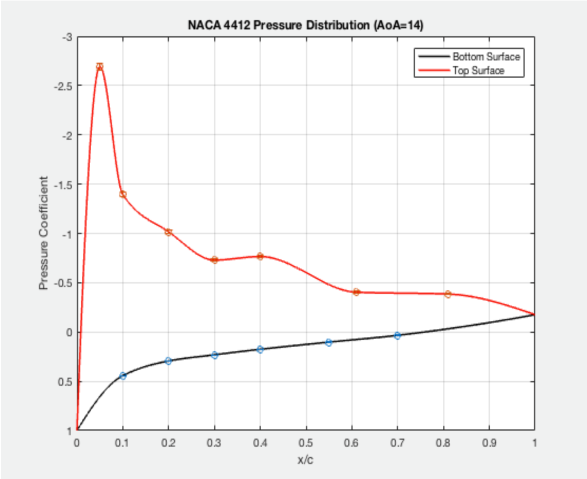

Figure 2 e). Fourteen degrees angle of attack.

B. Plotting Cl, Cd, and l/d Versus Alpha

i. NACA 0012

The lift coefficient vs alpha plot is much like what was expected from looking at the pressure distributions. As the enclosed area between the top and bottom surfaces increases, so does the lift coefficient. From the plot, the maximum lift coefficient is approximately 1.0 and the zero lift angle is zero. In the drag coefficient vs alpha plot, it is interesting to note that the maximum drag coefficient occurs at the same angle as the maximum lift coefficient. Both of these curve plots are close but not completely realistic to a full sized aircraft wing. Missing from these plots is the Reynolds Number. Even though viscous forces were neglected in the calculation, using $R = \frac{vc}{V}$, where V is the viscosity of the air, the Reynolds Number is approximately between 200,000 and 250,000. The Reynolds Number would help give the plots some relatively since the inertial forces can change how these lift and drag curves look. It is also helpful to know since these are scaled airfoils. These smaller airfoils are not going to yield the same results as if a full sized aircraft wing is being tested.

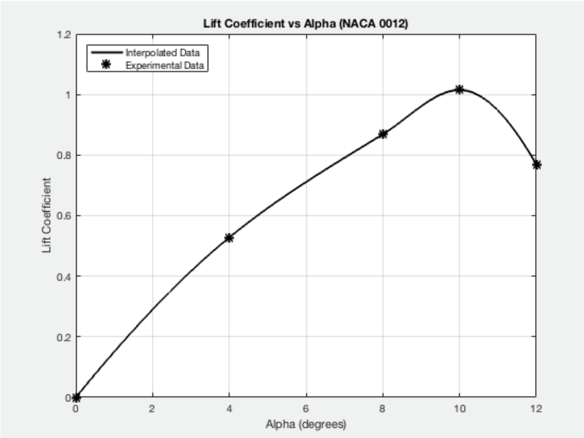

Figure 3 a). Lift Curve Vs Alpha

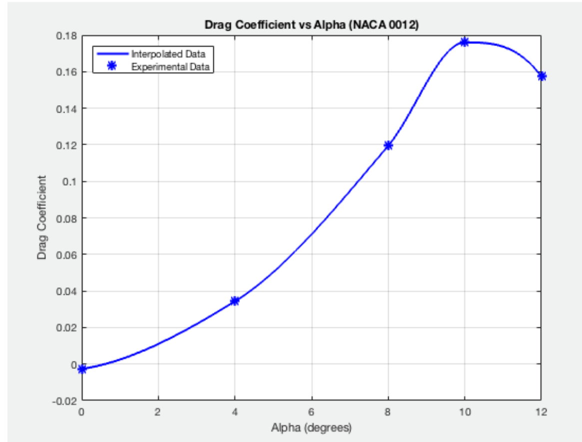

Figure 3 b). Drag Curve Vs Alpha

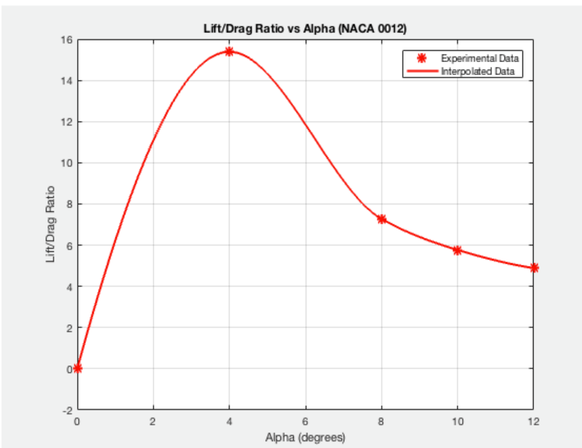

Figure 3 c). Lift to drag ratio Vs Alpha

The lift to drag ratio for NACA 0012 shows that at an angle of attack of 4 degrees there is a maximum amount of lift for the amount of drag induced. While the maximum amount of lift occurs at an angle of attack of 10 degrees, the lift to drag ratio at 10 degrees is low because the amount of drag is also at a maximum at 10 degrees. This plot can be helpful for pilots to gain the most amount of lift while reducing the amount of fuel consumed. In other words, the lift to drag ratio shows the most efficient angle of attack.

ii. NACA 4412

The lift curve for NACA 4412 is very similar to NACA 0012 in terms of shape. As stated earlier, there is a maximum lift coefficient at an angle of attack of 10 degrees, if more angle measurements had been taken between 10 and 14 degrees the plot may look slightly different with a higher maximum lift coefficient. The drag curve differs from NACA 0012 since it is constantly increasing and does not

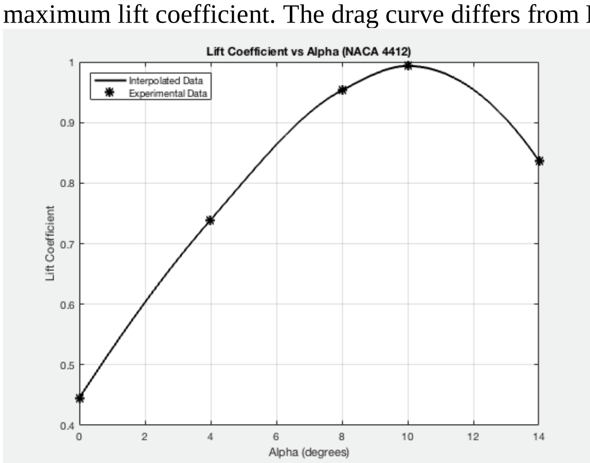

Figure 4 a). Lift Curve Vs Alpha

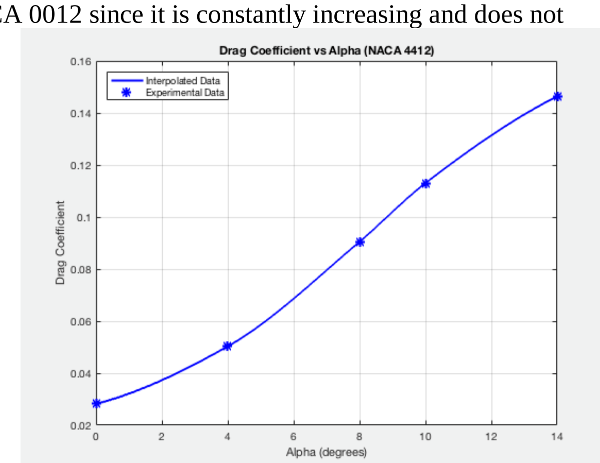

Figure 4 b). Drag Curve Vs Alpha

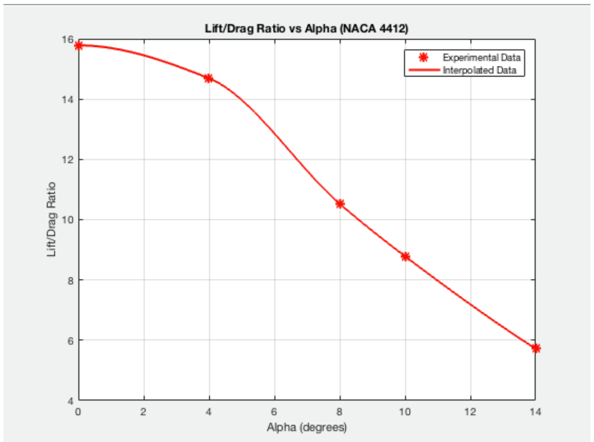

Figure 4 c). Lift to drag ratio Vs Alpha

decrease after the stall angle. Looking at the lift to drag ratio, the maximum lift to drag occurs at zero angle of attack and gradually decreases from there. The airfoils cambered shape helps it gain more lift at lower angles of attack than NACA 0012 with a symmetrical shape.

C. Uncertainties

The machine error for the pressure transducer is at $\pm 10\ Pa$ and the scannivalve at $\pm\ 3\ Pa$. Because of the size of the errors being small in relation to the dynamic and gauge pressures being measured, the uncertainties in the calculation are rather small. For the lift and drag coefficients, the uncertainties at an angle of attack of 8 degrees were calculated. The lift and drag for NACA 0012 respectively were 0.8687 & 0.1195. Accounting for the machine errors, we can expect that the lift coefficient is between 0.8577 & 0.8797 and the drag coefficient is between 0.1181 & 0.1209. For NACA 4412, the lift and drag coefficient respectively were 0.9530 & 0.0905. With uncertainties in measurements we can expect the lift coefficient to be between 0.9381 & 0.9680 and the drag coefficient to be between 0.0895 & 0.0915. To show the small variation in the coefficients, a plot of the pressure distribution for NACA 4412 at 4 degrees angle of attack is shown in figure 5 zoomed in on the error bars.

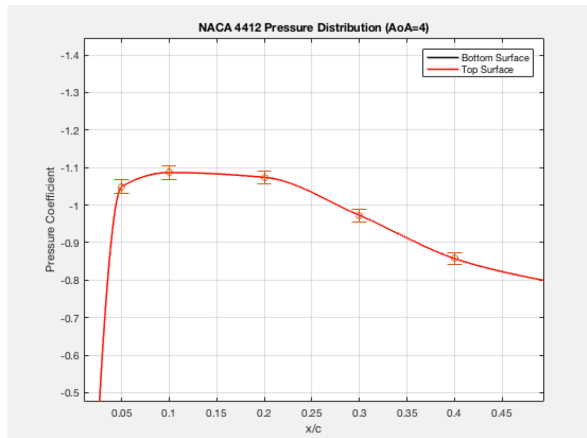

Figure 5. NACA 4412 pressure distribution errors.

III. Conclusion

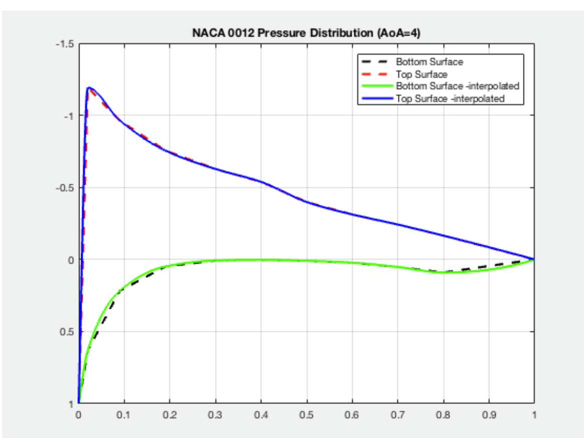

Figure 6. Linear Vs Interpolation

Results for NACA 0012 were about as expected but reiterating from earlier, the drag coefficient for NACA 4412 did not decrease after the stall angle which is the opposite from what would be expected because of the loss of velocity after losing lift.

One area of improvement for the results would be how the data between measured values are interpolated. MATLAB by default uses a linear fit between data points which in this case give a close approximation to the true enclosed area of the pressure distributions. To improve this approximation, using a different interpolation method between data points may improve the accuracy of the results. Alternatively, having a model airfoil with more pressure taps would also improve the accuracy of the results.

In NACA 0012, only the top surface was measured and the bottom surface was derived by taking measurements for a negative angle of attack and changing the sign on the scannivalve results. To improve the results of NACA 0012, taking measurements on both the top and bottom surfaces may help yield a better result.

The significance of this lab is to establish the *Thin Airfoil Theory*. The Thin Airfoil Theory is a way to approximate the coefficient of lift for a thin, small cambered airfoil by using $C_l = 2\pi\alpha$. This theory assumes that the wing span is infinite, meaning that induced drag is not accounted for in this approximation. It also works best for small cambered airfoils; the larger the camber in the airfoil, the worse the approximation is. Looking at the plot of both lift curves and the Thin Airfoil Theory approximation, it is apparent that NACA 0012, having no camber, gets the best approximation for the lift coefficient up until the stall angle. NACA 4412 having a larger camber, gets a less accurate approximation from this theory. For small or no cambered airfoils, the plot gives validity to the theory for accurately approximating lift coefficients.

The results of this lab show why NACA 4412 is a good airfoil for aircraft wings while NACA 0012 is a good elevator. NACA 4412 getting a maximum lift to drag ratio and zero angle of attack helps greatly with both take off and landing. NACA 4412 also allows pilots to ascend without having to be at a large angle of attack. NACA 0012 is a good elevator since the lift coefficient at zero angle of attack is zero and non-zero in most practical angle of attacks. This allows the airfoil to do its job when needed and then not affecting lift when cruising.

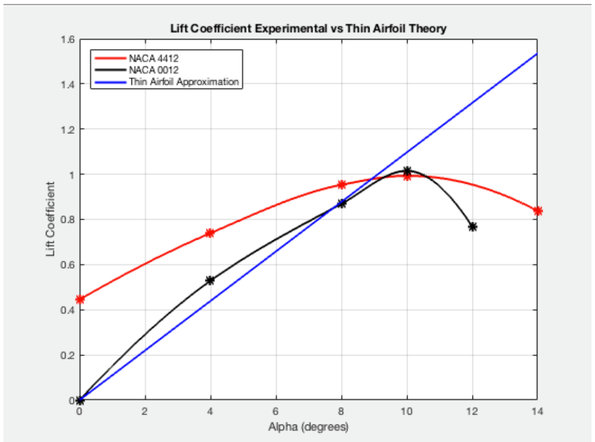

Figure 7. Lift Curves Vs Thin Airfoil

Appendix A

Table A-1

Bottom Surface	Alpha = 0	Alpha = 4	Alpha = 8	Alpha = 10	Alpha = 12
0.02c	-0.2186	0.6294	1.1128	1.2682	0.9139
0.09c	-0.3828	0.2176	0.6562	0.8433	0.4874
0.19c	-0.3742	0.0488	0.3805	0.5339	0.2268
0.30c	-0.3266	0.0073	0.2705	0.3957	0.1170
0.40c	-0.2744	0.0023	0.2171	0.3193	0.0432
0.50c	-0.2250	0.0093	0.1859	0.2683	-0.0129
0.60c	-0.1782	0.0220	0.1635	0.2301	-0.0631
0.70c	-0.1278	0.0522	0.1630	0.2111	-0.0894
0.80c	-0.0887	0.0902	0.1701	0.1963	-0.1430

Table A-1 a). NACA 0012, bottom surface.

Top Surface	Alpha = 0	Alpha = 4	Alpha = 8	Alpha = 10	Alpha = 12
0.02c	-0.2186	-1.1911	-2.0993	-2.7155	-3.0799
0.09c	-0.3828	-0.9639	-1.2777	-1.4113	-1.5758
0.19c	-0.3742	-0.7550	-1.0012	-1.1059	-1.1305
0.30c	-0.3266	-0.6274	-0.7888	-0.8551	-0.8538
0.40c	-0.2744	-0.5379	-0.6296	-0.6718	-0.6574
0.50c	-0.2250	-0.3982	-0.5056	-0.5299	-0.5075
0.60c	-0.1782	-0.3118	-0.4019	-0.4134	-0.3897
0.70c	-0.1278	-0.2419	-0.2954	-0.3029	-0.2938
0.80c	-0.0887	-0.1649	-0.1994	-0.2090	-0.2258

Table A-1 b). NACA 0012, top surface

Bottom Surface	Alpha = 0	Alpha = 4	Alpha = 8	Alpha = 10	Alpha = 14
0.10c	-0.3006	0.0455	0.2852	0.3540	0.4427
0.20c	-0.1599	0.0361	0.1868	0.2326	0.2940
0.30c	-0.0719	0.0623	0.1624	0.1937	0.2314
0.40c	-0.0297	0.0607	0.1327	0.1545	0.1754
0.55c	0.0041	0.0559	0.0975	0.1074	0.1027
0.70c	0.0320	0.0553	0.0683	0.0674	0.0338

Table A-1 c). NACA 4412, bottom surface.

Top Surface	Alpha = 0	Alpha = 4	Alpha = 8	Alpha = 10	Alpha = 14
0.05c	-0.3677	-1.0495	-1.8451	-2.1468	-2.6941
0.10c	-0.5848	-1.0868	-1.6207	-1.8321	-1.3952
0.20c	-0.7410	-1.0741	-1.3974	-1.4619	-1.0159
0.30c	-0.7308	-0.9728	-1.2278	-1.2182	-0.7291
0.40c	-0.6711	-0.8577	-0.9920	-1.0051	-0.7656
0.61c	-0.5075	-0.7233	-0.6517	-0.6165	-0.4032
0.81c	-0.3027	-0.3090	-0.3328	-0.2830	-0.3850

Table A-1 d). NACA 4412, top surface.

Table A-2

Alpha (degrees)	Cl
0	0.00
4	0.53
8	0.87
10	1.01
12	0.77

NACA 0012.

Alpha (degrees)	Cd
0	0.00
4	0.03
8	0.12
10	0.18
12	0.16

NACA 0012

Alpha (degrees)	Lift/Drag
0	0.00
4	15.38
8	7.27
10	5.77
12	4.89

NACA 0012

Alpha	Cl
0	0.44
4	0.74
8	0.95
10	0.99
14	0.84

NACA 4412

Alpha	Cd
0	0.03
4	0.05
8	0.09
10	0.11
14	0.15

NACA 4412

Alpha	Lift/Drag
0	15.78
4	14.69
8	10.53
10	8.79
14	5.72

NACA 4412

References

Computer Software
MATLAB Release 2016a, The MathWorks, Inc., Natick, Massachusetts, United States.

Lab Data
https://myasucourses.asu.edu/webapps/blackboard/content/listContent.jsp?course_id=_361719_1&content_id=_16797327_1

Books
Bertin, J. J., and Cummings, R. M., *Aerodynamics for Engineers*, 5th ed., Ch. 3, 5., Pearson 2008.

Appendix B

Sample Calculations and MATLAB Code

```
clc; clear;

load('aee360_lab2_data')
```

D. Initial Calculations

```
ambientPressure = 96.64*1000; % convert kPa to Pa

    % convert inch of water to Pa
    % 1 inch of water = 248.84 Pa
scanivalve_0012 = scanivalve_0012t.*248.84;
scanivalve_4412b = scanivalve_4412b.*248.84;
scanivalve_4412t = scanivalve_4412t.*248.84;

    % chord length
c = 100; % mm
    % LE to port # distance (mm)
x_0012 = [0,2, 9, 19, 30, 40, 50, 60, 70, 80,100];
    % x/c (mm/mm)
x_0012 = x_0012./c;
    % calculate thickness of airfoil
z_0012 = (0.12/0.2)*(0.296*sqrt(x_0012./c)-0.126*(x_0012./c)-0.3516*...
    (x_0012./c).^2+0.2843*(x_0012./c).^3-0.1015*(x_0012./c).^4);
    % use airfoils symmetry to get z-coordinates for top/bottom surfaces
z_0012t = z_0012./2;
z_0012b = -1.*(z_0012./2);
    % LE to port # horizontal distance (x/c)
x_4412b = [0.1, 0.2, 0.3, 0.4, 0.55, 0.7];
x_4412t = [0.05, 0.1, 0.2, 0.3, 0.4, 0.61, 0.81];
    % LE to port # vertical distance
z_4412b = [-0.0293, -0.0274, -0.0225, -0.0180, -0.0119, -0.0063];
z_4412t = [0.0449, 0.0643, 0.0874, 0.0975, 0.0980, 0.0797, 0.0458];
    % density of air in lab
rho = (ambientPressure/(287*(31+273.15)));
```

```
    % calculate pressure coefficient for both airfoils
cp_0012b = (-1.*scanivalve_0012(1:36))./(dynamicPressure_0012(1:36));
cp_0012t = (scanivalve_0012(37:81))./(dynamicPressure_0012(37:81));
cp_4412b = (scanivalve_4412b)./(dynamicPressure_4412b);
cp_4412t = (scanivalve_4412t)./(dynamicPressure_4412t);

% press_err = 10 Pa
% scani_err = 3 Pa

ambientPress_max = ambientPressure+10;
ambientPress_min = ambientPressure-10;

scanivalve_0012_max = scanivalve_0012+3;
scanivalve_0012_min = scanivalve_0012-3;
scanivalve_4412b_max = scanivalve_4412b+3;
scanivalve_4412b_min = scanivalve_4412b-3;
scanivalve_4412t_max = scanivalve_4412t+3;
scanivalve_4412t_min = scanivalve_4412t-3;

dynamicPressure_0012_max = dynamicPressure_0012+10;
dynamicPressure_0012_min = dynamicPressure_0012-10;

dynamicPressure_4412b_max = dynamicPressure_4412b+10;
dynamicPressure_4412b_min = dynamicPressure_4412b-10;
dynamicPressure_4412t_max = dynamicPressure_4412t+10;
dynamicPressure_4412t_min = dynamicPressure_4412t-10;

    % calculate uncertainties
cp_0012b_max = (-1.*scanivalve_0012_max(1:36))./...
    (dynamicPressure_0012_max(1:36));
cp_0012b_min = (-1.*scanivalve_0012_min(1:36))./...
    (dynamicPressure_0012_min(1:36));
cp_0012t_max = (scanivalve_0012_max(37:81))./...
    (dynamicPressure_0012_max(37:81));
cp_0012t_min = (scanivalve_0012_min(37:81))./...
    (dynamicPressure_0012_min(37:81));

cp_4412b_max = (scanivalve_4412b_max)./(dynamicPressure_4412b_max);
cp_4412b_min = (scanivalve_4412b_min)./(dynamicPressure_4412b_min);
cp_4412t_max = (scanivalve_4412t_max)./(dynamicPressure_4412t_max);
cp_4412t_min = (scanivalve_4412t_min)./(dynamicPressure_4412t_min);

    % calculate uncertainties for each alpha
cp_0012_0_max = [1,cp_0012t_max(1:9)',0];
cp_0012_0_min = [1,cp_0012t_min(1:9)',0];
cp_0012_4b_max = [1,-cp_0012b_max(28:36)',0];
cp_0012_4b_min = [1,-cp_0012b_min(28:36)',0];
cp_0012_4t_max = [1,cp_0012t_max(10:18)',0];
cp_0012_4t_min = [1,cp_0012t_min(10:18)',0];
cp_0012_8b_max = [1,-cp_0012b_max(19:27)',0];
cp_0012_8b_min = [1,-cp_0012b_min(19:27)',0];
cp_0012_8t_max = [1,cp_0012t_max(19:27)',0];
cp_0012_8t_min = [1,cp_0012t_min(19:27)',0];
cp_0012_10b_max = [1,-cp_0012b_max(10:18)',0];
```

```
cp_0012_10b_min = [1,-cp_0012b_min(10:18)',0];
cp_0012_10t_max = [1,cp_0012t_max(28:36)',0];
cp_0012_10t_min = [1,cp_0012t_min(28:36)',0];
cp_0012_12b_max = [1,-cp_0012b_max(1:9)',0];
cp_0012_12b_min = [1,-cp_0012b_min(1:9)',0];
cp_0012_12t_max = [1,cp_0012t_max(37:45)',0];
cp_0012_12t_min = [1,cp_0012t_min(37:45)',0];
    % add end points to airfoil 0012 to close the surface
    % _4b = 4 degrees AoA bottom surface
cp_0012_0 = [1,cp_0012t(1:9)',0];
cp_0012_4b = [1,-cp_0012b(28:36)',0];
cp_0012_4t = [1,cp_0012t(10:18)',0];
cp_0012_8b = [1,-cp_0012b(19:27)',0];
cp_0012_8t = [1,cp_0012t(19:27)',0];
cp_0012_10b = [1,-cp_0012b(10:18)',0];
cp_0012_10t = [1,cp_0012t(28:36)',0];
cp_0012_12b = [1,-cp_0012b(1:9)',0];
cp_0012_12t = [1,cp_0012t(37:45)',0];

cp_4412_0b = [1,cp_4412b(1:6)',0];
cp_4412_0t = [1,cp_4412t(1:7)',0];
cp_4412_4b = [1,cp_4412b(7:12)',0];
cp_4412_4t = [1,cp_4412t(8:14)',0];
cp_4412_8b = [1,cp_4412b(13:18)',0];
cp_4412_8t = [1,cp_4412t(15:21)',0];
cp_4412_10b = [1,cp_4412b(19:24)',0];
cp_4412_10t = [1,cp_4412t(22:28)',0];
cp_4412_14b = [1,cp_4412b(25:30)',0];
cp_4412_14t = [1,cp_4412t(29:35)',0];

cp_4412_0b_max = [1,cp_4412b_max(1:6)',0];
cp_4412_0b_min = [1,cp_4412b_min(1:6)',0];
cp_4412_0t_max = [1,cp_4412t_max(1:7)',0];
cp_4412_0t_min = [1,cp_4412t_min(1:7)',0];
cp_4412_4b_max = [1,cp_4412b_max(7:12)',0];
cp_4412_4b_min = [1,cp_4412b_min(7:12)',0];
cp_4412_4t_max = [1,cp_4412t_max(8:14)',0];
cp_4412_4t_min = [1,cp_4412t_min(8:14)',0];
cp_4412_8b_max = [1,cp_4412b_max(13:18)',0];
cp_4412_8b_min = [1,cp_4412b_min(13:18)',0];
cp_4412_8t_max = [1,cp_4412t_max(15:21)',0];
cp_4412_8t_min = [1,cp_4412t_min(15:21)',0];
cp_4412_10b_max = [1,cp_4412b_max(19:24)',0];
cp_4412_10b_min = [1,cp_4412b_min(19:24)',0];
cp_4412_10t_max = [1,cp_4412t_max(22:28)',0];
cp_4412_10t_min = [1,cp_4412t_min(22:28)',0];
cp_4412_14b_max = [1,cp_4412b_max(25:30)',0];
cp_4412_14b_min = [1,cp_4412b_min(25:30)',0];
cp_4412_14t_max = [1,cp_4412t_max(29:35)',0];
cp_4412_14t_min = [1,cp_4412t_min(29:35)',0];
```

E. Output i
Outputs for Cp values for each alpha

```
fprintf('Cp for Alpha = 0 in 0012:
\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n\n',cp_0012_0(2:10))
fprintf('Cp for Alpha = 4 in 0012:
\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n\n',cp_0012_4b(2:10))
fprintf('Cp for Alpha = 4 in 0012:
\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n\n',cp_0012_4t(2:10))
fprintf('Cp for Alpha = 8 in 0012:
\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n\n',cp_0012_8b(2:10))
fprintf('Cp for Alpha = 8 in 0012:
\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n\n',cp_0012_8t(2:10))
fprintf('Cp for Alpha = 10 in 0012:
\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n\n',cp_0012_10b(2:10))
fprintf('Cp for Alpha = 10 in 0012:
\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n\n',cp_0012_10t(2:10))
fprintf('Cp for Alpha = 12 in 0012:
\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n\n',cp_0012_12b(2:10))
fprintf('Cp for Alpha = 12 in 0012:
\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n\n',cp_0012_12t(2:10))

fprintf('Cp for Alpha = 0 in 4412:
\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n\n',cp_4412_0b(2:7))
fprintf('Cp for Alpha = 0 in 4412:
\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n\n',cp_4412_0t(2:8))
fprintf('Cp for Alpha = 4 in 4412:
\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n\n',cp_4412_4b(2:7))
fprintf('Cp for Alpha = 4 in 4412:
\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n\n',cp_4412_4t(2:8))
fprintf('Cp for Alpha = 8 in 4412:
\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n\n',cp_4412_8b(2:7))
fprintf('Cp for Alpha = 8 in 4412:
\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n\n',cp_4412_8t(2:8))
fprintf('Cp for Alpha = 10 in 4412:
\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n\n',cp_4412_10b(2:7))
fprintf('Cp for Alpha = 10 in 4412:
\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n\n',cp_4412_10t(2:8))
fprintf('Cp for Alpha = 14 in 4412:
\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n\n',cp_4412_14b(2:7))
fprintf('Cp for Alpha = 14 in 4412:
\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n%0.4f\n\n',cp_4412_14t(2:8))
```

F. Interplation & Plotting Cp Distributions

```
xx = linspace(0,1,1000);
    % interpolate data
p1 = pchip(x_0012, cp_0012_0,xx);
p1_min = pchip(x_0012, cp_0012_0_min,xx);
p1_max = pchip(x_0012, cp_0012_0_max,xx);
p2 = pchip(x_0012, cp_0012_4b,xx);
p2_min = pchip(x_0012, cp_0012_4b_min,xx);
p2_max = pchip(x_0012, cp_0012_4b_max,xx);
p3 = pchip(x_0012, cp_0012_4t,xx);
p3_min = pchip(x_0012, cp_0012_4t_min,xx);
```

```
p3_max = pchip(x_0012, cp_0012_4t_max,xx);
p4 = pchip(x_0012, cp_0012_8b,xx);
p4_min = pchip(x_0012, cp_0012_8b_min,xx);
p4_max = pchip(x_0012, cp_0012_8b_max,xx);
p5 = pchip(x_0012, cp_0012_8t,xx);
p5_min = pchip(x_0012, cp_0012_8t_min,xx);
p5_max = pchip(x_0012, cp_0012_8t_max,xx);
p6 = pchip(x_0012, cp_0012_10b,xx);
p6_min = pchip(x_0012, cp_0012_10b_min,xx);
p6_max = pchip(x_0012, cp_0012_10b_max,xx);
p7 = pchip(x_0012, cp_0012_10t,xx);
p7_min = pchip(x_0012, cp_0012_10t_min,xx);
p7_max = pchip(x_0012, cp_0012_10t_max,xx);
p8 = pchip(x_0012, cp_0012_12b,xx);
p8_min = pchip(x_0012, cp_0012_12b_min,xx);
p8_max = pchip(x_0012, cp_0012_12b_max,xx);
p9 = pchip(x_0012, cp_0012_12t,xx);
p9_min = pchip(x_0012, cp_0012_12t_min,xx);
p9_max = pchip(x_0012, cp_0012_12t_min,xx);

    % plot data
    % NACA 0012
    % 0 degrees
figure
plot(xx, p1,'k',xx,p1,'--r','LineWidth',1.5);
hold on
errorbar(x_0012(2:10),cp_0012_0(2:10),(cp_0012_0_max(2:10)-...
    cp_0012_0_min(2:10))./2,'o')
axis ij
grid on
title 'NACA 0012 Pressure Distribution (AoA=0)'
legend('Bottom Surface','Top Surface')
xlabel 'x/c'
ylabel 'Pressure Coefficient'
hold off
    % 4 degrees
figure
plot(xx, p2,'k',xx,p3,'r','LineWidth',1.5);
hold on
errorbar(x_0012(2:10),cp_0012_4b(2:10),(cp_0012_4b_max(2:10)-...
    cp_0012_4b_min(2:10))./2,'o')
errorbar(x_0012(2:10),cp_0012_4t(2:10),(cp_0012_4t_max(2:10)-...
    cp_0012_4t_min(2:10))./2,'o')
axis ij
grid on
title 'NACA 0012 Pressure Distribution (AoA=4)'
legend('Bottom Surface','Top Surface')
xlabel 'x/c'
ylabel 'Pressure Coefficient'
hold off
    % 8 degrees
figure
plot(xx, p4,'k',xx,p5,'r','LineWidth',1.5);
hold on
```

```
errorbar(x_0012(2:10),cp_0012_8b(2:10),(cp_0012_8b_max(2:10)-...
    cp_0012_8b_min(2:10))./2,'o')
errorbar(x_0012(2:10),cp_0012_8t(2:10),(cp_0012_8t_max(2:10)-...
    cp_0012_8t_min(2:10))./2,'o')
axis ij
grid on
title 'NACA 0012 Pressure Distribution (AoA=8)'
legend('Bottom Surface','Top Surface')
xlabel 'x/c'
ylabel 'Pressure Coefficient'
hold off
    % 10 degrees
figure
plot(xx, p6,'k',xx,p7,'r','LineWidth',1.5);
hold on
errorbar(x_0012(2:10),cp_0012_10b(2:10),(cp_0012_10b_max(2:10)-...
    cp_0012_10b_min(2:10))./2,'o')
errorbar(x_0012(2:10),cp_0012_10t(2:10),(cp_0012_10t_max(2:10)-...
    cp_0012_10t_min(2:10))./2,'o')
axis ij
grid on
title 'NACA 0012 Pressure Distribution (AoA=10)'
legend('Bottom Surface','Top Surface')
xlabel 'x/c'
ylabel 'Pressure Coefficient'
hold off
    % 12 degrees
figure
plot(xx, p8,'k',xx,p9,'r','LineWidth',1.5);
hold on
errorbar(x_0012(2:10),cp_0012_12b(2:10),(cp_0012_12b_max(2:10)-...
    cp_0012_12b_min(2:10))./2,'o')
errorbar(x_0012(2:10),cp_0012_12t(2:10),(cp_0012_12t_max(2:10)-...
    cp_0012_12t_min(2:10))./2,'o')
axis ij
grid on
title 'NACA 0012 Pressure Distribution (AoA=12)'
legend('Bottom Surface','Top Surface')
xlabel 'x/c'
ylabel 'Pressure Coefficient'
hold off
    % NACA 4412
    % interplate data
p1 = pchip([0,x_4412b,1],[1;cp_4412b(1:6);-0.1353],xx);
p2 = pchip([0,x_4412t,1],[1;cp_4412t(1:7);-0.1353],xx);
p3 = pchip([0,x_4412b,1],[1;cp_4412b(7:12);-0.12682],xx);
p4 = pchip([0,x_4412t,1],[1;cp_4412t(8:14);-0.12682],xx);
p5 = pchip([0,x_4412b,1],[1;cp_4412b(13:18);-0.13223],xx);
p6 = pchip([0,x_4412t,1],[1;cp_4412t(15:21);-0.13223],xx);
p7 = pchip([0,x_4412b,1],[1;cp_4412b(19:24);-0.10783],xx);
p8 = pchip([0,x_4412t,1],[1;cp_4412t(22:28);-0.10783],xx);
p9 = pchip([0,x_4412b,1],[1;cp_4412b(25:30);-0.17563],xx);
p10 = pchip([0,x_4412t,1],[1;cp_4412t(29:35);-0.17563],xx);
```

```
    % plot data
    % 0 degrees
figure
plot(xx, p1,'k',xx,p2,'r','LineWidth',1.5);
hold on
errorbar(x_4412b,cp_4412_0b(2:7),(cp_4412_0b_max(2:7)-cp_4412_0b_min(2:7))...
    ./2,'o')
errorbar(x_4412t,cp_4412_0t(2:8),(cp_4412_0t_max(2:8)-cp_4412_0t_min(2:8))...
    ./2,'o')
axis ij
grid on
title 'NACA 4412 Pressure Distribution (AoA=0)'
legend('Bottom Surface','Top Surface')
xlabel 'x/c'
ylabel 'Pressure Coefficient'
hold off
    % 4 degrees
figure
plot(xx, p3,'k',xx,p4,'r','LineWidth',1.5);
hold on
errorbar(x_4412b,cp_4412_4b(2:7),(cp_4412_4b_max(2:7)-cp_4412_4b_min(2:7))...
    ./2,'o')
errorbar(x_4412t,cp_4412_4t(2:8),(cp_4412_4t_max(2:8)-cp_4412_4t_min(2:8))...
    ./2,'o')
axis ij
grid on
title 'NACA 4412 Pressure Distribution (AoA=4)'
legend('Bottom Surface','Top Surface')
xlabel 'x/c'
ylabel 'Pressure Coefficient'
hold off
    % 8 degrees
figure
plot(xx, p5,'k',xx,p6,'r','LineWidth',1.5);
hold on
errorbar(x_4412b,cp_4412_8b(2:7),(cp_4412_8b_max(2:7)-cp_4412_8b_min(2:7))...
./2,'o')
errorbar(x_4412t,cp_4412_8t(2:8),(cp_4412_8t_max(2:8)-cp_4412_8t_min(2:8))...
    ./2,'o')
axis ij
grid on
title 'NACA 4412 Pressure Distribution (AoA=8)'
legend('Bottom Surface','Top Surface')
xlabel 'x/c'
ylabel 'Pressure Coefficient'
hold off
    % 10 degrees
figure
plot(xx, p7,'k',xx,p8,'r','LineWidth',1.5);
hold on
errorbar(x_4412b,cp_4412_10b(2:7),(cp_4412_10b_max(2:7)-...
    cp_4412_10b_min(2:7))./2,'o')
errorbar(x_4412t,cp_4412_10t(2:8),(cp_4412_10t_max(2:8)-...
    cp_4412_10t_min(2:8))./2,'o')
```

```
axis ij
grid on
title 'NACA 4412 Pressure Distribution (AoA=10)'
legend('Bottom Surface','Top Surface')
xlabel 'x/c'
ylabel 'Pressure Coefficient'
hold off
    % 14 degrees
figure
plot(xx, p9,'k',xx,p10,'r','LineWidth',1.5);
hold on
errorbar(x_4412b,cp_4412_14b(2:7),(cp_4412_14b_max(2:7)-...
    cp_4412_14b_min(2:7))./2,'o')
errorbar(x_4412t,cp_4412_14t(2:8),(cp_4412_14t_max(2:8)-...
    cp_4412_14t_min(2:8))./2,'o')
axis ij
grid on
title 'NACA 4412 Pressure Distribution (AoA=14)'
legend('Bottom Surface','Top Surface')
xlabel 'x/c'
ylabel 'Pressure Coefficient'
hold off
```

G. Lift & Drag Coefficient Calculations i

Calculating the lift and drag per unit span for NACA 0012 Calculating lift to drag ratios

```
x_0012 = [0,flip(x_0012),0, x_0012,0]';   % set x from TE to LE to TE
    % set z from bottom surface to top surface
z_0012 = [0, flip(z_0012b),0, z_0012t, 0]';

    % set cp same as above
    % 0 degrees
cp_0012_0 = [0;flip(cp_0012_0)';1;cp_0012_0';0];
    % convert from degrees to radians
alpha = 0*(pi/180);
    % integrate cp area numerically
cz_0012_0 = trapz(x_0012, -cp_0012_0);
cx_0012_0 = trapz(z_0012, cp_0012_0);
    % calculate the coefficients of lift/drag
lift_co_0012_0 = cz_0012_0*cos(alpha)-cx_0012_0*sin(alpha);
drag_co_0012_0 = cz_0012_0*sin(alpha)+cx_0012_0*cos(alpha);

    % 4 degrees
cp_0012_4 = [0;flip(cp_0012_4b)';1;cp_0012_4t';0];

alpha = 4*(pi/180); % convert from degrees to radians

cz_0012_4 = trapz(x_0012, -cp_0012_4);
cx_0012_4 = trapz(z_0012, cp_0012_4);

lift_co_0012_4 = cz_0012_4*cos(alpha)-cx_0012_4*sin(alpha);
drag_co_0012_4 = cz_0012_4*sin(alpha)+cx_0012_4*cos(alpha);
```

```
    % 8 degrees
cp_0012_8 = [0;flip(cp_0012_8b)';1;cp_0012_8t';0];

alpha = 8*(pi/180); % convert from degrees to radians

cz_0012_8 = trapz(x_0012, -cp_0012_8);
cx_0012_8 = trapz(z_0012, cp_0012_8);

lift_co_0012_8 = cz_0012_8*cos(alpha)-cx_0012_8*sin(alpha);
drag_co_0012_8 = cz_0012_8*sin(alpha)+cx_0012_8*cos(alpha);

    % 8 degrees
    % minimum coefficient values
cp_0012_8_min = [0;flip(cp_0012_8b_min)';1;cp_0012_8t_max';0];

alpha = 8*(pi/180); % convert from degrees to radians

cz_0012_8_min = trapz(x_0012, -cp_0012_8_min);
cx_0012_8_min = trapz(z_0012, cp_0012_8_min);

lift_co_0012_8_min = cz_0012_8_min*cos(alpha)-cx_0012_8_min*sin(alpha);
drag_co_0012_8_min = cz_0012_8_min*sin(alpha)+cx_0012_8_min*cos(alpha);

    % 8 degrees
    % maximum coefficient values
cp_0012_8_max = [0;flip(cp_0012_8b_max)';1;cp_0012_8t_min';0];

alpha = 8*(pi/180); % convert from degrees to radians

cz_0012_8_max = trapz(x_0012, -cp_0012_8_max);
cx_0012_8_max = trapz(z_0012, cp_0012_8_max);

lift_co_0012_8_max = cz_0012_8_max*cos(alpha)-cx_0012_8_max*sin(alpha);
drag_co_0012_8_max = cz_0012_8_max*sin(alpha)+cx_0012_8_max*cos(alpha);

    % uncertainties
delta_cl_0012_8 = (lift_co_0012_8_max-lift_co_0012_8_min)/2;
err_cl_0012_8 = [lift_co_0012_8-delta_cl_0012_8,lift_co_0012_8+...
    delta_cl_0012_8];

delta_cd_0012_8 = (drag_co_0012_8_max-drag_co_0012_8_min)/2;
err_cd_0012_8 = [drag_co_0012_8-delta_cd_0012_8,drag_co_0012_8+...
    delta_cd_0012_8];

    % 10 degrees
cp_0012_10 = [0;flip(cp_0012_10b)';1;cp_0012_10t';0];

alpha = 10*(pi/180); % convert from degrees to radians

cz_0012_10 = trapz(x_0012, -cp_0012_10);
cx_0012_10 = trapz(z_0012, cp_0012_10);

lift_co_0012_10 = cz_0012_10*cos(alpha)-cx_0012_10*sin(alpha);
drag_co_0012_10 = cz_0012_10*sin(alpha)+cx_0012_10*cos(alpha);
```

```
    % 12 degrees
cp_0012_12 = [0;flip(cp_0012_12b)';1;cp_0012_12t';0];

alpha = 12*(pi/180); % convert from degrees to radians

cz_0012_12 = trapz(x_0012, -cp_0012_12);
cx_0012_12 = trapz(z_0012, cp_0012_12);

lift_co_0012_12 = cz_0012_12*cos(alpha)-cx_0012_12*sin(alpha);
drag_co_0012_12 = cz_0012_12*sin(alpha)+cx_0012_12*cos(alpha);
```

H. Lift & Drag Curves i

Plotting the lift, drag, and lift to drag curves for NACA 0012

```
alpha = [0 4 8 10 12]; % define alpha angles
alphax = linspace(0,12,1000);

lift_co_0012 = [lift_co_0012_0,lift_co_0012_4,lift_co_0012_8,...
    lift_co_0012_10,lift_co_0012_12];

drag_co_0012 = [drag_co_0012_0, drag_co_0012_4, drag_co_0012_8,...
    drag_co_0012_10, drag_co_0012_12];

p = pchip(alpha, drag_co_0012, alphax);

figure
plot(alphax, p,'b', alpha, drag_co_0012,'b*', 'LineWidth',2, 'MarkerSize',8)
grid on
title 'Drag Coefficient vs Alpha (NACA 0012)'
xlabel 'Alpha (degrees)'
ylabel 'Drag Coefficient'
legend('Interpolated Data','Experimental Data','Location','NorthWest')

c = 0.1; % meters
    % find a mean dynamic pressure for each alpha
dynamicPressure_0012_0 = mean(dynamicPressure_0012(37:45));
dynamicPressure_0012_4 = mean(mean([dynamicPressure_0012(28:36),...
    dynamicPressure_0012(46:54)]));
dynamicPressure_0012_8 = mean(mean([dynamicPressure_0012(19:27),...
    dynamicPressure_0012(55:63)]));
dynamicPressure_0012_10 = mean(mean([dynamicPressure_0012(10:18),...
    dynamicPressure_0012(64:72)]));
dynamicPressure_0012_12 = mean(mean([dynamicPressure_0012(1:9),...
    dynamicPressure_0012(73:81)]));

dynamicPressure_0012 = [dynamicPressure_0012_0,dynamicPressure_0012_4,...
    dynamicPressure_0012_8,dynamicPressure_0012_10,dynamicPressure_0012_12];

lift_0012 = lift_co_0012.*dynamicPressure_0012*c;
drag_0012 = drag_co_0012.*dynamicPressure_0012*c;

lift_drag_ratio_0012 = lift_0012./drag_0012;
```

```
p = pchip(alpha, lift_drag_ratio_0012, alphax);

figure
plot(alpha, lift_drag_ratio_0012,'r*',alphax,p,'r','LineWidth',2,...
    'MarkerSize',8)
grid on
title 'Lift/Drag Ratio vs Alpha (NACA 0012)'
xlabel 'Alpha (degrees)'
ylabel 'Lift/Drag Ratio'
legend('Experimental Data','Interpolated Data')
```

I. Lift & Drag Coefficient Calculations ii

Calculating the lift and drag per unit span for NACA 4412 Calculating lift to drag ratios

```
x_4412 = [1, flip(x_4412b),0, x_4412t,1]';
z_4412 = [0,flip(z_4412b),0, z_4412t,0]';

    % 0 degrees
cp_4412_0 = [-0.1353;flip(cp_4412b(1:6));1;cp_4412t(1:7);-0.1353];

alpha = 0*(pi/180); % convert from degrees to radians

% integrate the enclosed area of the pressure distributions
cz_4412_0 = trapz(x_4412, -cp_4412_0);
cx_4412_0 = trapz(z_4412, cp_4412_0);

lift_co_4412_0 = cz_4412_0*cos(alpha)-cx_4412_0*sin(alpha);
drag_co_4412_0 = cz_4412_0*sin(alpha)+cx_4412_0*cos(alpha);

    % 4 degrees
cp_4412_4 = [-0.12682;flip(cp_4412b(7:12));1;cp_4412t(8:14);-0.12682];

alpha = 4*(pi/180); % convert from degrees to radians

% integrate the enclosed area of the pressure distributions
cz_4412_4 = trapz(x_4412, -cp_4412_4);
cx_4412_4 = trapz(z_4412, cp_4412_4);

lift_co_4412_4 = cz_4412_4*cos(alpha)-cx_4412_4*sin(alpha);
drag_co_4412_4 = cz_4412_4*sin(alpha)+cx_4412_4*cos(alpha);

    % 8 degres
cp_4412_8 = [-0.13223;flip(cp_4412b(13:18));1;cp_4412t(15:21);-0.13223];

alpha = 8*(pi/180); % convert from degrees to radians

    % integrate the enclosed area of the pressure distributions
cz_4412_8 = trapz(x_4412, -cp_4412_8);
cx_4412_8 = trapz(z_4412, cp_4412_8);

lift_co_4412_8 = cz_4412_8*cos(alpha)-cx_4412_8*sin(alpha);
drag_co_4412_8 = cz_4412_8*sin(alpha)+cx_4412_8*cos(alpha);
```

```
    % 8 degrees minimum coefficient
cp_4412_8_min = [-0.13223,flip(cp_4412b_min(13:18))',1,cp_4412t_max(15:21)'...
    ,-0.13223];

alpha = 8*(pi/180); % convert from degrees to radians

cz_4412_8_min = trapz(x_4412, -cp_4412_8_min);
cx_4412_8_min = trapz(z_4412, cp_4412_8_min);

lift_co_4412_8_min = cz_4412_8_min*cos(alpha)-cx_4412_8_min*sin(alpha);
drag_co_4412_8_min = cz_4412_8_min*sin(alpha)+cx_4412_8_min*cos(alpha);

    % 8 degrees maximum coefficient
cp_4412_8_max = [-0.13223,flip(cp_4412b_max(13:18))',1,cp_4412t_min(15:21)'...
    ,-0.13223];

alpha = 8*(pi/180); % convert from degrees to radians

cz_4412_8_max = trapz(x_4412, -cp_4412_8_max);
cx_4412_8_max = trapz(z_4412, cp_4412_8_max);

lift_co_4412_8_max = cz_4412_8_max*cos(alpha)-cx_4412_8_max*sin(alpha);
drag_co_4412_8_max = cz_4412_8_max*sin(alpha)+cx_4412_8_max*cos(alpha);

    % uncertainties for NACA 4412
delta_cl_4412_8 = (lift_co_4412_8_max-lift_co_4412_8_min)/2;
err_cl_4412_8 = [lift_co_4412_8-delta_cl_4412_8,lift_co_4412_8+...
    delta_cl_4412_8];

delta_cd_4412_8 = (drag_co_4412_8_max-drag_co_4412_8_min)/2;
err_cd_4412_8 = [drag_co_4412_8-delta_cd_4412_8,drag_co_4412_8+...
    delta_cd_4412_8];

    % 10 degrees
cp_4412_10 = [-0.10783;flip(cp_4412b(19:24));1;cp_4412t(22:28);-0.10783];

alpha = 10*(pi/180); % convert from degrees to radians

% integrate the enclosed area of the pressure distributions
cz_4412_10 = trapz(x_4412, -cp_4412_10);
cx_4412_10 = trapz(z_4412, cp_4412_10);

lift_co_4412_10 = cz_4412_10*cos(alpha)-cx_4412_10*sin(alpha);
drag_co_4412_10 = cz_4412_10*sin(alpha)+cx_4412_10*cos(alpha);

    % 14 degrees
cp_4412_14 = [-0.17563;flip(cp_4412b(25:30));1;cp_4412t(29:35);-0.17563];

alpha = 14*(pi/180); % convert from degrees to radians

% integrate the enclosed area of the pressure distributions
cz_4412_14 = trapz(x_4412, -cp_4412_14);
cx_4412_14 = trapz(z_4412, cp_4412_14);
```

```
lift_co_4412_14 = cz_4412_14*cos(alpha)-cx_4412_14*sin(alpha);
drag_co_4412_14 = cz_4412_14*sin(alpha)+cx_4412_14*cos(alpha);
```

J. Lift & Drag Curves ii
Plotting the lift, drag, and lift to drag curves for NACA 4412

```
alpha = [0 4 8 10 14];
alphax = linspace(0,14,1000);

lift_co_4412 = [lift_co_4412_0,lift_co_4412_4,lift_co_4412_8,...
    lift_co_4412_10,lift_co_4412_14];

p = pchip(alpha, lift_co_4412, alphax);

figure
plot(alphax,p,'k',alpha,lift_co_4412,'k*', 'LineWidth', 2, 'MarkerSize',8)
grid on
title 'Lift Coefficient vs Alpha (NACA 4412)'
xlabel 'Alpha (degrees)'
ylabel 'Lift Coefficient'
legend('Interpolated Data','Experimental Data','Location','NorthWest')

drag_co_4412 = [drag_co_4412_0, drag_co_4412_4, drag_co_4412_8,...
    drag_co_4412_10, drag_co_4412_14];
p = pchip(alpha, drag_co_4412, alphax);

figure
plot(alphax, p,'b', alpha, drag_co_4412,'b*', 'LineWidth',2, 'MarkerSize',8)
title 'Drag Coefficient vs Alpha (NACA 4412)'
xlabel 'Alpha (degrees)'
ylabel 'Drag Coefficient'
legend('Interpolated Data','Experimental Data','Location','NorthWest')
grid on

    % find mean dynamic pressure for each alpha
dynamicPressure_4412_0 = mean(mean([dynamicPressure_4412b(1:6);...
    dynamicPressure_4412t(1:7)]));
dynamicPressure_4412_4 = mean(mean([dynamicPressure_4412b(7:12);...
    dynamicPressure_4412t(8:14)]));
dynamicPressure_4412_8 = mean(mean([dynamicPressure_4412b(13:18);...
    dynamicPressure_4412t(15:21)]));
dynamicPressure_4412_10 = mean(mean([dynamicPressure_4412b(19:24);...
    dynamicPressure_4412t(22:28)]));
dynamicPressure_4412_14 = mean(mean([dynamicPressure_4412b(25:30);...
    dynamicPressure_4412t(29:35)]));

dynamicPressure_4412 = [dynamicPressure_4412_0,dynamicPressure_4412_4,...
    dynamicPressure_4412_8,dynamicPressure_4412_10,dynamicPressure_4412_14];

lift_4412 = lift_co_4412.*dynamicPressure_4412*c;
drag_4412 = drag_co_4412.*dynamicPressure_4412*c;
```

```
lift_drag_ratio_4412 = lift_4412./drag_4412;

p = pchip(alpha, lift_drag_ratio_4412, alphax);

figure
plot(alpha, lift_drag_ratio_4412,'r*',alphax,p,'r','LineWidth',2,...
    'MarkerSize',8)
grid on
title 'Lift/Drag Ratio vs Alpha (NACA 4412)'
xlabel 'Alpha (degrees)'
ylabel 'Lift/Drag Ratio'
legend('Experimental Data','Interpolated Data')
```

K. Output ii

```
fprintf(['The lift coefficients for NACA 0012 are:\nalpha=0: %0.2f\nalpha=4: ',...
    '%0.2f\nalpha=8: %0.2f\nalpha=10: %0.2f\nalpha=12: %0.2f\n\n'],...
    lift_co_0012)
fprintf(['The drag coefficients for NACA 0012 are:\nalpha=0: %0.2f\nalpha=4: ',...
    '%0.2f\nalpha=8: %0.2f\nalpha=10: %0.2f\nalpha=12: %0.2f\n\n'],...
    drag_co_0012)
fprintf(['The lift/drag ratios for NACA 0012 are:\nalpha=0: %0.2f\nalpha=4: ',...
    '%0.2f\nalpha=8: %0.2f\nalpha=10: %0.2f\nalpha=12: %0.2f\n\n'],...
    lift_drag_ratio_0012)

fprintf(['The lift coefficients for NACA 4412 are:\nalpha=0: %0.2f\nalpha=4: ',...
    '%0.2f\nalpha=8: %0.2f\nalpha=10: %0.2f\nalpha=14: %0.2f\n\n'],...
    lift_co_4412)
fprintf(['The drag coefficients for NACA 4412 are:\nalpha=0: %0.2f\nalpha=4: ',...
    '%0.2f\nalpha=8: %0.2f\nalpha=10: %0.2f\nalpha=14: %0.2f\n\n'],...
    drag_co_4412)
fprintf(['The lift/drag ratios for NACA 4412 are:\nalpha=0: %0.2f\nalpha=4: ',...
    '%0.2f\nalpha=8: %0.2f\nalpha=10: %0.2f\nalpha=14: %0.2f\n\n'],...
    lift_drag_ratio_4412)

fprintf(['With uncertainties we can expect the cofficient of lift for NACA ',...
    '0012 when AoA=8 to be between %0.4f & %0.4f\n'], err_cl_0012_8)
fprintf(['With uncertainties we can expect the cofficient of drag for NACA ',...
    '0012 when AoA=8 to be between %0.4f & %0.4f\n\n'], err_cd_0012_8)

fprintf(['With uncertainties we can expect the cofficient of lift for NACA ',...
    '4412 when AoA=8 to be between %0.4f & %0.4f\n'], err_cl_4412_8)
fprintf(['With uncertainties we can expect the cofficient of drag for NACA ',...
    '4412 when AoA=8 to be between %0.4f & %0.4f\n\n'], err_cd_4412_8)
```

L. Thin Airfoil Theory

Compare results with thin airfoil theory

```
alpha_4412 = [0 4 8 10 14];
alphax_4412 = linspace(0,14,1000);

lift_co_4412 = [lift_co_4412_0,lift_co_4412_4,lift_co_4412_8,...
```

```
    lift_co_4412_10,lift_co_4412_14];

alpha_0012 = [0 4 8 10 12];
alphax_0012 = linspace(0,12,1000);

lift_co_0012 = [lift_co_0012_0,lift_co_0012_4,lift_co_0012_8,...
    lift_co_0012_10,lift_co_0012_12];

p1 = pchip(alpha_0012, lift_co_0012, alphax_0012);

p2 = pchip(alpha_4412, lift_co_4412, alphax_4412);
alpha_rad = alphax_4412.*(pi/180);
cl_approx = 2*pi*alpha_rad;

figure
h1 = plot(alphax_4412,p2,'r',alpha_4412,lift_co_4412,'r*', 'LineWidth', 2, 'MarkerSize',8);
hold on
h2 = plot(alphax_0012,p1,'k',alpha_0012,lift_co_0012,'k*', 'LineWidth', 2, 'MarkerSize',8);
h3 = plot(alphax_4412, cl_approx,'b','LineWidth', 2);
grid on
title 'Lift Coefficient Experimental vs Thin Airfoil Theory'
xlabel 'Alpha (degrees)'
ylabel 'Lift Coefficient'
legend([h1(1), h2(1),h3],'NACA 4412','NACA 0012','Thin Airfoil
Approximation','Location','NorthWest')
hold off
```