Using an Error Budget Approach to Estimate Muzzle Velocity Variation

By Jeff Siewert

Background

Dedicated long-range shooters know that reducing variation in muzzle velocity is key to shooting small groups at extended ranges. Many factors influence muzzle velocity variation, and variation in muzzle velocity influences group size at the target; both directly and indirectly. To obtain a reasonable estimate of the muzzle velocity variation, shooters need to identify the factors involved in changing muzzle velocity, calculate the sensitivity to each factor, and then multiply the sensitivity by the measured or calculated error. The product of each factor is then squared, summed, and the square root taken to estimate the expected muzzle velocity variation. Conversely, since the advent of (relatively) inexpensive and accurate chronographs, the total true variation can be accurately measured (with sufficient samples), and upper bounds placed on some of the unknown velocity variation components.

The advantage of the error budget approach to estimating muzzle velocity standard deviation is that it can identify which factors are largest so you can decide which factors to attack to reduce the muzzle velocity variation.

From a targeting standpoint, here are the effects on target:

Direct:

- Vertical Stringing due to change in Time of Flight to the target from MV variation
- Horizontal impact differences due to changes drift due to yaw of repose (AKA "spin drift")

Indirect:

- Increased sensitivity to cross winds due to MV variation increases scatter in fall of shot the horizontal axis.
- Changes in "initialization jump" will cause increased scatter in vertical fall of shot when firing in a cross wind.

How many shots are needed for a good MV Sigma measurement? Figure 1 shows the error in measuring standard deviation as a fraction of the true "average" muzzle deviation value and the sample size (number of shots fired). To get the error of muzzle velocity standard deviation below 20% (on average) the shooter will have to fire 14 rounds; to reduce it to 15%, 24 rounds are required.

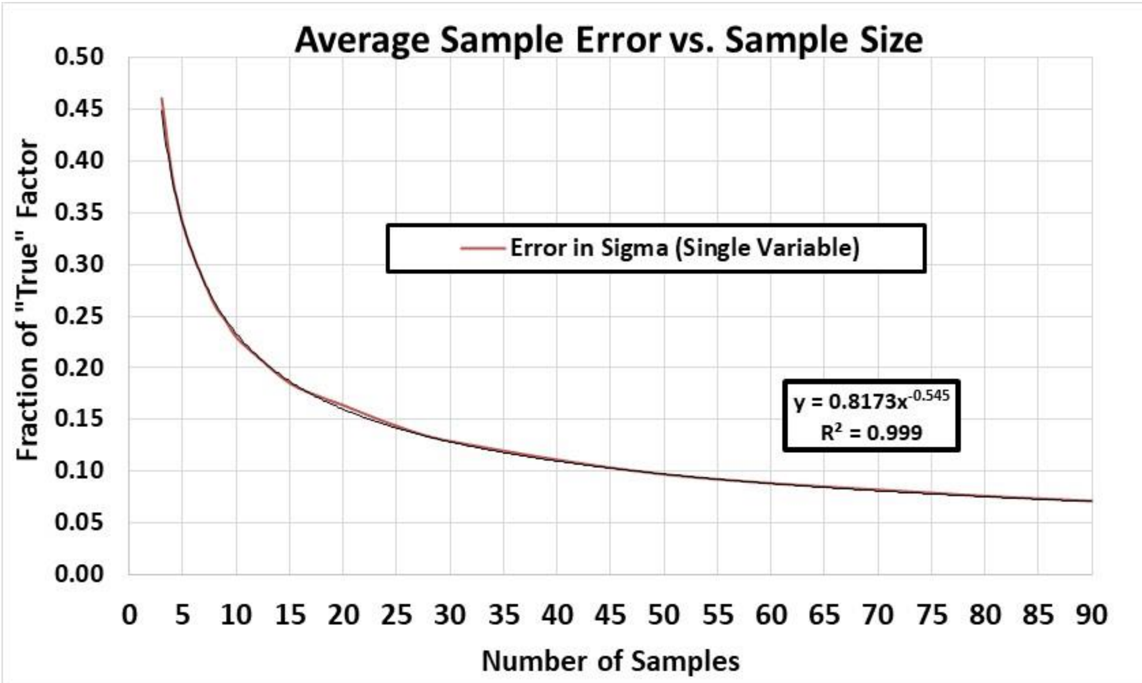

Figure 1: Error in MV Standard Deviation vs. Sample Size

Low muzzle velocity Standard Deviation means an accurate average MV measurement, which provides inputs for ballistic calculators.

The Error Budget Approach

Error budgets are typically used for assessing fall of shot of various gun systems at extended range. Errors usually fall into one of two categories, bias errors and random errors. The same is true when applying an error budget approach to estimating muzzle velocity variations; there are some factors (e.g. case volume and shape) that influence the muzzle velocity and the variations, while other factors influence only the muzzle velocity variations. For the purposes of this article, the bias effects will be ignored, and the focus will be on the factors influencing the random error components affecting muzzle velocity.

To estimate the expected muzzle velocity variation, each factor affecting muzzle velocity must be identified, the sensitivity of bullet muzzle velocity must be determined, and lastly, an estimate of the error of each factor needs to be made. This approach is useful because it allow comparison among the various factors, allowing them to be prioritized. Further, if the muzzle velocity variation is measured via chronograph, it allows the factors to be “calibrated” to reflect real world conditions.

Below are listed the factors influencing muzzle velocity variation and how knowledgeable shooters can quantify both sensitivity and error for each:

Charge weight variation

Shot-to-shot differences in propellant charge weight directly affect the energy available in the case and can thus affect muzzle velocity. Shooters can consult reloading manuals to determine the slope of muzzle velocity with charge weight (FPS/grain) and choosing the powder with lowest muzzle velocity sensitivity to changes in charge weight is a good idea. This sensitivity factor has a positive slope as MV increases with increasing charge weight.

Table 1 shows the minimum and maximum charge weights for various propellants when firing 140g bullet in the 6.5mm Creedmoor cartridge.

Hornady Reloading Manual, 10th Edition. 6.5 Creedmoor, 140-143gg Bullet

Propellant Designation	Min Charge, Grains	Min Charge MV, FPS	Max Charge, Grains	Max Charge MV, FPS		Propellant Geometry	Max-Min MV Sens., FPS/grain	Max Charge Efficiency, FPS/grain
Norma 203-B	32.5	2400	37.3	2650		Tubular	52.1	71.0
Varget	32.7	2400	36.4	2600		Tubular	54.1	71.4
Alliant RL-15	33.5	2400	37.9	2650		Tubular	56.8	69.9
BIG GAME	35.1	2400	41.4	2750		Tubular	55.6	66.4
Alliant RL-17	35.3	2400	41.3	2750		Tubular	58.3	66.6
Norma URP	35.3	2400	41.4	2750		Tubular	57.4	66.4
Hybrid 100V	35.6	2400	41.3	2750		Ball	61.4	66.6
WIN 760	35.9	2400	41.3	2700		Ball	55.6	65.4
IMR 4350	36.1	2400	42.0	2750		Tubular	59.3	65.5
H 4350	35.6	2400	41.5	2700		Tubular	50.8	65.1
IMR 4451	36.6	2400	41.3	2650		Tubular	53.2	64.2
Superformance	38.2	2400	44.7	2800		Ball	61.5	62.6

Faster Relative Quickness
Slower Relative Quickness

Table 1: Min and Max Charge Weights for 6.5mm Creedmoor, 140g Bullet

The H4350 propellant is highlighted with a red circle because it is the powder with the lowest muzzle velocity sensitivity to changes in powder weight for this cartridge and bullet weight, shown in the second column from the right.

As far as charge weight error goes, the reloader can quickly estimate the error in dispensing propellant. If the propellant is being dispensed volumetrically, a sample of 25 charges, weighed and recorded will provide an idea of the charge weight variation with volumetric charging by entering the data into a spreadsheet and performing a statistical query for standard deviation. If the powder is being "trickled" to final weight, the error would be one-half the resolution of the scale in question, typically 0.05 grain.

Chamber volume variation

An increase in internal case volume (by itself) typically causes a decrease in peak chamber pressure developed, so this sensitivity factor of FPS/cubic inch has a negative slope. Since a given lot of cartridge cases have likely all been formed on the same tooling, the variation in internal volume caused by case-to-case changes in wall thickness are typically quite small, percentwise. Most of the variation in case internal volume arises due to case-to-case changes in the internal head thickness, but that doesn't mean the case weight is reasonably well correlated to internal volume. Figure 2 shows the case weight vs. internal volume for 25-06 cartridge case made by several case manufacturers. Except for the converted military cases, the cases don't show very strong correlation (the R^2 factor would be >0.5 or < -0.5 for good correlation) of volume with case weight.

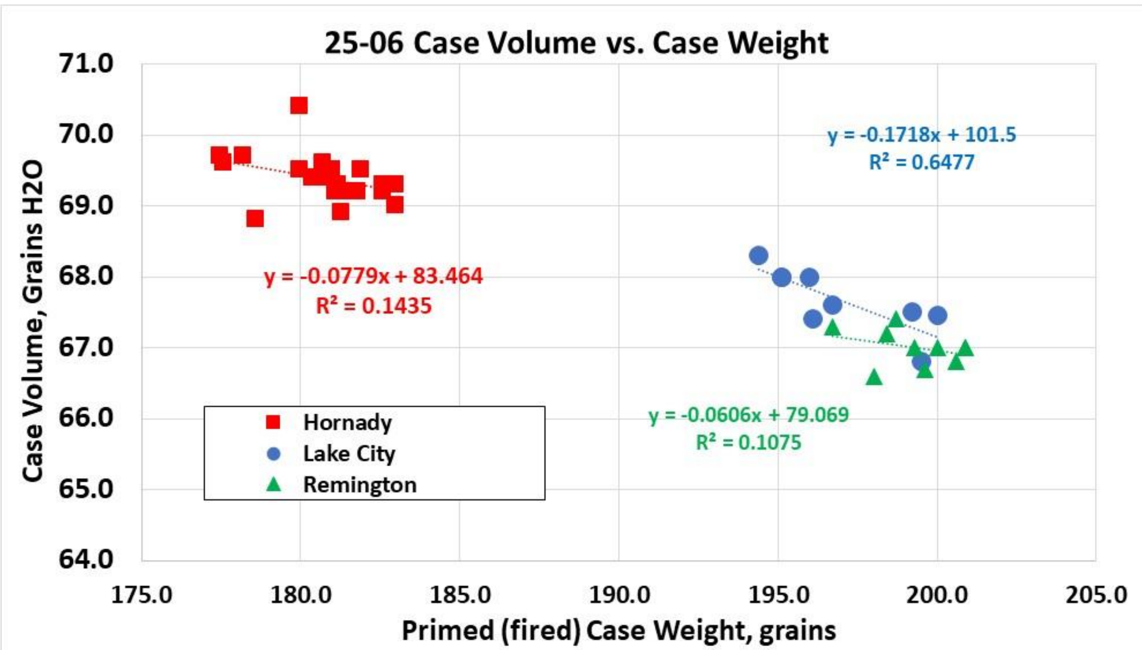

Figure 2: Case Weight vs. Internal Volume, 25-06 Cases

The sensitivity to changes in case volume can be determined computationally by interior ballistics codes such as QuickLOAD or numerical integration codes. For purposes of this paper, I simulated the performance of a 140g bullet in a 6.5mm Creedmoor cartridge with H4350 propellant. I adjusted the case volume in QuickLOAD until the free volume in the case matched what I obtained in a solid modeling program, with the volume of the flash hole included in the assessment. I ran a baseline simulation with peak pressure and muzzle velocity in reasonable agreement, and then changed the case volume in QuickLOAD until the free volume matched the desired free volume change. I then ran the simulation and recorded the new muzzle velocity. The change in muzzle velocity divided by the change in free volume in the case is the "case volume" sensitivity number. I repeated this process in the Baer-Frankle numerical integration interior ballistics code. As will be shown later, this factor is likely to cause the largest swing in muzzle velocity of factors assessed.

Interestingly, the muzzle velocity sensitivity to changes in internal case volume in QuickLOAD is approximately twice the sensitivity determined by Baer-Frankle numerical integration interior ballistics. This is a computational "quirk" which bears more investigation.

The magnitude of the case volume error case-to-case within a lot can be determined by weighing a batch of cases (use the same lot of cases or at least the same headstamp) and converting the standard deviation in brass case weight in grains to cubic inches by dividing by a factor of 252.892.

Bullet mass variation

As bullet weight increases, the muzzle velocity would be expected to decrease, so this factor also has a negative slope with bullet weight. But this factor isn't as sensitive as first appearances might indicate because increased bullet weight also increases peak pressure, partially compensating the muzzle velocity for the increase in bullet weight. The sensitivity to changes in bullet weight can be determined computationally by interior ballistics codes such as QuickLoad or numerical integration codes. Remarkably, Quickload and numerical integration code yield the same sensitivity for bullet mass variation: 5.0 FPS / grain for Alliant Reloader 17 in the 6.5mm Creedmoor firing 140g ELD-M.

Peak Engraving Pressure Variation

Variation in peak engraving pressure is very likely to be one of the largest factors in muzzle velocity error budget computation. Unfortunately, the shot-to-shot variation in this parameter is determined by random changes in the bullet-barrel interface, and the reloader can't do much to control the variation of this parameter. This factor has a positive slope because increases in resistance pressure reduce the rate of change of volume behind the bullet as it travels down the bore, increasing the peak pressure attained and also the muzzle velocity.

The sensitivity to changes in resistance pressure can be determined computationally by interior ballistics codes such as QuickLOAD or numerical integration codes. The FPS/1% change in resistance pressure is a useful sensitivity factor; however, measurements indicate the smallest the engraving pressure (force) variation is likely to be is approximately 5%. The author made engraving pressure measurements on 5.56mm and 7.62mm projectiles in the 2000-2004 timeframe on cup and core bullets, as well as monolithic bullets.

The variation in engraving pressure as a function of in-bore travel and bullet type for several 150g, 30 caliber bullets is shown in Figure 3. The lowest engraving pressure variation is approximately 5%.

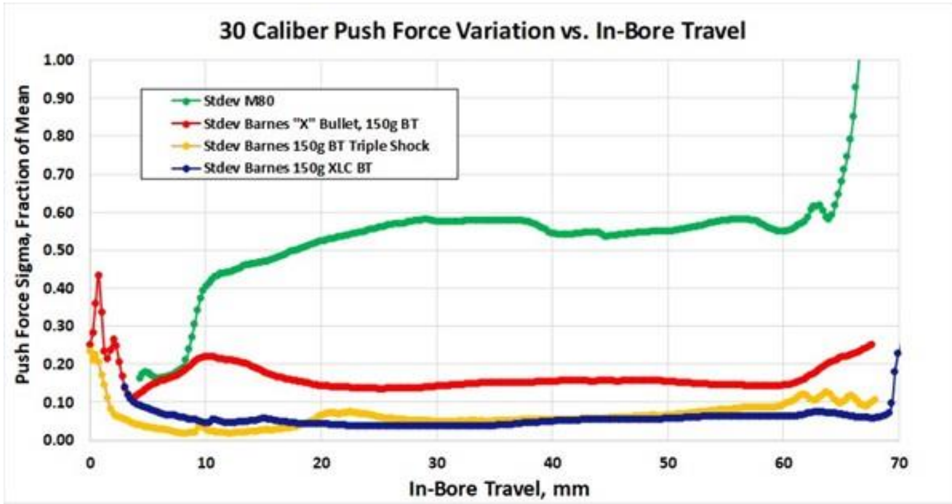

Figure 3: Push Force Variation vs Travel as Fraction of Mean Peak Engraving Pressure

Shot Start Pressure Variation

Variation in shot start pressure (the pressure required to move the bullet from the case mouth) have a positive slope because an increase in shot start pressure also increases the peak pressure attained, and the muzzle velocity along with it. Factor units of FPS/1% change in shot start pressure are used.

The sensitivity to changes in shot start pressure cannot be determined computationally by interior ballistics codes such as QuickLOAD but can by numerical integration codes. The muzzle velocity sensitivity to shot start was studied with a Baer-Frankle lumped parameter interior ballistics code, and a sensitivity of 0.15 FPS per 1% change in shot start pressure was obtained.

A brief study of a seating pressure for a typical small caliber system (30-06 with 178g bullet, see the article on Push Force testing) indicated a variation of 20% in shot start pressure was typical. Unnervingly, there was virtually no correlation between the case mouth diameter, case wall thickness and the measured seating pressure in both 25 and 30 caliber, so this is another factor affecting muzzle velocity beyond the control of the reloader. The shot start pressure is also expected to be less than shown in the accompanying plots due to the yielding of the case neck during bullet insertion.

Reloaders can minimize the shot start pressure by annealing the case mouth but should be aware that “0.002 inches of neck tension” is a measure of dimension, not shot start pressure, unless one can generate a non-linear stress-strain curve for the brass and the thickness of the case neck is known.

The variation in peak shot start pressure for 25 caliber and 30 caliber is shown in Figure 4.

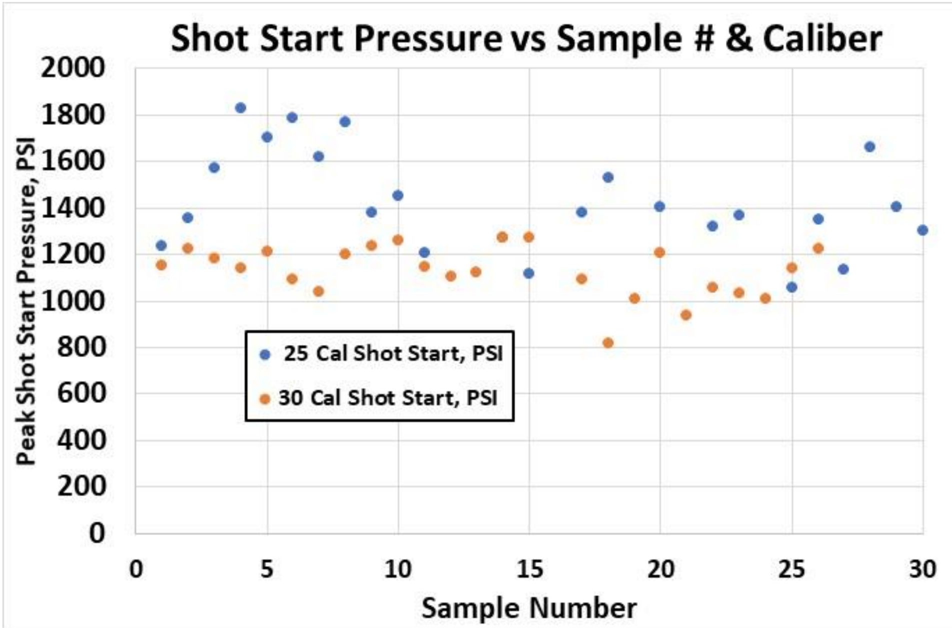

Figure 4: Peak Shot Start Pressure vs Sample Number for 25 & 30 cal Systems

Figure 5 shows the average and standard deviation in shot start pressure vs. caliber for the 25 caliber and 30 caliber cartridges tested. It should be noted that there was essentially zero correlation between measured shot start pressure and the interference between the cartridge case mouth inside diameter and the bullet, nor any with wall thickness.

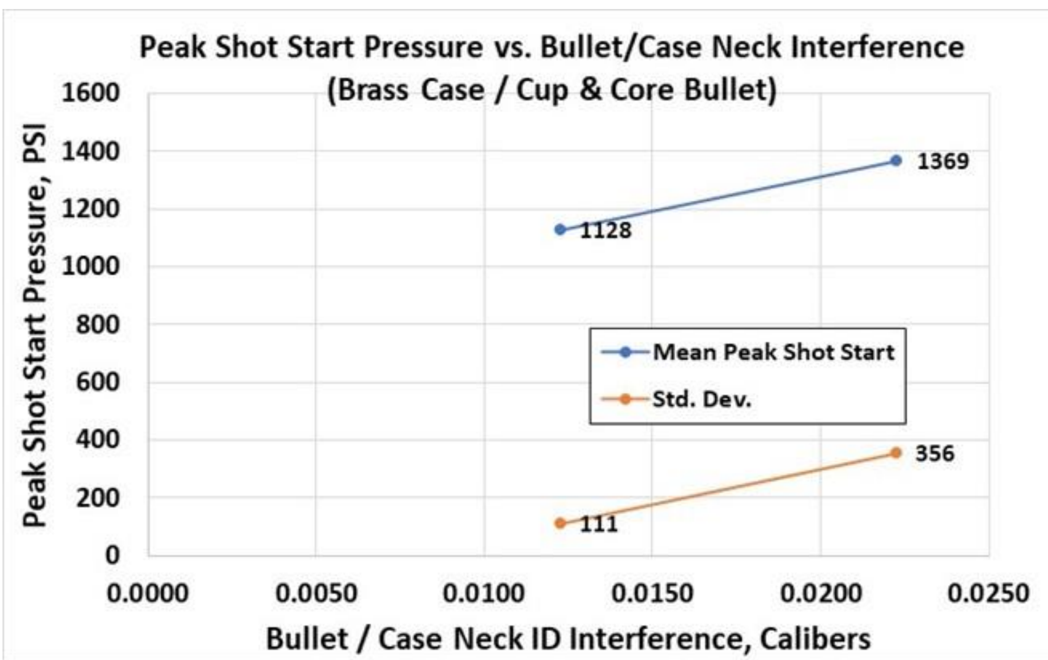

Figure 5: Avg & Std Deviation of Shot Start Pressure vs Bullet/Case Neck Interference for 25 & 30 Cal cartridges

Free Run Variation

Shot-to-shot changes in free run of the bullet to the start of engraving that don't affect the volume in the cartridge case (e.g. minor changes in bullet ogive shape) affect the muzzle velocity. An increase in free run typically results in a decrease in muzzle velocity due to a reduction in the peak pressure developed for a given shot. Thus, this factor also has a negative slope.

The sensitivity to changes in free run to first engraving cannot be determined computationally by interior ballistics codes such as QuickLOAD but can by numerical integration codes. The muzzle velocity sensitivity to free run to first engraving was studied with a Baer-Frankle lumped parameter interior ballistics code, and a sensitivity of 3.2 FPS per 0.010" change in shot free run was obtained WITHOUT a change in internal volume in the case. This yields a sensitivity of 320 FPS/inch of free run change. Be aware, this is assumed to be a linear effect for a very narrow range of free run changes from the baseline say no more than 0.010 inches; in the real world, a free run change of 0.030-0.050 would certainly be expected to have a larger change on peak pressure and resulting muzzle velocity.

Cartridge Overall Length

Changes in overall loaded length of the cartridge have two factors affecting peak pressure and muzzle velocity "fighting" one another. An increase in overall loaded length reduces the free run to the start of engraving which would tend to push peak pressure and muzzle velocity up, while also increasing the free volume in the case, which would tend to reduce peak pressure and resulting muzzle velocity. This effect cannot be accurately studied with QuickLOAD due to the lack of a "free run to the start of engraving" input in the code. Whether this factor has a positive or negative slope depends on many factors: case volume, bullet weight, propellant type and charge weight, primer, forcing cone details, free run to the start of engraving, etc. QuickLOAD can assess the change in free volume in the case part of this effect, but the effect of an increased run to the lands (e.g. "long throat") cannot be accurately assessed.

The effect of a change in overall cartridge length was assessed using Baer-Frankle, taking care to simultaneously modify (decrease) the free volume in the case and increase the free run to the rifling. For the 140g bullet in 6.5mm Creedmoor cartridge with 41.3 grains of H4350, the sensitivity to changes in seating depth was determined to be -70 FPS/inch of overall length change. As with free run variation, this is assumed to be a linear effect for a very narrow range of free run changes from the baseline say no more than 0.010 inches; in the real world, a free run change of 0.030-0.050 would certainly be expected to have a larger change on peak pressure and resulting muzzle velocity.

Bullet Diameter Variation

The diameter of the bullet affects the average resistance pressure of the bullet as it passes through the forcing cone and is engraved by the rifling. If the bullet is larger in diameter than the groove diameter of the barrel, a dramatic increase in resistance pressure is seen compared to bullets that only interfere with the lands of the barrel. This is because the barrel grooves are typically much wider than the lands, and bullet interference with the bore around the whole circumference of the bullet moves more material than the bullet just being engraved by the lands.

Figure 6 shows the test data on average peak pressure and peak pressure standard deviation for a 90 grain .243 caliber copper alloy bullet with varying outside diameters with primer, powder and cases from the same lot. The effect of this change cannot be studied directly with QuickLOAD or Baer-Frankle unless the user has measured the effect of changes in bullet diameter on the peak engraving force.

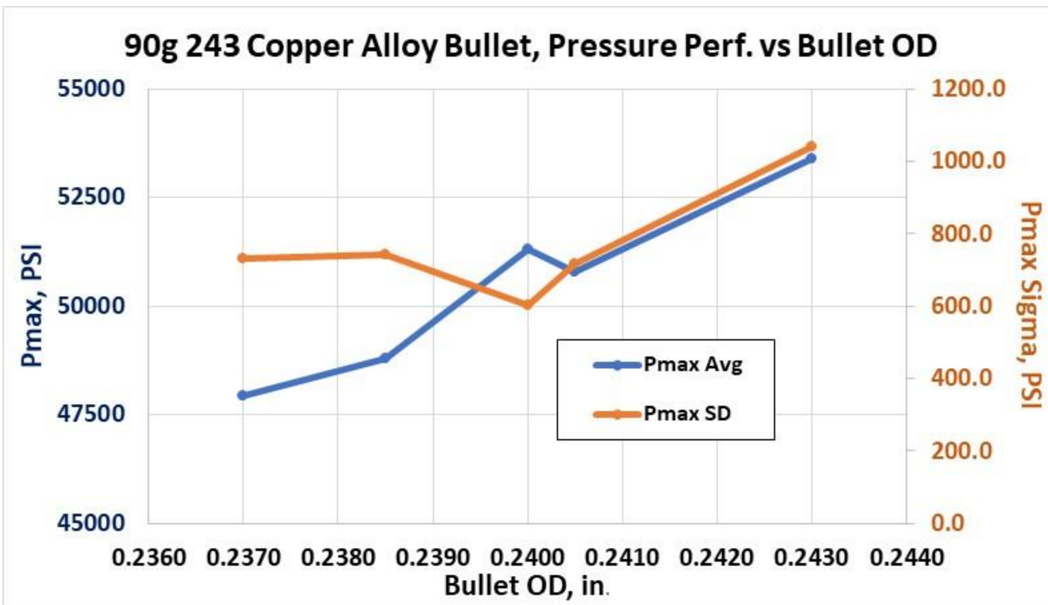

Figure 6: Peak Pressure & MV Std Deviation for 6mm bullet vs Bullet Diameter

Figure 7 shows the effect on average muzzle velocity and muzzle velocity standard deviation (sigma) for the 243 caliber, 90 grain copper alloy bullet as a function of bullet diameter.

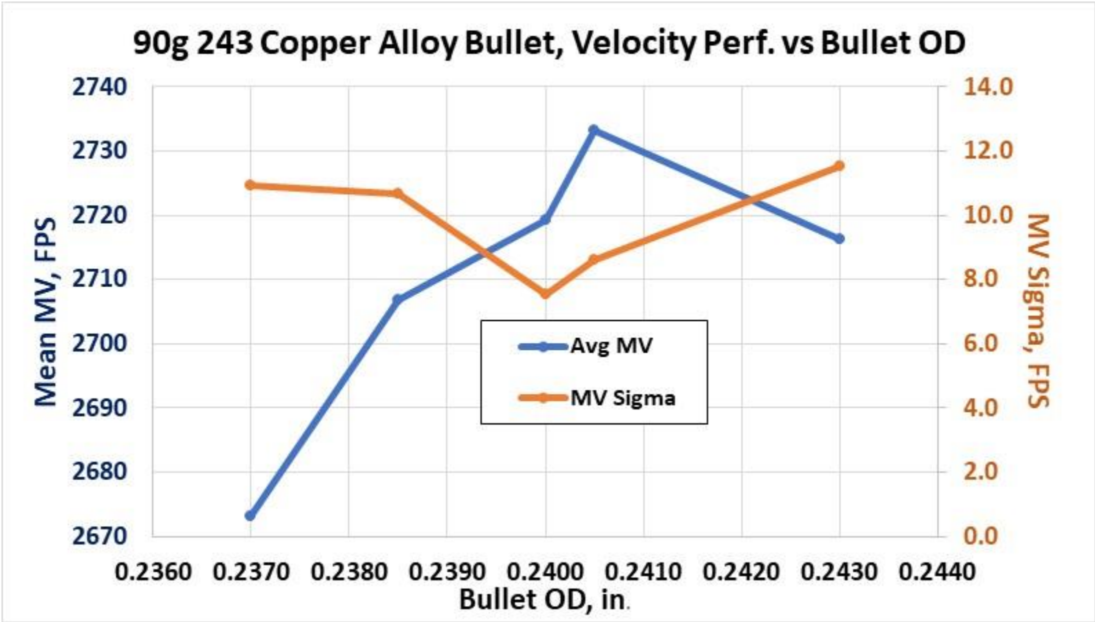

Figure 8: Mean MV & MV Std Deviation for 6mm bullet vs Bullet Diameter

While the effect of changes in bullet diameter on muzzle velocity will likely change with caliber, there is no data on this effect in other calibers, so it was decided to use this data in lieu of nothing.

Primer/Powder Interaction Variation

This factor is a real "wildcard" in the effort to estimate the magnitude of the muzzle velocity variation because there aren't any "knobs" in any interior ballistics code that allow the user to study this effect. The user typically only gets feedback on this parameter by testing and finding out that the muzzle velocity variability is much larger than is typical for the cartridge. Shown below in Figure 9 and Figure 10 are peak pressure vs. muzzle velocity data for a cartridge with good primer-propellant interaction and poor primer-propellant interaction respectively.

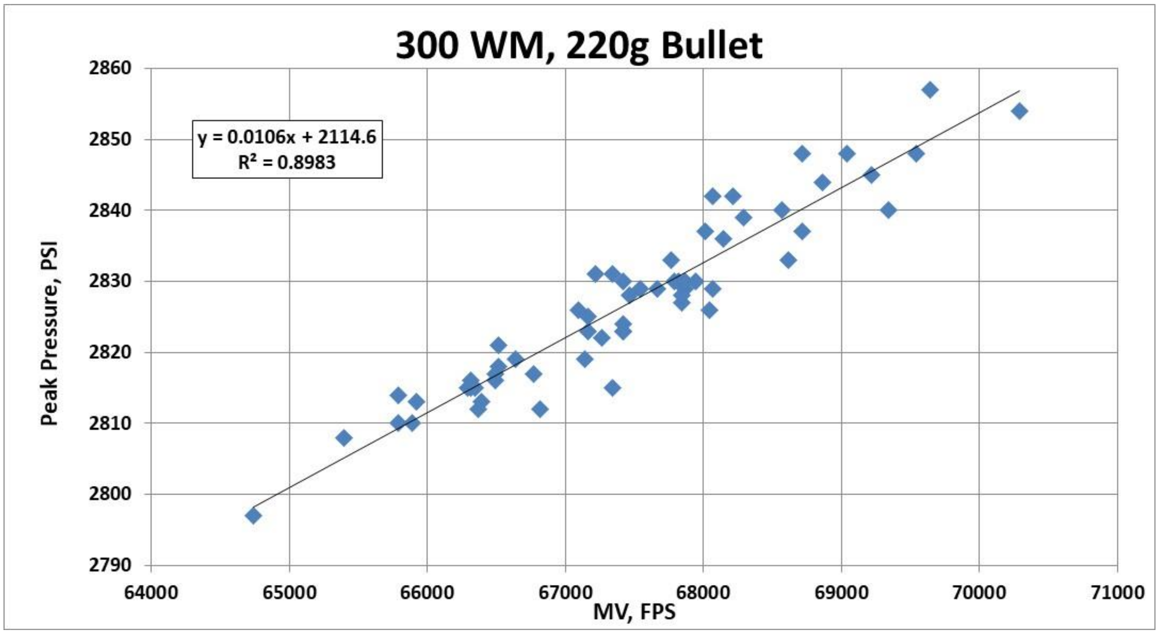

Figure 9: Peak Pressure vs MV for 300 WM

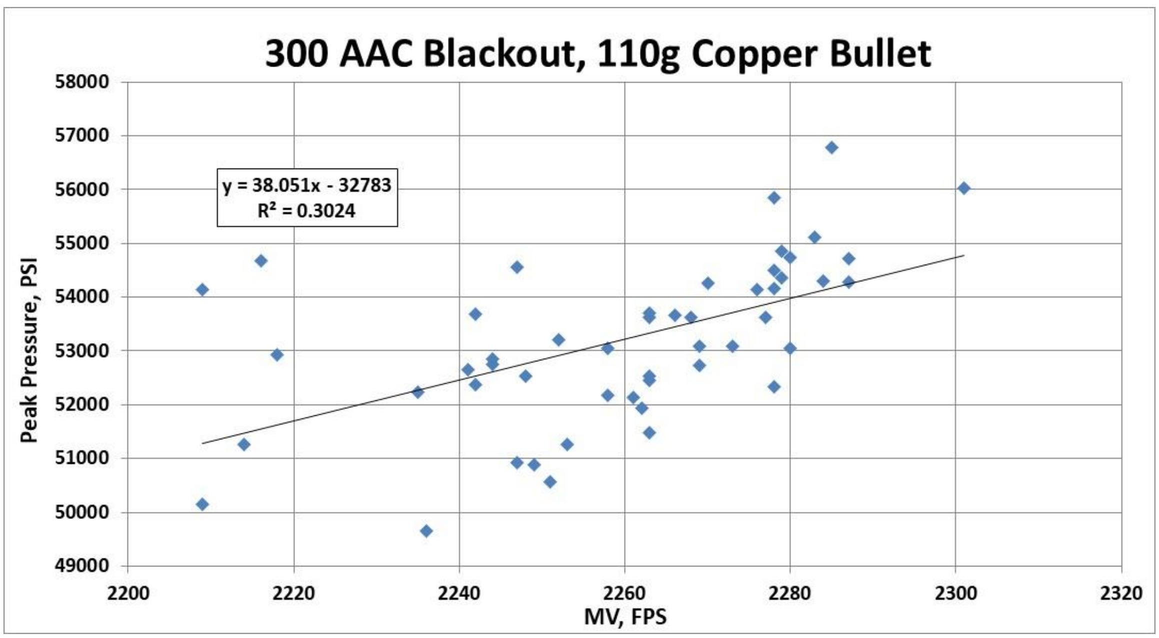

Figure 10: Peak Pressure vs. MV for 300 Blackout

For the 300 WM data illustrated in Figure 9, the velocity standard deviation is about 12.5 FPS, while for the 300 Blackout data shown in Figure 10, the velocity standard deviation is almost 22 FPS. The point being that how efficiently and consistently primers can ignite the propellant in the cartridge case is dependent on many factors which are not under the control of the reloader other than to choose different combinations of primer, case and powder.

Misc. Factors

Miscellaneous factors that influence the average muzzle velocity of a cartridge will also likely influence the muzzle velocity standard deviation to some extent. Factors affecting the muzzle velocity and the muzzle velocity variation include, but are not limited to:

- Peak Chamber pressure
- Case shape (bottleneck vs. straight wall, degree of necking)
- Barrel Temperature (cold vs. very hot)
- Barrel forcing cone geometry (shallow vs. steep)
- Expansion ratio of the system ((Case vol + barrel vol) / case vol))
- Primer type (standard or magnum)
- Powder Geometry (tubular vs. spherical)

Rifling geometry would normally be on this list, but a comparison of mean velocity and velocity variation was conducted in 30 caliber non-magnum cartridge with "standard" rifling and polygonal rifling geometry and these two extremely different rifling geometries were found to not influence the velocity variation sensitivity factor provided the total bore area remained the same.

Estimating the Muzzle Velocity Standard Deviation

To estimate the muzzle velocity standard deviation, the product of each sensitivity factor multiplied by the error in the factor is squared, summed with the square of the other error products, and then the square root is taken. The equation is shown below:

$$(MVsigma)^2 = CW^2 + CVol^2 + BM^2 + PE^2 + SS^2 + FR^2 + BD^2 + PI^2 + Misc^2$$

Equation 1: Error Budget Calculation

Where:

CW = Charge weight sensitivity – variation product
CVol = Chamber volume sensitivity – variation product
BM = Bullet mass sensitivity – variation product
PE = Peak Engraving sensitivity - variation product
SS = Shot Start sensitivity – variation product
FR = Free Run sensitivity – variation product
BD = Bullet Diameter sensitivity – variation product
PI = Primer/Powder Interaction sensitivity – variation product
Misc = Effect of miscellaneous factors

With the sensitivity factors computed and the errors for each factor determined, it is possible to estimate the expected standard deviation in muzzle velocity. Table 2 shows a list of muzzle velocity sensitivity factors and errors and the estimated muzzle velocity standard deviation using the Baer-Frankle numerical integration interior ballistics code and QuickLOAD. The error products causing a velocity shift more than 0.5 FPS are highlighted for both computational approaches.

140g ELD-M, RL-17, 26" inbore Travel		Baer- Frankle				QuickLoad	
Random Errors	Sensitivity Units	Sensitivity	Error Est.	Error Product	Sensitivity	Error Est.	Error Product
Charge Mass	FPS/Grain	45.40	0.05	2.27	45.40	0.05	2.27
Bullet Mass	FPS/Grain	-5.00	0.15	-0.75	-5.00	0.15	-0.75
Chamber Vol	FPS/1 inch vol change	-2765.71	0.00115	-3.17	-5828.57	0.00115	-6.68
Shot Start Press	FPS/ 1% Value Change	-0.02	22.00	-0.44			
Peak Engraving Press	FPS/ 1% Value Change	0.73	8.85	6.46	0.60	8.85	5.31
Free Run	FPS/ 1.0" Change	-427.00	0.0020	-0.85			
OCL	FPS/inch (0.010" Sim)	-70.00	0.0013	-0.09	-200.00	0.0013	-0.26
Primer/Powder Interaction	FPS/ Factor	1.00	1.00	1.00			
Jacket Residue	FPS/ Factor	1.00	1.00	1.00			
Bullet Dimensions	FPS/1" Change	-1050.00	0.00030	-0.315			
			Est. MV Sigma, FPS	7.78			8.86

Table 2: Muzzle Velocity Standard Deviation Estimate For Baer-Frankle & Quickload

Table 2 clearly shows the effect of each of the factors on muzzle velocity variation. Serious long-range shooters are clearly correct to segregate cases by internal volume (e.g. case weight) because with either interior ballistics approach, variability in case volume is the largest contributor to variation in muzzle velocity. Whether the sensitivity to muzzle velocity is as large as indicated by QuickLOAD remains to be seen, however.

Past there, the variation in peak engraving pressure is the next largest factor in determining muzzle velocity variability, but this factor is likely to not be within the control of the shooter/reloader. Charge weight is next most important, and most dedicated shooters are already trickling powder to obtain accurate charge weights with low variation.

Summary:

The error budget approach is a useful tool to identify and quantify the factors contributing to muzzle velocity variation. Additional experiments will be needed to refine the muzzle velocity sensitivity to changes in case internal volume.

Bibliography:

1. Small Caliber Engraving Force Measurements: https://ndia.dtic.mil/2004/2004arms.html