Why Don't All My Bullets Land in the Same Place?
Or
What Causes Dispersion?

By Jeff Siewert

As the title of this article suggests, you probably want to know more about the reasons all the bullets fired from your rifle don't land in one hole. To have this discussion, we need to talk about two related concepts, error budgets and dispersion. The error budget sets the stage for the dispersion discussion and it is helpful to use the concepts contained herein when trying to troubleshoot a rifle/firearm that won't shoot as well as desired.

Error Budget Definition

An error budget is the "laundry list" of error sources that contribute to dispersion and the magnitudes of each of these sources. A notional error budget for two shots is shown in Figure 1.

The **barrel related** components to the jump error budget are:

1. muzzle pointing variation (the shot-to-shot change in bore centerline at the target at bullet release from the muzzle)
2. barrel crossing velocity variation (the transverse velocity of the barrel muzzle as the bullet is released from the barrel)

Both of these dispersion sources are caused by an interaction between the shot-to-shot variations in projectile passage through a non-straight barrel bore and the bending frequency of the barrel.

The remaining jump error budget factors are **projectile** or shooter related; and they are:

1. projectile cross velocity variations (caused by the product of the radial offset of the projectile center of gravity from the bore centerline and the bullet spin rate)
2. aerodynamic jump variations (arising from the product of the projectile in-bore tilt relative to the bore axis and exit spin rate at muzzle exit)
3. sabot discard externally applied impulse variations (an induced angular rate, if a sabot is used)
4. muzzle velocity variation & resulting gravity drop variation
5. projectile drag/mass variation (a trajectory effect)
6. variations in external winds / atmospheric conditions
7. gun cant variations (imperfect alignment of the gun relative to the gravity vector shot-to-shot)
8. shot-to-shot aiming variations

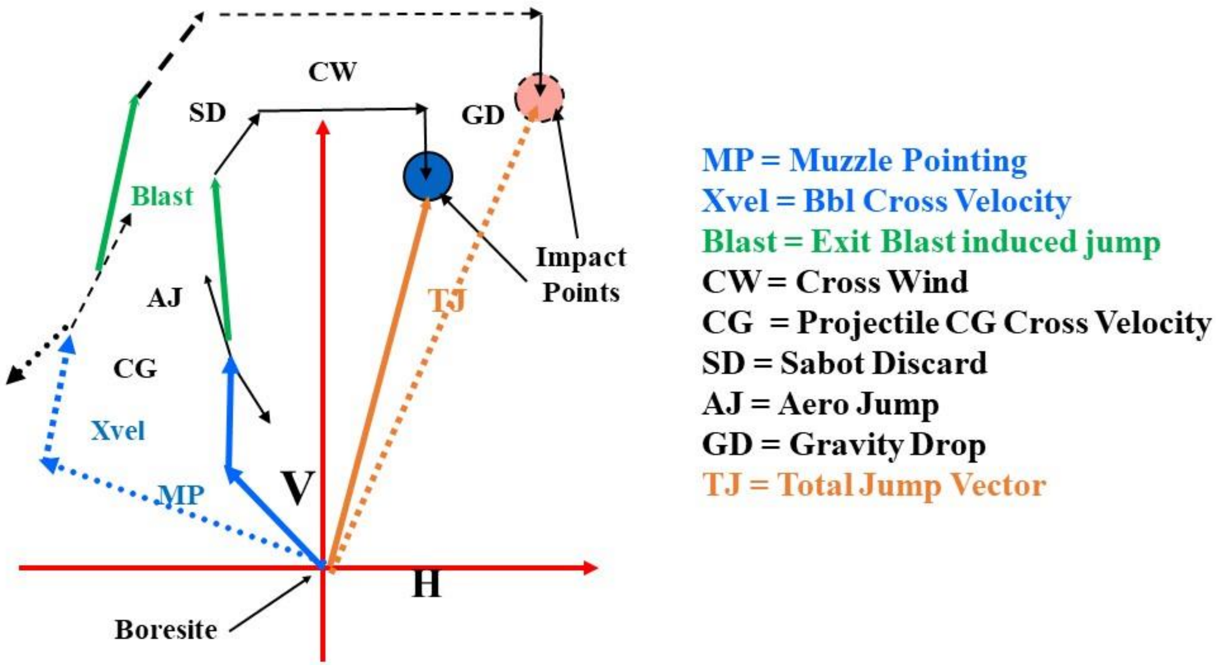

Figure 1: Notional Short-Range Dispersion Error Budget

Figure 2 shows the (greatly exaggerated) in-bore tilt of the projectile and center of gravity offset mentioned in projectile dispersion items #1 and #2 just above. While the actual in-bore angle and center of gravity offset for typical projectiles are quite small (on the order of 0.05 deg. based on analysis of dispersion and radar data), they aren't **exactly** zero for a large percentage of bullets fired toward the target.

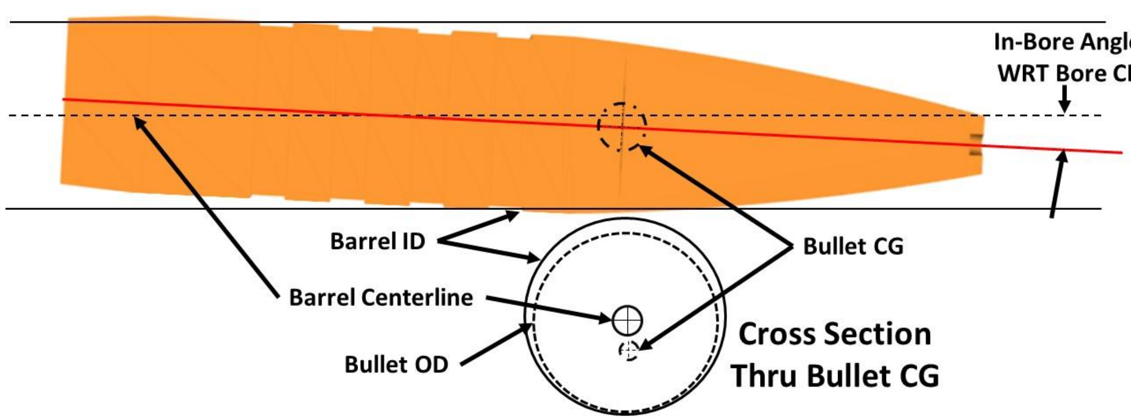

Figure 2: Illustration of Projectile In-Bore Tilt and CG Offset

This discussion is about "short range" dispersion (the short range "random" portion of the targeting problem). The majority of the time, the direction and magnitude of each of the vectors comprising the error budget is not precisely known, so they are added in a "root-sum-square" manner. That is, the individual errors are squared, the results are summed in an algebraic manner, then the square root of the whole sum is taken.

$$\sigma_{total} = ((\sigma_{aero\ jump} + \sigma_{x\ vel})^2 + \sigma_{sabot\ discard}^2 + \sigma_{mv\ var}^2 + \sigma_{drag\ var}^2 + \sigma_{x\ wind}^2 + \sigma_{cant}^2 + \sigma_{aim}^2 + \sigma_{blast}^2)^{0.5}$$

Equation 1: Dispersion Error Sum

Looking at Equation 1, it is easy to see why reducing dispersion (σ_{total}) to zero is nearly impossible (e.g. it's tough to put all the bullets through one hole). By changing the overall cartridge length, the aero jump and cross velocity terms can be changed, but that still leaves a half-dozen or so other terms in the equation that likely have been unaffected. The user should evaluate for him or herself whether the other factors will have a significant effect on group size at the target distance being used.

The "blast" dispersion component arises from an angular rate induced at projectile release from the muzzle. This angular rate is typically quite small, (0.5-1.5 rad/sec resulting in angular departure of ~ 0.03-0.05 mils) but this factor appears to prevent dispersion from going to zero. The effect of this component is covered in a separate article.

One thing is certain; dispersion as measured by angle, never gets smaller with an increase in range. For that to happen, an all-knowing entity would have to exist that paid attention to the intended line of flight of the projectile, and apply precisely the correct force at the correct location on the bullet for the precise duration to direct the projectile back onto the originally intended flight path. While I believe in God, I'm quite sure he has better things to do with his/her time than babysit a group of projectiles on the way to a given target.

The two, barrel related components, the barrel pointing variability (shot-to-shot) get lumped in with the cross velocity variability (x_{vel} in the equation above). Comparing the barrel pointing variability to the barrel cross velocity variability at bullet exit, the pointing variability is usually the larger of the two barrel related "jump" components.

The notional dispersion error budget shown in Figure 1 is for two shots; slightly different orientation and magnitude should be expected for **each** of the error budget vectors on subsequent shots. **IT IS THE VARIABILITY OF COMPONENTS OF THE ERROR BUDGET SHOT-TO-SHOT THAT IS RESPONSIBLE FOR THE IMPACT POINT SCATTER AT THE TARGET SHOT-TO-SHOT KNOWN AS DISPERSION. ALL OF THE ERROR BUDGET COMPONENTS EXCEPT THE GRAVITY DROP ARE OF RANDOM MAGNITUDE AND ORIENTATION SHOT-TO-SHOT.**

For each shot fired, there are minor variations in the length and orientation of the vectors shown in Figure 1; an error budget vector sum for a second projectile is shown with the dashed lines in Figure 1. Scatter of a group of projectiles at the target is the result of the summation of the random length error budget vectors of each shot. It is the fact that the dispersion error budget is comprised of so many factors that prevents the dispersion from going to zero when just one thing in the gun/ammunition system is improved. If the firing setup is fully instrumented with eddy current probes on the barrel to measure barrel pointing and cross velocity, along with orthogonal flash xrays of the bullet at first maximum yaw, the root-sum-square approach to determining dispersion is dispensed with, the errors are added up in a vector sense, as shown in Figure 1.

Analytically, the effect some of the above factors have on dispersion can be studied via specialized dynamics codes. In the spring of 2008, I was determined to better understand the interaction between small caliber projectiles and the gun barrel and used a gun dynamics code written by Arrow Tech Associates in the search for a physical explanation for the dramatic dispersion reduction observed by Barnes Bullets on their Triple Shock X bullets with the external cannelures cut in the projectile shank relative to their original "X" bullets. The search for an explanation led to execution of balloting simulations and parametrically study the effect of contact stiffness between the projectile and the barrel as the bullet runs down the bore.

The gun dynamics code used for this investigation has been in use since 1973, and virtually every high-performance projectile used by the US military since then has been examined with this code. The analytical model of the 30 caliber, 168g Barnes Triple Shock Bullet (Cannelure Bullet), the original "X" bullet (Solid Shank Bullet) and the 300 Win Mag test fixture in which these projectiles were fired is shown in Figure 3. It should be noted that the model of the bullets and barrel are **<u>not</u>** shown to scale.

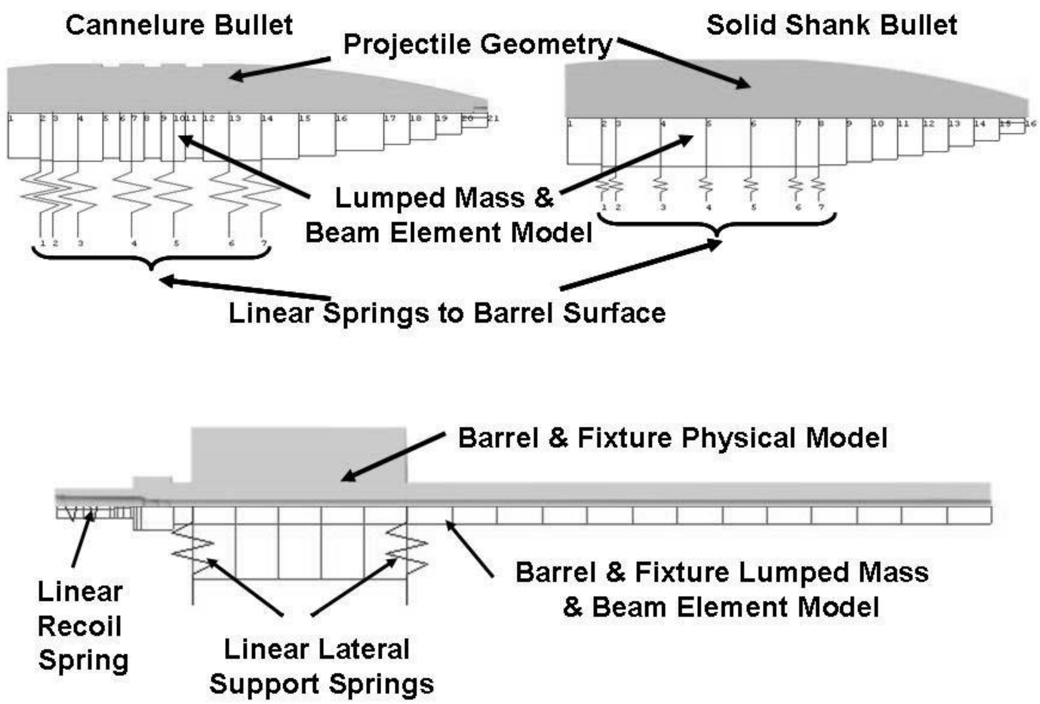

Figure 3: Balloting Model of Barnes 30 Caliber Bullets and 300 Win Mag Test Fixture

A total of 500 sets of random initial conditions varying within reasonable limits were simulated for each one of the parametric values for bourrelet stiffness values to study the expected dispersion of a projectile with a defined radial stiffness fired in the 300 Win Magnum test fixture. Figure 4 shows the predicted variation in the dispersion error budget components and total dispersion for the Barnes 168g triple shock as a function of support stiffness between the projectile and the barrel in the 300 Win Mag test fixture. Note the non-linear behavior of dispersion error budget components, <u>other than</u> the angle in the gun multiplied by the spin rate component, with increasing projectile bourrelet stiffness. I should point out that this analytical model doesn't take into account the shot-to-shot effects of barrel heating and copper deposition in the barrel, which surely adversely affect dispersion. However, this model still yields valuable insights as to the expected trends for changes in the various important interface parameters. Figure 4 shows the expected dispersion of a "solid" copper projectile at about 52 million lb/in. and the same projectile with circumferential grooves (e.g. cannelures) cut in the exterior at about 25 million lb/in. The difference in dispersion is about 40%, which is a significant improvement in dispersion performance for the lower stiffness projectile.

Subsequent comparative balloting simulations of 168g Barnes "X" Bullet and the 168g Triple Shock with identical in-bore angle distributions, using ANSYS to determine the radial stiffness of the solid copper projectiles, indicates the "X" bullet would be expected to exhibit about 30% more dispersion than the Triple Shock. This is in general agreement with the observed change in dispersion seen with the incorporation of the cannelures of the Triple Shock design.

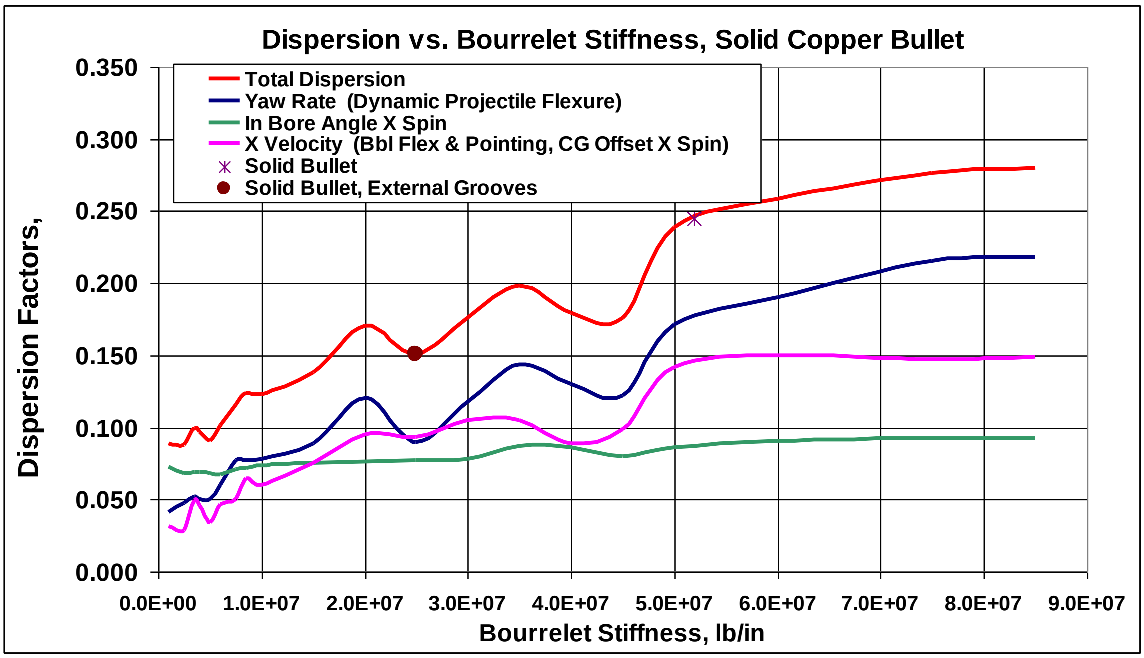

Figure 4: Dispersion Error Budget for 30 Caliber 168g Barnes Triple Shock in 300 Win Mag Test Fixture

As is evident in Figure 4, the dispersion and error budget components are decidedly non-linear functions of the spring (bourrelet) stiffness connecting the bullet to the barrel. The truly surprising thing about this was not so much the non-linear response of the bullet with changes in support stiffness, but that increased dispersion is predicted with increasing spring stiffness. Previous experience with medium and large caliber projectiles have repeatedly shown poor dispersion with very low projectile-barrel contact stiffness, and reduced dispersion with increasing contact stiffness, so the increase in dispersion with increased bourrelet stiffness exhibited by small caliber projectiles came as a bit of a surprise. The increased dispersion with increased support stiffness is simply a case of too much of a good thing.

Figure 4 should be contrasted to a similar analysis run on a more conventional lead core – copper jacket construction projectile. The dispersion error budget for a lead core – copper jacket construction bullet is shown in Figure 5. Note the expected difference (reduction) in dispersion and bourrelet stiffness for the lead core – copper jacket bullet compared to the solid copper projectile shown in Figure 4.

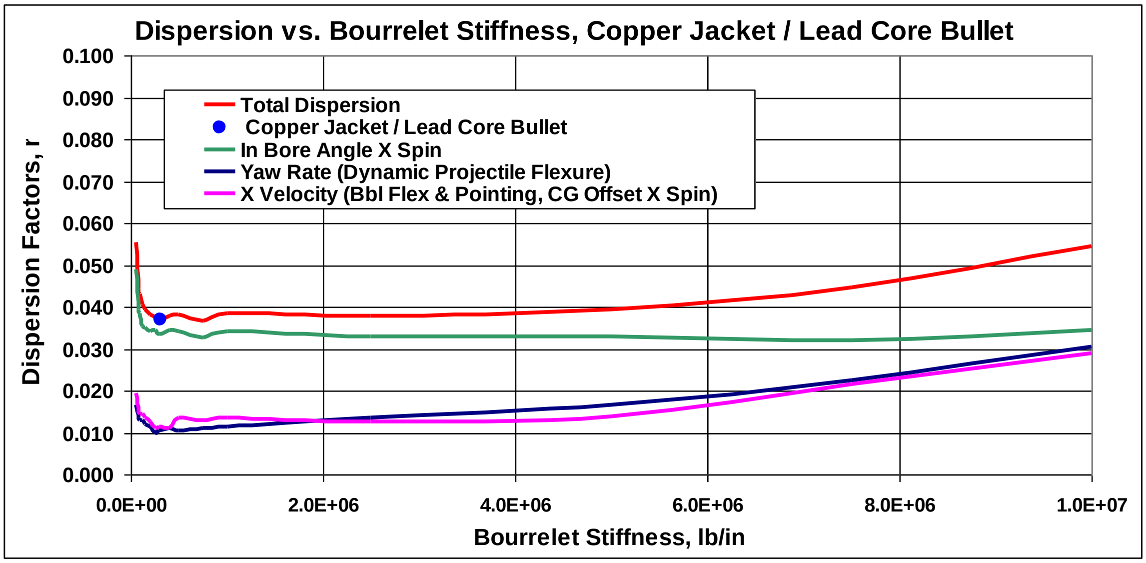

Figure 5: Error Budget for 30 Caliber 168g Copper Jacket / Lead Core Bullet in 300 Win Mag Test Fixture

By comparing the expected dispersion error budget for the lead core – copper jacket bullet and the solid copper bullet, we can see that the lead core – copper jacketed bullet is expected to shoot much smaller dispersion (on average) than a solid copper bullet. This is primarily due to the structural "feedback" between the solid copper projectile and the barrel structure, and is predicated on the lead and copper bullet material acting in a structurally "linear" manner and not yielding during launch. As it turns out, this expectation **should be mostly unfounded** due to the low yield strength of the lead core. Lead's low yield strength should make the true dispersion of lead core/copper jacket bullets higher than is indicated in Figure 5 because of the yielding of the lead projectile core from launch acceleration. The projectile axis tilt that results from that yielding would be expected to increase the variability of projectile yaw rate and cross velocity shot-to-shot. However, analysis of dispersion data from a well-known match bullet manufacturer indicates the simulated dispersion shown in Figure 5 is quite close to that seen in actual production.

Dispersion (NOT ACCURACY, Dammit!)

On more than a few occasions, folks use the term "accuracy" when they really mean to indicate something about dispersion (AKA group size). **Dispersion is NOT ACCURACY!** and Figure 6 shows why folks in the know don't confuse the two terms. Accuracy refers to the distance between the mean point of impact of a group of projectiles and the **aim point**, while dispersion is a measure of the group size.

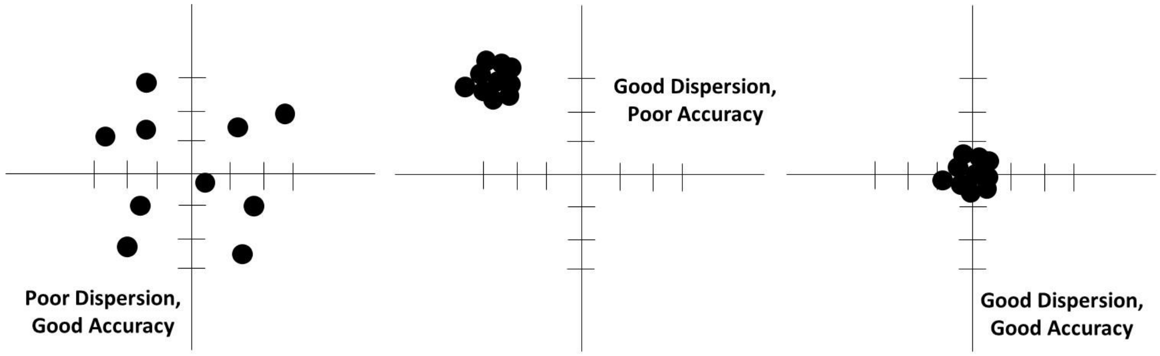

Figure 6: **Graphical Definition of Dispersion and Accuracy**

When firing one bullet, an accurate bullet and rifle is desired; a "system" that hits very close to the point of aim. When firing a group of bullets, the size of the group (the dispersion) is desired to be small, so that it is easy to adjust the cross hairs of the scope to coincide with the center of the group (mean point of impact) with little error. That way, when pulling the trigger on a single shot, the shooter will have a high degree of confidence that the bullet will go where it's intended to. Say what you will about an accurate rifle, load, etc.; what is desired are small shot groups and the center of the group(s) is very close to the aim point. Once the system dispersion is acceptably small, it is then quite a simple matter to adjust the sights to the mean point of impact, subsequently making the firearm system accurate. Conversely, if the dispersion is excessive, the shooter doesn't have much of a clue where the next bullet is going to go relative to the aim point, and confidence in your firearm/sighting/ammunition system is lost. Further, sight adjustments to the mean point of impact while not pointless, are quite sketchy due to the mean point of impact jumping around so much. This leads to a reduction in effective range for the firearm/ammunition system combination, regardless of the other parameters considered.

At short ranges, there are essentially two sources of dispersion for small caliber projectiles, variable projectile angular rate at muzzle exit and variable cross velocity. This is because the effect of the remaining components of the error budget (MV variation, drag variation, wind variation, etc) are so small. For both angular rate and cross velocity, the variability of these factors refers to changes in both magnitude and direction. Variable magnitude pretty much speaks for itself; it's changes in the size of the disturbance being considered. The variable direction can be thought of as changing "around the clock" if the bore centerline as the pivot point of the clock hands looking down range. In some instances, in small caliber, the tilt of a projectile in the bore with respect to the gun bore centerline also causes a projectile center of gravity offset with respect to the bore centerline, leading to an unbalanced load operating on the gun barrel structure at the local, instantaneous projectile center of gravity. Once the projectile is free of the restraints provided by the internal surface of the barrel, the CG offset multiplied by the spin rate imparts a "cross" velocity to the projectile, perpendicular to the forward velocity vector. Barrel pointing variations and CG offset induced cross velocity add in a vector sense, further contributing to dispersion.

For medium and large caliber guns, a closed form equation known as the "Jump Equation" is used to estimate the dispersion of projectiles. The jump equation is shown in Equation 2. The success of this equation for medium and large caliber projectiles can be directly attributed to the fact that materials comprising the structure of these bullets typically operate well within their elastic range. When small caliber projectiles experience an in-bore angle with respect to the bore centerline, as they leave the barrel, they are imparted with an initial angular rate equal to the in-bore angle multiplied by the exit spin rate, and are hence subject to the same laws of physics as large and medium caliber bullets.

As shown in Equation 2, the jump equation has two main parts, a "sensitivity term" (how far with the flight path deviate for a given disturbance) and a launch disturbance term. The factors include an aerodynamic term, a mass properties term, a "scale" term, an angular rate term, and a cross velocity term. It does **not** include a barrel pointing variability term, crucial to accurately predicting dispersion for small caliber projectiles, primarily because small caliber projectiles shoot such small dispersion to begin with.

$$\Theta_j = \left[\left[\overbrace{\underbrace{\left(\frac{C_{N\alpha} - C_D}{C_{m\alpha}}\right)}_{\text{Aero Term}}\underbrace{\left(\frac{I_y - I_x}{md^2}\right)}_{\text{Mass Prop. Term}}\underbrace{\left(\frac{d}{V_m}\right)}_{\text{"Scale" Term}}}^{\text{Bullet Sensitivity to Launch Disturbance}}\overbrace{\underbrace{\left(\alpha_g \bullet p_m\right)}_{\text{Angular Rate Term}}\right] \mp \left[\underbrace{\Delta_{CG} \bullet \frac{p_m}{V_m}}_{\text{Cross Vel. Term}}\right]}^{\text{Magnitude of Launch Disturbance}}\right]$$

Equation 2: Closed Form Jump Equation

Where:

Θ_j = Projectile Jump Angle with respect to the barrel centerline
$C_{N\alpha}$ = Normal Force Coefficient Derivative per sine angle of attack
C_D = Drag Force Coefficient
$C_{m\alpha}$ = Pitching Moment Coefficient Derivative per sine angle of attack
I_x = Projectile Polar Moment of Inertia
I_y = Projectile Transverse Moment of Inertia
m = Projectile Mass
α_g = Projectile angle in the tube, radians
p_m = Projectile exit spin rate, radians per second
d = Projectile Reference Diameter
V_m = Muzzle Velocity
Δ_{CG} = Radial Distance from the tube centerline to the projectile center of gravity

The aerodynamic term (AERO's in Equation 2) is essentially the inverse of the "static margin" of the projectile, the separation between the location of the center of gravity (CG) and location of the aerodynamic Normal force center of pressure (CP). In this instance, the larger the separation between CG and CP (larger $C_{m\alpha}$), the smaller dispersion performance we expect from the projectile.

This means by selecting long, sleek bullets with low density front ends (aluminum or plastic), smaller groups should result. This is an effect of selecting bullets which are less "jump sensitive" and thereby reducing the jump magnitude of so-called "fliers".

The mass properties term has the projectile transverse moment (I_y) and polar moment (I_x) of inertia in the numerator, and the mass and diameter in the denominator. For small caliber bullets that might typically be fired from rifles, the ratio of I_y to I_x is on the order of 5:1 to 8:1, so the transverse moment of inertial is **much** more important to dispersion performance than the polar moment of inertia. This can be thought of as the expected oscillation frequency of a dumbbell (e.g. the projectile) if it were suspended from the ceiling by a wire acting as a torsional pendulum. If the weights of the dumbbell are at the very ends of the bar connecting them, the mass (projectile) has a high transverse moment of inertia, and hence the oscillation frequency is very low. If the weights are near the middle of the suspending wire, the transverse moment of inertia is low, and the resulting oscillation frequency is high. Low frequency oscillation is detrimental to small dispersion because it takes a long time for the bullet to complete a single oscillation and the bullet spends this time at an angle of attack which causes it to swerve off the intended line of flight.

This means by selecting long, sleek bullets with low density front ends (aluminum or plastic), smaller groups should result. This is caused by reducing the jump magnitude of so-called "fliers" which happen to all shooters, regardless of their skill behind the rifle or at the reloading bench.

The scale term is the projectile diameter divided by muzzle velocity. This is "intuitively obvious"; the larger the bullet diameter, the larger the reference area of the bullet on which the aerodynamic forces operate. The higher the velocity, the less time the aerodynamic forces have to swerve the bullet.

This means, all things being equal, the bullet with higher muzzle velocity is expected to shoot smaller groups.

As previously discussed, the angular rate term is equal to the projectile angle in the gun multiplied by the exit spin rate.

The cross velocity term is the center of gravity offset with respect to the bore centerline multiplied by the exit spin divided by the velocity. Using consistent units, we get a small number from this term.

While it is critically important to load consistent ammunition, by selecting a well-designed, well manufactured bullet and testing to determine the optimal free run to the rifling, the shooter can shoot smallest possible groups at short range. Making the same bullet shoot small groups at long range is then a matter of getting the muzzle velocity variation as small as possible.