MATH FUNDEMENTALS 3- GEOMETRY

1. What is geometry?
2. What does geometric deal with?
3. Give the abbreviation or symbol for the following terms.
   A. Angle
   B. Approximately
   C. Area
   D. Area of base of polyhedra
   E. Base
   F. Circumference
   G. Clockwise
   H. Congruent
   I. Counterclockwise
   J. Degrees
   K. Diameter
   L. Height
   M. Length
   N. Measure of
   O. Parallel
   P. Perpendicular
   Q. Pi
   R. Radius
   S. Right angle
   T. Side or slant heights
   U. Similar
   V. Surface area
   W. Triangle
   X. Volume
   Y. Width
4. Describe and give examples of the following lines.
   A. Line
   B. Line segment
   C. Intersecting lines
   D. Parallel lines
   E. Ray
   F. Perpendicular lines
   G. Transversal
   H. Skew lines
5. How are angles formed?
6. What is a vertex?
7. Where is the vertex located in the angle name?

8. Describe and draw an example of the following angles:
    A. Acute angles
    B. Right angles
    C. Obtuse angles
    D. Straight angles
    E. Reflex angles
    F. Adjacent angles
    G. Vertical angles
    H. Congruent angles
    I. Complementary angles
    J. Supplementary angles
    K. Same side interior angles
    L. Same side exterior angles
    M. Corresponding angles
    N. Alternate exterior angles
    O. Alternate interior angles
9. What is a polygon?
10. How many sides does a polygon have to have?
11. How do you name a polygon?
12. What does "poly" mean?
13. Explain and draw an example of the following polygons:
    A. Regular polygon
    B. Irregular polygon
    C. Composite figure
    D. Congruent figure
    E. Similar figures
    F. Diagonal
    G. Convex polygon
    H. Concave polygon
    I. Perimeter
    J. Area
14. What is a triangle?
15. What is the sum of the interior angles?
16. How are triangles classified?
17. How do you name a triangle?
18. Draw and label a diagram showing the area of a triangle.
19. Recreate a chart explaining and displaying how to classify a triangle.
20. What is the Pythagorean Theorem? Show examples
21. How can you determine if any three line segments form a triangle?
22. What is a quadrilateral?
23. What is the sum of the interior angles?
24. What is the height of a quadrilateral?

25. How do you name a quadrilateral?
26. What are the symbols for:
    A. Parallel
    B. Perpendicular
    C. Congruent
27. Draw a chart showing the following quadrilaterals:
    A. Parallelogram
    B. Rectangle
    C. Rhombus
    D. Square
    E. Trapezoid
    F. Isosceles trapezoid
    G. Trapezium
    H. Kite

    Be sure to include an example, and a definition as well as the quadrilateral explanation.
28. How do you find the area of any polygon?
29. How do you find the number of diagonals in any polygon?
30. Explain and draw a diagram of the following:
    A. Pentagon
    B. Decagon
    C. Hexagon
    D. Hendecagon
    E. Heptagon
    F. Octagon
    G. Nonagon
    H. Dodecagon
31. What is a 'n-gon"?
32. What is a circle?
33. What is the complete rotation of a circle?
34. Explain and draw a diagram of the following:
    A. Center
    B. Diameter
    C. Radius
    D. Chord
    E. Central angle
    F. Arc
    G. Inscribed angle
    H. Intercepted arc
    I. Semicircle
    J. Concentric circles
35. Describe in detail the following circle measurements:
    A. Pi

B. Circumference
C. Area of a circle

36. What is an ellipse?
37. Draw and label a diagram showing the area of an ellipse.
38. What are space figures?
39. What is a polyhedron?
40. What 3 things does a polyhedron have?
41. Explain, draw, and label the following polyhedron:
    A. Regular polyhedron
    B. Net of a polyhedron
    C. Convex polyhedron
    D. Concave polyhedron
    E. Surface area
    F. Volume
42. What is a prism?
43. How is a prism named?
44. Explain, draw, and label the following polyhedral-prisms:
    A. Rectangular prism
    B. Triangular prism
    C. Cube
    D. Pentagonal prism
    E. Hexagonal prism
45. What is a pyramid?
46. What are the lateral faces?
47. What is the height of a pyramid?
48. How is a pyramid named?
49. Draw, label, and explain:
    A. Rectangular pyramid
    B. Triangular pyramid
    C. Square pyramid
50. Draw, label, and explain the following space figures with curved surfaces:
    A. Cylinder
    B. Cone
    C. Hemisphere
    D. Sphere
51. What is a transformation?
52. Draw, label, and explain the following transformations:
    A. Rotation
    B. Reflection
    C. Translation
    D. Dilation

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Math Fundamentals 3

Geometry - points, lines, angles, planes, solids & space figures!

Boost math confidence & test scores!

What Is Geometry?

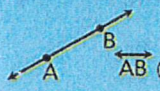
Geometry comes from the Greek, *geōmetrein*, meaning "to measure the Earth"; geometry is the branch of mathematics dealing with the **properties, measurement** and **relationship** of points, lines, planes, angles, solids and space figures

Abbreviation & Symbol Key: Use this key to understand abbreviations & symbols used in this guide

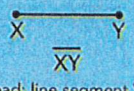
∠ : angle	B : area of base of polyhedra	CW : clockwise	° : degrees	l : length	⊥ : perpendicular	h : right angle	SA : surface area
≈ : approximately equal	b : base	≅ : congruent	d : diameter	m : measure of	π : Pi	s : side (polygons) or slant height (space figures)	Δ : triangle
A : area	C : circumference	CCW : counterclockwise	h : height	∥ : parallel	r : radius	~ : similar	V : volume
							w : width

LINES

Line
A series of points extending indefinitely in opposite directions; it has no endpoints; any two points on the line name the line

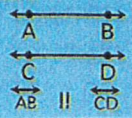


Line Segment
A part of a line with two endpoints; the endpoints name the segment

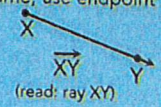


Intersecting Lines
Lines that share exactly one point or all points

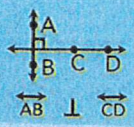
Parallel Lines
Lines with no points in common; never intersect; equidistant from each other; symbol: ∥



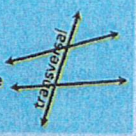
Ray
A part of a line with one endpoint that extends indefinitely in one direction; the endpoint and one other point name the ray; in name, use endpoint first



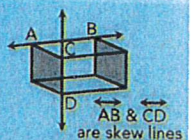
Perpendicular Lines
Lines intersecting at exactly a 90° angle; symbol: ⊥



Transversal
When two or more lines are intersected by another line at different points, the intersecting line is called the transversal



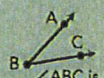
Skew Lines
Lines in different planes; not parallel and never intersect



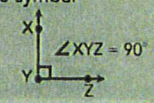
ANGLES

Angle Facts: Angles are formed by the union of two rays with a common endpoint, called the **vertex** (plural is vertices); use three capital letters to name an angle; the vertex must be the middle letter in the name

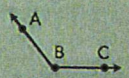
Acute Angles
Less than 90°; greater than 0°



Right Angles
Exactly 90°; rays are ⊥; square in corner is right angle symbol

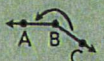


Obtuse Angles
Between 90° and 180°

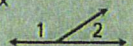


Straight Angles
Exactly 180°

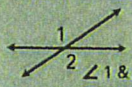
Reflex Angles
Between 180° and 360°



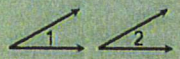
Adjacent Angles
Two angles sharing a common ray and a common vertex



Vertical Angles
Non-adjacent angles formed by intersecting lines; share only a vertex; ∠1 ≅ ∠2

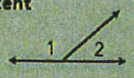


Congruent Angles
Angles equal in measure; congruent (≅)



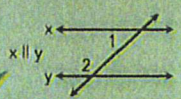
Complementary Angles
Two angles that total 90°; **do not have to be adjacent**

Supplementary Angles
Two angles that total 180°; **do not have to be adjacent**



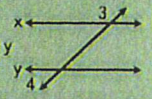
Angles formed by two or more lines with a transversal

Same-Side Interior Angles



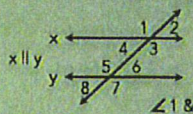
THINK: Angles inside the lines and on same side of transversal; **total 180° IF lines ∥**

Same-Side Exterior Angles



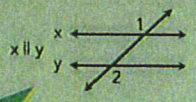
THINK: Angles outside the lines and on same side of transversal; **total 180° IF lines ∥**

Corresponding Angles



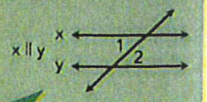
THINK: Every other angle on same side of transversal; **≅ IF lines ∥**

Alternate Exterior Angles



THINK: Angles outside the lines and opposite sides of transversal; **≅ IF lines ∥**

Alternate Interior Angles



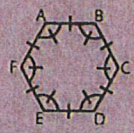
THINK: Angles inside the lines and opposite sides of transversal; **≅ IF lines ∥**

QuickStudy

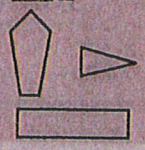
POLYGONS

Polygon Facts: Polygons are **closed-plane figures**; sides are line segments; must have **three or more sides**; name a polygon by using the capital letters at the vertices; classified by number of sides; "**poly-**" means "many"

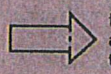
Regular Polygon
All sides **and** all angles ≅



Irregular Polygon
Any polygon that is not regular; some angles and sides NOT ≅

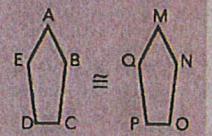


Composite Figure
Polygons united to make one figure; when figuring P, be sure to include **ALL** line segments: "walk around the whole figure"; to find the area of composite figures, find the area for each polygon and add together



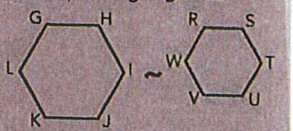
find area of the rectangle and area of the triangle; then, add the areas together

Congruent Figures
Two or more polygons with ALL sides and ALL angles equal



ABCDE ≅ MNOPQ

Similar Figures
Polygons that are the same shape, may or may not be the same size; corresponding sides are in proportion, and corresponding angles ≅



GHIJKL ~ RSTUVW

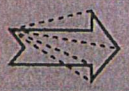
Diagonal
A line segment joining two non-adjacent sides of a polygon

dashed segments are diagonals

Convex Polygon
All diagonals are inside polygon

THINK: All line segments point out of polygon

Concave Polygon
Some diagonals are outside polygon



THINK: At least two line segments point inside polygon

Perimeter (P)
Distance around a figure

THINK: Walking around the figure; **unit of measure: unit(s)**

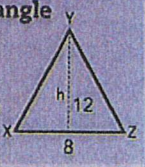
Area (A)
Total square units within a figure

THINK: Coloring inside the figure; **unit of measure: square unit(s)**

TRIANGLES (3-sided Polygons)

Triangle Facts: A triangle is a **three-sided polygon;** sum of interior angles = **180°**; triangles are classified by BOTH the measure of their sides and by the measures of their angles; name a triangle with three capital letters and the triangle symbol: △ XYZ

Area of Triangle
$A = \frac{1}{2}bh$
$A = \frac{1}{2}(8)(12)$
$A = 4(12)$
$A = 48$ square units



CLASSIFYING TRIANGLES
(MUST have term from each column!)

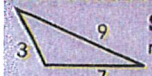
By Sides

Equilateral
all sides equal; if equilateral, it is also equiangular and acute

Isosceles
two equal sides called legs; two base angles equal

∠1 is a vertex angle
∠2 & ∠3 are base angles

Scalene
no sides equal



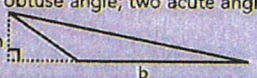
By Angles

Equiangular
all angles equal; all angles measure 60°; all angles acute; if equiangular, it is also equilateral

Acute
all angles acute

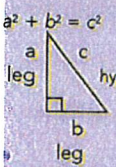
Right
one right angle; two acute angles

Obtuse
one obtuse angle; two acute angles



Pythagorean Theorem – use ONLY with right triangles;
sum of the legs² = hypotenuse²

$a^2 + b^2 = c^2$



$a^2 + b^2 = c^2$
$4^2 + 3^2 = c^2$
$16 + 9 = c^2$
$25 = c^2$
$\sqrt{25} = \sqrt{c^2}$
$5 = c$

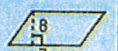
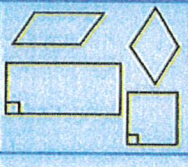
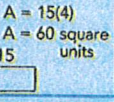
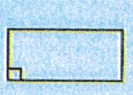
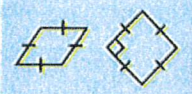
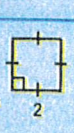
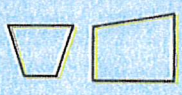
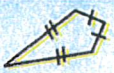
$a^2 + b^2 = c^2$
$a^2 + 3^2 = 5^2$
$a^2 + 9 = 25$
$a^2 = 25 - 9$
$a^2 = 16$
$\sqrt{a^2} = \sqrt{16}$
$a = 4$

quick tip! To determine if any three line segments can form a triangle: Sum of lengths of any two line segments > length of third line segment

QUADRILATERALS (4-sided Polygons)

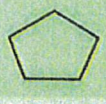
Quadrilateral Facts: A quadrilateral is a **four-sided polygon;** sum of interior angles = **360°**; the height of a quadrilateral is a ⊥ line segment joining opposite sides (bases); four capital letters at vertices name the quadrilateral: ▱ ABCD

Symbols: parallel: ||; perpendicular: ⊥; congruent: ≅

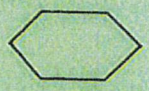
Quadrilateral	Examples	Definition
Parallelogram $A = bh$ $A = 7(8)$ $A = 56$ square units 		A parallelogram always has opposite sides ||; opposite angles ≅; consecutive angles are supplementary
Rectangle $A = bh$ or $A = lw$ $A = 15(4)$ $A = 15(4)$ $A = 60$ square units $A = 60$ square units 		A rectangle is **always** a parallelogram; **PLUS** all angles are right angles; **AND** diagonals are ≅
Rhombus $A = bh$ $A = 5(7)$ $A = 35$ square units 		A rhombus is **always** a parallelogram; **PLUS** all sides ≅ and all diagonals ⊥; a rhombus is also a square **IF** all angles ≅
Square $A = s^2$ $A = 2^2$ $A = 4$ square units 		A square is **always** a parallelogram, a rectangle and rhombus; **PLUS** it has four equal sides and four right angles; **AND** diagonals ≅ and ⊥
Trapezoid $A = \frac{1}{2}h(b_1 + b_2)$ $A = \frac{1}{2}(6)(9 + 10)$ $A = 3(19)$ $A = 57$ square units 		A trapezoid is **never** a parallelogram; it has exactly one pair of || sides, called bases; its height is the length of a ⊥ line segment between the bases; its legs are sides that are **not** ||
Isosceles Trapezoid To find area, follow instructions for trapezoid		Trapezoid with ≅ base angles AND ≅ legs
Trapezium To find area, split into separate triangles; sum of areas of triangles = area of trapezium		A trapezium is a quadrilateral with **NO** || sides
Kite To find area, follow instructions for trapezium		Trapezium with two sets of adjacent and ≅ sides

OTHER POLYGONS

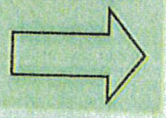
Pentagon
5 sides
sum of interior angles = 540°



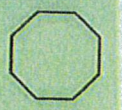
Hexagon
6 sides
sum of interior angles = 720°



Heptagon (or Septagon)
7 sides
sum of interior angles = 900° (this example is a concave polygon)



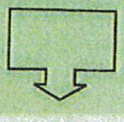
Octagon
8 sides
sum of interior angles = 1,080°



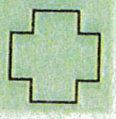
Area of Any Polygon
To find A of any polygon that does not have its own formula, split it into triangles and/or quadrilaterals, and then find the sum of the A of those polygons

Decagon
10 sides
sum of interior angles = 1,440° (this example is a concave polygon)

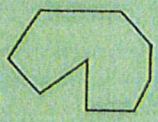
Hendecagon
11 sides
sum of interior angles = 1,620° (this example is a concave polygon)



Dodecagon
12 sides
sum of interior angles = 1,800° (this example is a concave polygon)



Nonagon
9 sides
sum of interior angles = 1,260° (this example is a concave polygon)



n-gon
"n" number of sides
sum of interior angles = 180(n - 2)
n-gon is a polygon with the number of sides represented by "**n**"

quick tip! To find the number of diagonals in any polygon, let n = number of sides of polygon, then $\frac{n^2-3n}{2}$ = number of diagonals in polygon, so, for a dodecagon, $\frac{12^2-3(12)}{2}$

$\frac{144-36}{2}$

$\frac{108}{2}$

54 = number of diagonals in a dodecagon

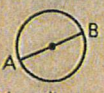
CIRCLES & ELLIPSES

Circle Facts: A circle is a **closed-plane figure** with **all points** the **same distance from a center point**; complete rotation of a circle is **360°**

Center
The point equally distant from all points in the circle; names the circle

Circle A

Diameter
Any line segment passing through the center of the circle, with endpoints on the circle; a diameter is also a chord



$\overline{AB}$ is a diameter & also a chord

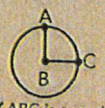
Radius
Any line segment between center of circle and point on circle; plural of radius is radii
2 radii = diameter

$\overline{xy}$ is a radius

Chord
Any line segment with endpoints on the circle; a diameter is also a chord (but every chord isn't a diameter)

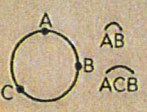
$\overline{AB}$ $\overline{xy}$ are chords

Central Angle
Angle inside the circle; vertex is the center of the circle; sides of angle are radii of circle



∠ABC is a central angle

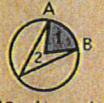
Arc
Part of a circle with two endpoints; use two letters to name an arc with a shorter curve; use three letters to name an arc with a longer curve



Inscribed Angle
Angle with vertex on edge of the circle; sides are chords

∠1 is an inscribed angle

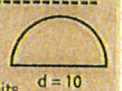
Intercepted Arc
Arc shared by a central angle and an inscribed angle; the measure of the inscribed angle is always half of the measure of the central angle



$m\angle 2 = \frac{1}{2}\, m\angle 1$

Semicircle
Half of a circle
$C = \pi r + d$ **or** $C = \frac{1}{2}\pi d + d$
$C = \pi 5 + 10$ $C = \frac{1}{2}\pi 10 + 10$
$C \approx 25.7$ units $C \approx 25.7$ units

$A = \frac{1}{2}\pi r^2$
$A = \frac{1}{2}\pi (5^2)$
$A \approx 39.3$ square units



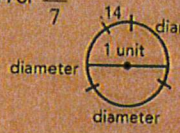
Concentric Circles
Circles that share the same center

Circle Measurements

$Pi(\pi) = \frac{\text{circumference}}{\text{diameter}}$

The ratio of the circumference to the diameter; number of times the diameter goes around the circle; uses the Greek letter Pi(π); Pi is an irrational number (a non-terminating, non-repeating decimal)

$Pi(\pi) \approx 3.14$ or $\frac{22}{7}$



first diameter = 1 unit
second diameter = 1 unit
third diameter = 1 unit
rest of circle length ≈ .14
$1+1+1+.14 = 3.14 \approx \pi$

Circumference
Distance around the circle
$C = 2\pi r$ or $C = \pi d$
$C = 2\pi 3$ $C = \pi 6$
$C \approx 18.8$ units $C \approx 18.8$ units

Area of a Circle
Total square units within a circle
$A = \pi r^2$
$A = \pi 3^2$
$A \approx 28.3$ square units

quick tip! When figuring area and circumference, be sure to check whether the problem/diagram is using radius or diameter;

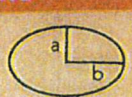
If it gives diameter, use $C = \pi d$ for circumference

If it gives radius, use $C = 2\pi r$

For area, always use radius: If problem gives diameter, divide it by 2 to find radius

Ellipse Facts: An ellipse is a **closed-plane figure** that is **oval-shaped**

Area of an Ellipse
$A = \pi ab$



$A = \pi ab$

quick tip! Confused about which formula is for area of a circle and which is for circumference of a circle? **REMEMBER** that the answer to an area problem is **square units (units²)** AND the formula for area has r^2 in it ($A = \pi r^2$)!

quick tip! $unit(s)^2$ = square units
$unit(s)^3$ = cubic units

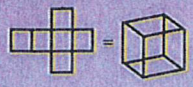
SPACE FIGURES: 3-D figures that have faces, edges and vertices, OR that are curved

Polyhedra

Polyhedra Facts: A polyhedron is a **space figure** whose surfaces or faces are flat (plural of polyhedron is polyhedra); polyhedra have **faces, edges** and **vertices; base(s)** are **special faces** whose polygon shape names the polyhedron; all **polyhedra** are three-dimensional (**3-D**)

Regular Polyhedra
Faces are all ≅ regular polygons; same number of edges meet at each vertex

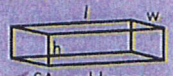
Net of a Polyhedron
Two-dimensional (2-D) pattern that can be folded to form a solid; A of net = SA of polyhedron



Convex Polyhedron
All line segments that connect any two points on the surface are completely inside the figure

Concave Polyhedron
A line segment that connects any two points on the surface; also contains a point outside the figure

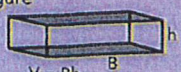
Surface Area
Sum of square units on all faces; A of net



SA = add areas of all faces

THINK: Wrapping a present; **unit of measure: square unit(s)**

Volume
Total cubic units within a figure



V = Bh
V = total cubic units
h = height
B = area of the base

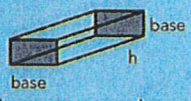
THINK: Filling a container; **unit of measure: cubic unit(s)**

Polyhedra - Prisms

Prism Facts: A prism is a **polyhedron** with two ≅, ∥, polygon bases; faces are parallelograms; a prism is named by the shape of its bases

Rectangular Prism
$V = Bh$ or $V = lwh$
$SA = 2lh + 2lw + 2wh$

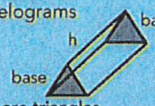
Two bases are rectangles; other faces are parallelograms



bases are rectangles

Triangular Prism
$V = Bh$
$SA = 2B + Ph$
P = perimeter of the base

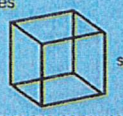
Two bases are triangles; other faces are parallelograms



bases are triangles

Cube
$V = Bh$ or $V = s^3$
$SA = 6s^2$

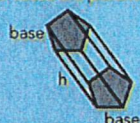
Prism with all square faces; a cube has: 6 faces, 8 vertices and 12 edges



all faces are congruent

Pentagonal Prism
$V = Bh$
SA = sum of area of faces

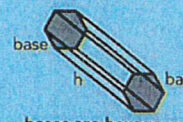
Two bases are pentagons; other faces are parallelograms



bases are pentagons

Hexagonal Prism
$V = Bh$
SA = sum of area of faces

Two bases are hexagons; other faces are parallelograms



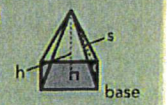
bases are hexagons

Polyhedra - Pyramids

Pyramid Facts: A pyramid is a **polyhedron** with one polygon base; lateral faces are triangles that meet at a point (vertex); the slant height(s) of the pyramid is the height of any of the lateral faces; a pyramid is named by the shape of its base

Rectangular Pyramid
$V = \frac{1}{3}Bh$
SA = sum of area of faces

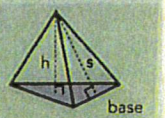
One rectangular base; all other faces are triangles



base is a rectangle

Triangular Pyramid
$V = \frac{1}{3}Bh$
SA = sum of area of faces

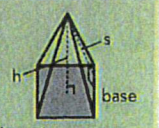
One triangular base; all other faces are triangles



base is a triangle

Square Pyramid
$V = \frac{1}{3}Bh$
SA = sum of area of faces

One square base; all other faces are triangles

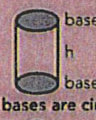


base is a square

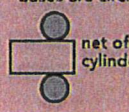
Space Figures with Curved Surfaces

Cylinder
$V = Bh$ or $V = \pi r^2 h$
$SA = 2\pi rh + 2\pi r^2$

Two ≅, ∥, circular bases joined by a curved surface (net of curved surface is a rectangle)



bases are circles



net of a cylinder

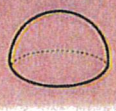
Cone
$V = \frac{1}{3}Bh$ or $V = \frac{1}{3}\pi r^2 h$
$SA = \pi rs + \pi r^2$

One circular base joined by a curved surface with one vertex

base is a circle

Hemisphere
$V = \frac{2}{3}\pi r^3$
$SA = 2\pi r^2 + \pi r^2$

Half of a sphere



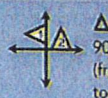
Sphere
$V = \frac{4}{3}\pi r^3$
$SA = 4\pi r^2$

A set of all points at a fixed distance from its center

CLASSIFYING TRANSFORMATIONS

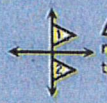
Transformation Facts: Any operation that changes the **size, shape** or **position** of an image from its original figure; *pre-image = original figure*; **image = transformation**

Rotation
A "turn" either CW or CCW around a given point; measured in degrees; images ≅



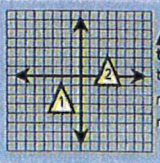
Δ1 rotated 90° CW (from origin) to Δ2

Reflection
A "fold" across a given line (line of reflection); pre-image and image are symmetrical; images ≅



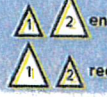
Δ1 reflected to Δ2

Translation
A "slide"; moves figure right/left &/or up/down; a "rule" describes number of units right/left &/or up/down; images ≅



Δ1 translated to Δ2
rule: up 3, right 5

Dilation
An enlargement or a reduction in size from the original figure; images ~



enlarged

reduced

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