Survival Heater Power Derivation - Artifact Loon LLC

This is a derivation of the formulas for predicting night heat loss that can be correlated from SmallWorld data. Not all of it (particularly the convection correlation) can be fully derived from first-principles; assumptions are made where appropriate

Starting assumptions

- Steady-state
- Nighttime
- "Traditional" photon harvester setup with white plate facing downward and insulated top/sides
- Isothermal photon harvester
- Ignore gap pad thermal resistance
- Ignore heat loss through cables and upper insulation

Nomenclature

T_{SURF} = Temperature of the photon harvester surface, generally assumed to be equal to the heater setpoint but it is acknowledged that this is imperfect and will be part of the data correlation.
T_{DWIR} = Downward IR temperature
T_{AIR} = Ambient air temperature
A = Area of the photon harvester plate
σ = The Stefan-Boltzmann constant, 5.67 x 10^{-8} W/m^2K^4
ε_{IR} = IR emissivity. Evaluated at 0.92 for our standard white paint
α_{IR} = IR absorptivity
h = The convection heat transfer coefficient. It tends to be on the order of ~1 W/m^2K for small payload sized objects on Loon.
Nu_L = Nusselt number = hL_C/k
Ra_L = Rayleigh number = $(g * \beta * (T_{SURF} - T_{AIR}) * c_P * \rho^2 * L_C^3) / (\mu * k)$
L_C = Characteristic length of the system. Not directly evaluated here but would be equal to 4 * the area of the photon harvester / its perimeter
k = Thermal conductivity of air. A function of temperature but not pressure.
μ = Absolute viscosity of air. A function of temperature but not pressure.
c_P = Specific heat of air. Constant at 1007 J/kg-K
β = Expansivity of air, equal to 1/T for an ideal gas, using absolute temperature (i.e. K, not C)
ρ = Density of air. A function of temperature and pressure; related by the ideal gas law
Pr = Prandtl number of air; assumed at a constant 0.7
Note that by convention, for convection correlations fluid properties are evaluated at the average of the plate temperature and ambient air temperature
g = Gravity. 9.75 m/s^2 average in Loon's range

Energy balance

1st law of thermodynamics:
$\dot{Q}_{IN} = \dot{Q}_{OUT}$ (Because we assumed steady-state)
$\dot{Q}_{INPUT\ RADIATION} + \dot{Q}_{HEATERS} = \dot{Q}_{EMITTED\ RADIATION} + \dot{Q}_{CONVECTION}$

Stefan-Boltzmann law and the definition of convection give us:
$\alpha_{IR} A \sigma T_{DWIR}^4 + \dot{Q}_{HEATERS} = \varepsilon_{IR} A \sigma T_{SURF}^4 + hA(T_{SURF} - T_{AIR})$

Radiation evaluation: use Kirchoff's Law of Thermal Radiation to take advantage of the fact that $\alpha_{IR} = \varepsilon_{IR}$

$\dot{Q}_{HEATERS} = \varepsilon_{IR} A \sigma (T_{SURF}^4 - T_{DWIR}^4) + hA(T_{SURF} - T_{AIR})$
$\dot{Q}_{HEATERS} = \varepsilon_{IR} A \sigma (T_{SURF}^4 - T_{DWIR}^4) + Nu * (k / L_C) * A(T_{SURF} - T_{AIR})$

Convection evaluation: Use the fact that for natural convection, $Nu = C * Ra^{1/4}, * Pr^n$, and the Prandtl number is constant so it can just get lumped into C_1 for us. We don't actually know what this constant *is* at this point because a flat plate with an insulated flange around it is a nonstandard shape, but all common shapes' laminar natural convection correlation (spheres, flat plates, cylinders) is proportional to $Ra^{1/4}$.

$\dot{Q}_{HEATERS} = \varepsilon_{IR} A \sigma (T_{SURF}^4 - T_{DWIR}^4) + C_1 * Ra^{1/4} * (k / L_C) * A(T_{SURF} - T_{AIR})$
$\dot{Q}_{HEATERS} = \varepsilon_{IR} A \sigma (T_{SURF}^4 - T_{DWIR}^4) + C_1 * [(g * \beta * (T_{SURF} - T_{AIR}) * c_P * \rho^2 * L_C^3) / (\mu * k)]^{1/4} * (k / L_C) * A(T_{SURF} - T_{AIR})$

Now consider that g, c_P, L_C, and A are all constants. These can be all rolled into C_1, since this will ultimately be evaluated individually based on data for each component. We have to do this anyway, so we may as well concern ourselves with fewer variables.

$\dot{Q}_{HEATERS} = \varepsilon_{IR} A \sigma (T_{SURF}^4 - T_{DWIR}^4) + C_1 * [(\beta * (T_{SURF} - T_{AIR}) * \rho^2) / (\mu * k)]^{1/4} * k * (T_{SURF} - T_{AIR})$

Now we can simplify a little bit and use the definition of β. For an ideal gas it is 1/T, evaluated at the average of the air and surface temperatures:

$\dot{Q}_{HEATERS} = \varepsilon_{IR} A \sigma (T_{SURF}^4 - T_{DWIR}^4) + C_1 * [(\beta * (T_{SURF} - T_{AIR}) * \rho^2) / (\mu * k)]^{1/4} * k * (T_{SURF} - T_{AIR})$

$\dot{Q}_{HEATERS} = \varepsilon_{IR} A \sigma (T_{SURF}^4 - T_{DWIR}^4) + C_1 * [\rho^2 / (T_{SURF} + T_{AIR}) / \mu]^{1/4} * k^{3/4} * (T_{SURF} - T_{AIR})^{5/4}$

Grouping air properties and temperatures separately this looks like:

$\dot{Q}_{HEATERS} = \varepsilon_{IR} A \sigma (T_{SURF}^4 - T_{DWIR}^4) + C_1 * (\rho^2 * k^3 / \mu)^{1/4} * [(T_{SURF} - T_{AIR})^5 / (T_{SURF} + T_{AIR})]^{1/4}$

We can even lump $\varepsilon_{IR}A\sigma$ into a single constant since the system actually "sees" the foam flange around it a little bit which makes the pure-radiation assumption a little less than perfect, so we might as well do a correlation for this as well. Call this C_2, so now the whole thing is

$$\dot{Q}_{HEATERS} = C_2 * (T_{SURF}^4 - T_{DWIR}^4) + C_1 * (\rho^2 * k^3 / \mu)^{1/4} * [(T_{SURF} - T_{AIR})^5 / (T_{SURF} + T_{AIR})]^{1/4}$$

Of these parameters:

- ρ *does* need to be evaluated in data-gathering. Density varies by about a factor of 3 in our flight range and so the resulting $\rho^{1/2}$ affects the convection term by up a factor of about 1.7.
- $(T_{SURF} + T_{AIR})^{-1/4}$ (a vestige of the β term) varies minimally and only impacts one term of our results by about 3%. Optional to evaluate as opposed to evaluating it as a constant at -40C and -70C, though one might as well since it's fairly easy.
- The $(k^3 / \mu)^{1/4}$ term is interesting. Both of these prop\6erties vary with temperature and not pressure in our altitude range. A linear curve fit for these properties in our temperature range shows:
    - $k = 0.0242 + 8.4 \times 10^{-5} * T$ (T in degrees C; end units are W/mK)
    - $\mu = 1.74 \times 10^{-5} + 5.41 \times 10^{-8} * T$ (T in degrees C; end units are Pa-s)
    - Evaluating this term between average air temperatures of -70C to -45C gives a variation of 6%. Not too big a variation but it's "almost free" to evaluate so we might as well.

It remains to evaluate C_1 and C_2 for each, and there is ample data in Smallworld to do this. In data-gathering, ideally one restricts themselves to nighttime steady-state data: no maneuvers, and the temperatures were the same within a few degrees for the past 30 minutes.