CARMA

YUIMA developers

4/2/2020

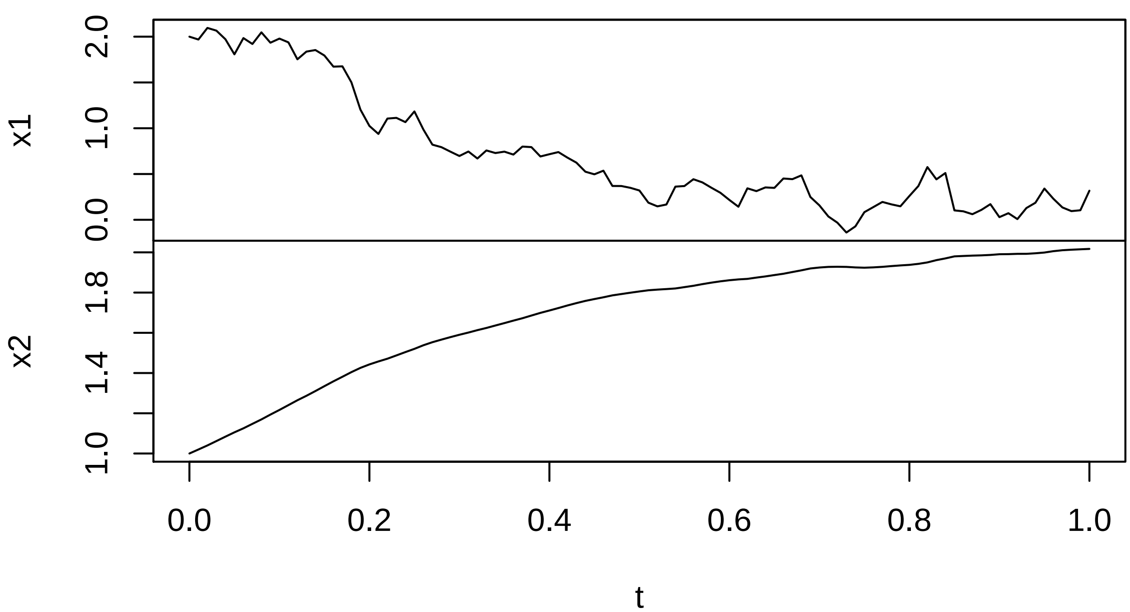
Q

Can YUIMA manage with statistical inference for the following model?

```
# yuima.model object
model <- setModel(drift = c("-3*x1","x1*x2"),
                  diffusion = matrix(c("1","0"),2,1),
                  state.variable = c("x1","x2"),
                  solve.variable = c("x1","x2"),xinit = c(2,1))
# simulated yuima object
sim <- simulate(model)
```

```
## Warning in yuima.warn("'delta' (re)defined."):
## YUIMA: 'delta' (re)defined.
```

```
# plot
plot(sim)
```



A

The simple answer to your question is No! Inference functions in YUIMA cannot treat the model.

Long answer:

Consider the following stochastic differential equations (SDEs):

$$\mathrm{d}X_{1,t} = -\alpha X_{1,t}\mathrm{d}t + \sigma \mathrm{d}W_t$$
$$\mathrm{d}X_{2,t} = -X_{1,t}X_{2,t}\mathrm{d}t.$$

This model is a transformation of a CARMA model. See also CARMA in YUIMA. Observe that

$$\mathrm{d}\left(\frac{X_{1,t}}{\sigma}\right) = -\alpha \left(\frac{X_{1,t}}{\sigma}\right)\mathrm{d}t + \mathrm{d}W_t$$
$$\mathrm{d}\left(-\frac{\log X_{2,t}}{\sigma}\right) = \left(\frac{X_{1,t}}{\sigma}\right)\mathrm{d}t.$$

By this observation, if we set

$$\tilde{X}_t = \begin{pmatrix} -\frac{\log X_{2,t}}{\sigma} \\ \frac{X_{1,t}}{\sigma} \end{pmatrix},$$

then

$$\mathrm{d}\tilde{X}_t = \begin{pmatrix} 0 & 1 \\ 0 & -\alpha \end{pmatrix} \tilde{X}_t \mathrm{d}t + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \mathrm{d}W_t$$

and the output process log $X_{2,t}$ (do not forget log!) is

$$Y_t = \begin{pmatrix} -\sigma \\ 0 \end{pmatrix}^{\top} X_t.$$

This is a CARMA(p, q) process with $p = 2, q = 0$. However, it does not satisfy the stationarity condition (2.5) of Brockwell, Davis and Yang 2011 since the matrix in front of $\tilde{X}$ is degenerate. Consequently, YUIMA cannot manage this process because the construction of the quasi-likelihood rests on the ergodic property of the process. Iacus and Mercuri 2015.

If the original model is modified as

$$\mathrm{d}\tilde{X}_t = \begin{pmatrix} 0 & 1 \\ -\alpha_2 & -\alpha_1 \end{pmatrix} \tilde{X}_t \mathrm{d}t + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \mathrm{d}W_t$$

with $\alpha_2 \neq 0$, then it can be treated.

```
set.seed(123)

# yuima.carma object
model <- setCarma(p = 2, q = 0)

# parameters
parameter <- list(a1 = 2.0, a2 = 0.2, b0 = -1.0)

# yuima.sampling object
samling <- setSampling(Terminal = 5, n = 1e3)
```

```
## Warning in yuima.warn("'delta' (re)defined."):
## YUIMA: 'delta' (re)defined.
```

```
# simulated yuima.object
sim <- simulate(model,
                true.parameter = parameter,
                sampling = samling,
                xinit = 2)

# estimation
opt <- qmle(sim,
            start = list(a1=1, a2 = 0.5, b0 = -1))
```

```
##
## Starting qmle for carma ...

## Warning in yuima.warn("quasi likelihood is too small to calculate."):
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```

```r
summary(opt)
```

```
## Quasi-Maximum likelihood estimation
##
## Call:
## qmle(yuima = sim, start = list(a1 = 1, a2 = 0.5, b0 = -1))
##
## Coefficients:
##      Estimate Std. Error
## b0 -1.3244297 0.03313313
## a2  0.0913724 0.27070104
## a1  1.8078168 3.21574021
##
## -2 log L: -12969.99
##
## Carma(2,0) model: Stationarity conditions are satisfied.
```

Outside YUIMA, the model can be treated as an integrated diffusion process Gloter 2006. Currently, it is not implemented in YUIMA but it is interesting to develop in this direction.