Roll No. ....

(01/23-II)

5199

B.A./B.A. (Hons.)/B.Sc. EXAMINATION

(Third Semester)

MATHEMATICS

BM-231

Advances Calculus

Time : Three Hours *Maximum Marks :* B.Sc. : 40, B.A. : 27

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory.

Compulsory Question

1. (a) Differentiate $\sqrt{\dfrac{1-x}{1+x}}$ w.r.t. x. 2(1)

(b) Evaluate : 2(1)

$$\lim_{x\to 0}\frac{\left(\tan^{-1}x\right)^2}{\log\left(1+x^2\right)}$$

(c) If u be a homogeneous function of degree n in x and y, then : 2(2)

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=nu$$

(d) Find the equation of tangent line at the point $t = 1$ to the curve : 2(1)

$$x=1+t,\ y=-t^2,\ z=1+t^2$$

Unit I

2. (a) Using $\varepsilon-\delta$ definition, prove that $f(x)=\begin{cases}x^2\cos\frac{1}{x}, & x\neq 0\\ 0, & x=0\end{cases}$ is a continuous function at $x = 0$. 4(2½)

(b) Verify Lagrange's mean value theorem for $f(x)=\sin x$ in $\left[\frac{\pi}{2},\frac{5\pi}{2}\right]$. 4(3)

3. (a) Evaluate : 4(2½)

$$\lim_{x\to 0}\left(\frac{1}{x^2}-\cot^2 x\right)$$

(b) Find the values of a, b, c so that :

$$\lim_{x\to 0}\frac{ae^x-b\cos x+ce^{-x}}{x\sin x}=2$$ 4(3)

Unit II

4. (a) Show that the function : 4(2½)

$$f(x,y)=\begin{cases}\frac{xy}{\sqrt{x^2+y^2}}; & (x,y)\neq(0,0)\\ 0; & (x,y)=(0,0)\end{cases}$$

is continuous at (0, 0).

(b) If z is a function of x and y, given by the equation $x^xy^yz^z=c$, where c is a constant, show that at $x=y=z$,

$$\frac{\partial^2 z}{\partial x\partial y}=-[x\log ex]^{-1}.$$ 4(3)

5. (a) If $u=\sin^{-1}\frac{x^2+y^2}{x+y}$, show that

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\tan u.$$ 4(2½)

(b) Expand $e^x \sin y$ in powers of x and y as far as terms of third degree. 4(3)

Unit III

6. (a) Show that the function given by :

$$f(x, y)=\begin{cases}\frac{xy\left(x^2-y^2\right)}{x^2+y^2}; & (x, y)\neq(0, 0)\\ 0; & (x, y)=(0, 0)\end{cases}$$

is differentiable at the origin. 4(2½)

(b) State and prove Young's Theorem. 4(3)

7. (a) Examine the function $f(x, y)=x^3+y^3-63(x+y)+12xy$ for maxima and minima. 4(2½)

(b) Find the maximum and minimum distance of the point (3, 4, 12) from the sphere $x^2+y^2+z^2=1$ using Lagrange's method of multipliers. 4(3)

Unit IV

8. (a) Express the curve :

$$\vec{r}=e^{2t}\cos t\hat{i}+e^{2t}\sin t\hat{j}+e^{2t}\hat{k}, \quad -\infty<t<\infty$$

in the normal form. 4(2½)

(b) Find the equation of osculating plane of the curve : 4(3)

$$x=a\cos t, y=a\sin t, z=bt$$

9. (a) Show that the radius R of the spherical curvature is given by : 4(2½)

$$R^2=\rho^4\sigma^2(\vec{r}''')^2-\sigma^2$$

(b) Find the envelop of the sphere : 4(3)

$$(x-a\cos\theta)^2+(y-a\sin\theta)^2+z^2=b^2$$