Roll No. ....

(01/23-II)

5161

B.A./B.A.(Hons.)/B.Sc. EXAMINATION

(First Semester)

MATHEMATICS

BM-113

Solid Geometry

Time : Three Hours *Maximum Marks :* B.Sc. : 40, B.A. : 27

Note : Attempt *Five* questions in all, selecting *one* question from each Section. Q. No. **1** is compulsory consisting of five parts.

(Compulsory Question)

1. (a) Define the conjugate points. 1½(1)

(b) To prove that one and only one conic of a confocal system will touch a given straight line. 1½(1)

(c) Prove that the plane $ax+by+cz=0$, cuts the cone $yz+zx+xy=0$ in perpendicular lines if $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0$. 1½(1)

(d) Find the condition that the plane $lx+my+nz=p$ should touch the conicoid $ax^2+by^2+cz^2=1$. 2(1)

(e) Find the condition that the line :

$$\frac{x-\alpha}{l}=\frac{y-\beta}{m}=\frac{z-\gamma}{n}$$

is a generator of the conicoid $ax^2+by^2+cz^2=1$. 1½(1)

Section I

2. Show that the coinc :

$$14x^2-4xy+11y^2-44x-58y+71=0$$

represents an ellipse and hence find its eccentricity, lengths and equation of the axis and foci. 8(5½)

3. (a) Find the locus of mid-point of the chords of the confocal $\frac{x^2}{a^2+\lambda}+\frac{y^2}{b^2+\lambda}=1$, which touch the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. 4(3)

(b) Prove that two points on the conic $\frac{l}{r}=1+e\cos\theta$, whose vectorial angles are α, β will be extremities of a diameter if $\frac{e+1}{e-1}=\tan\frac{\alpha}{2}\cdot\tan\frac{\beta}{2}$. 4(2½)

Section II

4. (a) Obtain the equation of the sphere, having the circle $x^2+y^2+z^2=9$, $x-2y+2z-5=0$ as the great circle. Find the centre and radius also. 4(3)

(b) Find the limiting points of the co-axial system of spheres :

$$x^2+y^2+z^2-20x+30y-40z+29+k(2x-3y+4z)=0.$$ 4(2½)

5. (a) Show that the line $\frac{x}{l}=\frac{y}{m}=\frac{z}{n}$, where $l^2+2m^2-3n^2=0$, is a generator of the cone $x^2+2y^2-3z^2=0$. 4(3)

(b) Find the equation of the right circular cylinder of radius 3 and axis as the line $\frac{x-1}{2}=\frac{y}{2}=\frac{z-3}{1}$. 4(2½)

Section III

6. (a) The normal at any point P of a central conicoid meets the thee principal planes at G_1, G_2, G_3. Show that $PG_1 : PG_2 : PG_3 = a^{-1} : b^{-1} : c^{-1}$. 4(3)

(b) Find the equations of the polar of the line $\frac{x-1}{5}=\frac{y-3}{7}=\frac{z+5}{2}$ w.r.t. the conicoid $x^2+3y^2-7z^2-21=0$ in symmetrical form. 4(2½)

7. (a) Find the equation of the enveloping cone from the point (x_1, y_1, z_1) to the conicoid $ax^2+by^2+cz^2=1$. 4(3)

(b) Prove that the normals from (α, β, γ) to the paraboloid $\frac{x^2}{a^2}+\frac{y^2}{b^2}=2z$ lie on the cone $\frac{\alpha}{x-\alpha}-\frac{\beta}{y-\beta}+\frac{a^2-b^2}{z-\gamma}=0$. 4(2½)

Section IV

8. (a) Prove that the section of the conicoid $ax^2+by^2+cz^2=1$ by the plane whose normal lies on the cone $\frac{x^2}{a}+\frac{y^2}{b}+\frac{z^2}{c}=0$ is a parabola. 4(3)

(b) Find the locus of the point of intersection of perpendicular generators of the hyperbolic paraboloid. 4(2½)

9. (a) Prove that the tangent planes to two confocals at any common point are at right angels. 4(3)

(b) Reduce the following equations to standard form :

$$4x^2 + y^2 + 4z^2 - 4yz + 8zx - 4xy + 2x - 4y + 5z + 1 = 0.$$ 4(2½)