Roll No. ....

(05/25)

5179

B.A./B.A. (Hons.)/B.Sc. EXAMINATION

(For Batch 2011 to 2023 Only)

(Second Semester)

MATHEMATICS

BM-121

Number theory and Trigonometry

Time : Three Hours *Maximum Marks* : $\begin{cases} \text{B.Sc.} : 40 \\ \text{B.A.} : 27 \end{cases}$

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. 1 is compulsory.

(Compulsory Question)

1. (a) Find a such that $a \equiv 7 \pmod 5$. 2(1)

(b) If x is any real number, then $\left[\frac{[x]}{n}\right] = \left[\frac{x}{n}\right]$, where n is a positive integer. 2(1)

(c) If $\sin(u + iv) = x + iy$, prove that

$$\frac{x^2}{\sin^2 u} - \frac{y^2}{\cos^2 u} = 1.$$ 2(1½)

(d) Show that :

$$\log(1 + \cos 2\theta + i\sin 2\theta) = \log(2\cos\theta) + i\theta.$$ 2(1½)

Unit I

2. (a) Show that there are infinitely many primes of the form $4n + 3$. 4(2½)

(b) Find all the solutions in positive integers of $5x + 3y = 52$. 4(3)

3. (a) If p be a prime number, then show that :

$$1^{p-1} + 2^{p-1} + 3^{p-1} + \ldots + (p-1)^{p-1} + 1 \equiv 0 \pmod{p}.$$ 4(2½)

(b) Find the g.c.d. of 858 and 325 and express it in the form $m.858 + n.325$. 4(3)

Unit II

4. (a) Find all the integers that give the remainder 2,6,5 when divided by 5,7 and 11, respectively. 4(2½)

(b) Find the highest power of 180 contained in 102 ! 4(3)

5. (a) Prove that $\frac{(2n)!}{(n!)^2}$ is even, where n is a natural number. 4(2½)

(b) Is the congruence $x^2 \equiv 150 \pmod{1009}$ solvable ? 4(3)

Unit III

6. (a) If $\sin\psi = \iota\tan\theta$, prove that :

$$\cos\theta + \iota\sin\theta = \tan\left(\frac{\pi}{4}+\frac{\psi}{2}\right).$$ 4(2½)

(b) Prove that :

$$16\sin^5\theta = \sin 5\theta - 5\sin 3\theta + 10\sin\theta.$$ 4(3)

7. (a) If $z = x + \iota y$, where x and y are real, find the real and imaginary parts of $\frac{\cos z}{1+z}$. 4(2½)

(b) If $\tan(\theta + \iota\phi) = \cos\alpha + \iota \sin\alpha$, where the letters denote real quantities, prove that $\theta = \frac{n\pi}{2} + \frac{\pi}{4}$, where n is any integer. 4(3)

Unit IV

8. (a) Prove that $\tanh\frac{u}{2} = \tan\frac{\theta}{2}$, if $u = \log\tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)$. 4(2½)

(b) Separate into real and imaginary parts $\tanh^{-1}(x + \iota y)$. 4(3)

9. (a) Find the sum of series $\sin\alpha + \frac{1}{2}\sin 2\alpha + \left(\frac{1}{2}\right)^2 \sin 3\alpha + \ldots\infty$. 4(2½)

(b) Sum the series :

$$\cos\theta - \frac{1}{2}\cos 2\theta + \frac{1}{3}\cos 3\theta - \ldots\infty.$$ 4(3)

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