Roll No. ....

(05/25)

5219

B.A./B.A. (Hons.)/B.Sc. EXAMINATION

(Fourth Semester)

MATHEMATICS

BM-241

Sequence and Series

Time : Three Hours *Maximum Marks :* $\begin{cases} \text{B.Sc.} : 40 \\ \text{B.A.} : 27 \end{cases}$

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. 1 is compulsory.

(Compulsory Question)

1. (a) If A and B are subsets of R, then $A \subseteq B \Rightarrow A' \subseteq B'$. 2(1)

(b) Show that : 2(1)

$$\lim_{n \to \infty} \frac{1}{n}(1 + 2^{1/2} + 3^{1/3} + \ldots.. + n^{1/n}) = 1.$$

(c) Show that the seircs : 2(1½)

$$\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\ldots\ldots$$

converges to $\frac{1}{2}$.

(d) Test the convergence of the series :

$$1-\frac{1}{4.3}+\frac{1}{4^2.5}-\frac{1}{4^3.7}+\ldots.$$ 2(1½)

Unit I

2. (a) Prove that set of rationals is not order complete. 4(2½)

(b) Prove that the intersection of an arbitrary family of closed sets is closed. 4(3)

3. (a) The derived set of any set is a closed set. 4(2½)

(b) Let A and B be two subsets of R. Show that $(A \cap B)^\circ = A^\circ \cap B^\circ$. 4(3)

Unit II

4. (a) State and prove Cauchy's second theorem on limits. 4(2½)

(b) Prove that the sequence $<a_n>$ defined by $a_1 = \sqrt{7}$ and $a_{n+1} = \sqrt{7 + a_n}$ converges to the positive root of the equation $x^2 - x - 7 = 0$. 4(3)

5. (a) Prove that : $\lim_{n \to \infty} \left(\frac{n^n}{n!} \right)^{\frac{1}{n}} = e.$ 4(2½)

(b) Discuss the convergence of the series :

$$\sum_{n=1}^{\infty} \left[\sqrt{n^2 + 1} - \sqrt{n^2 - 1} \right].$$ 4(3)

Unit III

6. (a) State and prove Raabe's test. 4(2½)

(b) Test the convergence of the series : 4(3)

$$1 + \frac{x^2}{2} + \frac{x^4}{4} + \frac{x^6}{6} + \ldots\ldots\ldots (x > 0).$$

7. (a) State and prove Cauchy's integral Test. 4(2½)

(b) Test the convergence of the series : 4(3)

$$1^p + \left(\frac{2}{3}\right)^p + \left(\frac{2.4}{3.5}\right)^p + \left(\frac{2.4.6}{3.5.7}\right)^p + \ldots\ldots .$$

Unit IV

8. (a) State and prove Leibnitz's test for the convergence of alternating series. 4(2½)

(b) Discuss the convergence and absolute convergence of the series. 4(3)

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}\left[\sqrt{n+1} - \sqrt{n-1}\right].$$

9. (a) Test the convergence of the series : 4(2½)

$$\sum_{n=3}^{\infty} \frac{(n^3+1)^{1/3} - n}{\log n}.$$

(b) Prove that : $\prod_{n=1}^{\infty}\left(1+\frac{x}{n}\right)e^{-x/n}$ 4(3)

is absolutely convergent for all real x.

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