

Roll No.

(12/24)

5241

B.A./B.Sc. EXAMINATION

(For Batch 2011 & Onwards)

(Fifth Semester)

MATHEMATICS

BM-353

Numerical Analysis

Time : Three Hours Maximum Marks : $\begin{cases} \text{B.Sc.: 30} \\ \text{B.A. : 20} \end{cases}$

Note : Attempt *Five* questions in all, selecting *one* question from each Section. Q. No. 1 is compulsory. All questions carry equal marks.

1. (i) Evaluate $\Delta^n \sin^2 x$.
- (ii) Find the third divided difference with arguments 2, 4, 9, 10 of the function $f(x) = x^3 - 2x$.

- (iii) Give the table for writing Bessel formula.
- (iv) Write the properties of normal distribution and normal probability curve.
- (v) Give any four properties of eigen values.
- (vi) Write the Newton-Cotes quadrature formula for numerical integration and hence give the formula for Simpson's three-eighth rule.

Section I

2. (i) Find $y(6)$ and $y(-1)$, if :

$$y(0) = 2, y(1) = 9, y(2) = 28, y(3) = 65, \\ y(4) = 126, y(5) = 217.$$

- (ii) Find the missing value in the following data :

$$\begin{array}{cccccc} x & : & 45 & 50 & 55 & 60 & 65 \\ y & : & 3.0 & --- & 2.0 & --- & -2.4 \end{array}$$

3. (i) Given $\sum_{1}^{10} f(x) = 500426,$

$$\sum_{4}^{10} f(x) = 329240, \sum_{7}^{10} f(x) = 175212 \text{ and}$$

$$f(10) = 40365. \text{ Find the value of } f(1).$$

- (ii) The values of the function $f(x)$ for values of x are given as :

$$\begin{array}{cccc} x & : & 1 & 2 & 7 & 8 \\ f(x) & : & 4 & 5 & 5 & 4 \end{array}$$

Find the value of $f(6)$ and also the value of x for which $f(x)$ is maximum or minimum.

Section II

4. (i) Derive the Gauss backward interpolation formula.

- (ii) Apply Sterling formula to obtain the value of y_{28} , given $y_{20} = 49225, y_{25} = 48316, y_{30} = 47236, y_{35} = 45926, y_{40} = 44306.$

5. (i) A bag contains 3 white and 4 red balls. Three balls are drawn one by one with replacement. Find the probability distribution of the number of red balls.
- (ii) The sum of mean and variance of a binomial distribution is 15 and the sum of their squares is 117. Find the distribution.

Section III

6. (i) Given that :

x	y
1.0	7.989
1.1	8.403
1.2	8.781
1.3	9.129
1.4	9.451
1.5	9.750
1.6	10.031

Find $\frac{dy}{dx}$ at $x = 1.1$ and 1.6 .

- (ii) From the following table, determine for what value of x the value of y is minimum ? Also find the minimum value of y .

x	y
3	0.205
4	0.240
5	0.259
6	0.262
7	0.250
8	0.205

7. (i) Find all the eigen values of the given matrix A, and point out the smallest eigen

$$\text{value } A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

(b) Transform the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$

to tri-diagonal form by Given's method.

Section IV

8. (i) Evaluate :

$$\int_0^6 \frac{1}{1+x^2} dx$$

by using Trapezoidal and Simpson's one-third rule.

(ii) Evaluate :

$$\int_0^3 (x^2 + 2x) dx$$

using Gauss's quadrature formula.

9. (i) Given $\frac{dy}{dx} = \frac{y-x}{y+x}$ with initial condition $y = 1$ at $x = 0$. Find y for $x = 0.1$ by Euler's modified method (5 steps).

(ii) Given that $\frac{dy}{dx} = x^2 + y^2$ and $y = 1$ when

$x = 0$. Find approximate value of y at $x = 0.2$ in two steps of 0.1 each, using Runge-kutta method.