Roll No. ....

(12/24)

5240

B.A./B.Sc. EXAMINATION

(For Batch 2011 & Onwards)

(Fifth Semester)

MATHEMATICS

BM-352

Groups of Rings

Time : Three Hours *Max. Marks* : B.A. : 26, B.Sc. : 40

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory. All questions carry equal marks.

(Compulsory Question)

1. (a) Show that a group upto order 2 is abelian. 1.5

(b) Define a cyclic group. 1.5

(c) State Lagrange's theorem. 1.5

(d) Let S = {1, 2, 3, 4, 5, 6}. Represent the permutation (2 3 1 4 5) as a product of transpositions. 2

(e) Define principal ideal domain. 1.5

Unit I

2. (a) A necessary and sufficient condition for a non-empty subset H of a group (G, .) to be a subgroup is that H must be closed with respect to multiplication. 4

   (b) Prove that G = {1, 2, 3, 4} is a group under multiplication modulo 5. 4

3. (a) State and prove Lagrange's Theorem. 4

   (b) Prove that every quotient group of a cyclic group is cyclic. 4

Unit II

4. (a) The necessary and sufficient condition for a homomorphism f to be one-one is that kernel $f = \{e\}$, where e is identity of domain. 4

   (b) If f is an automorphism of a group G and a ∈ G be an element, then show that $f(N(a)) = N(f(a))$. 4

5. (a) Let Z (G) be the centre of group G. If G/Z is cyclic, then prove that G is abelian. 4

   (b) Find the centre of permutation group S_3. 4

Unit III

6. (a) Every finite non-zero integral domain is a field. Prove. 4

   (b) Every field is a principal ideal ring. Prove. 4

7. (a) Let $f : R \rightarrow R'$ be a ring homomorphism. Let S be an ideal of R. The $f(S)$ is a ideal of $f(R)$. 4

   (b) Prove that the quotient field F of an integral domain is the smallest field of R. 4

Unit IV

8. (a) The ring of Gaussian integers is a Euclidean domain (ring). 4

   (b) Show that every non-zero prime ideal of a principal ideal domain is maximal. 4

9. (a) If R is an integral domain, then R[x] is also an integral domain. 4

   (b) Prove that Every Euclidean ring is a unique factorization domain. 4