Roll No. ....

(12/24)

5239

B.A./B.Sc. EXAMINATION

(For Batch 2011 & Onwards)

(Fifth Semester)

MATHEMATICS

BM-351

Real Analysis

Time : Three Hours *Maximum Marks :* $\begin{cases} \text{B.Sc.} : 40 \\ \text{B.A.} : 27 \end{cases}$

Note : Attempt *Five* questions in all, selecting *one* question from each Section. Q. No. 1 is compulsory. Marks within brackets are for B.A. students.

(Compulsory Question)

1. (a) If $f(x) = \dfrac{1}{x^2}$ on [1, 4] and P = {1, 2, 3, 4} be a partition of [1, 4] then compute L(*f*, P).

(b) If $a > 0$, $b > 0$; prove that :

$$\int_0^\infty \frac{e^{-ax} - e^{-bx}}{x} dx = \log\left(\frac{b}{a}\right).$$

(c) Show that in discrete metric space X, every subset of X is open.

(d) Define complete metric space.

(e) Prove that an isometry is a uniformly continuous function.

(f) State Bolzano Weierstrass property in metric spaces.

Section I

2. (a) If a function 'f' is defined on $[0, a]$, $a > 0$ by $f(x) = x^{3n}$, then show that f is Riemann integrable on $[0, a]$ and $\int_0^a f\, dx = \frac{a^4}{4}$.

(b) Using definition, evaluate $\int_0^{\pi/2} \sin x\, dx$.

3. (a) If 'f' is integrable on $[a, b]$ and 'c' is a real number, then prove that cf is integrable on $[a, b]$.

(b) State and prove Fundamental theorem of integral calculus.

Section II

4. (a) Show that the integral $\int_0^{\pi/2} \frac{\sin^m x}{x^n} dx$ is convergent if an only if $n < m + 1$.

(b) Discuss the convergence of Gamma function.

5. (a) State and prove Abel's test for convergence of an improper integral.

(b) Evaluate $\int_0^\infty \frac{\log(1+\alpha^2 x^2)}{1+x^2} dx$; where α is a parameter.

Section III

6. (a) Let X be an arbitrary non-empty set and $d: X \times X \rightarrow R$ be a function such that $d(x,y)=\begin{cases} 0 & \text{if } x=y \\ 1 & \text{if } x \neq y \end{cases}$; show that (X, *d*) is a metric space.

(b) Prove that every derived set is a closed set.

7. (a) Let *d* and *d** be two metrices on the same set X such that $d^*(x, y) = \frac{M d(x,y)}{1+d(x,y)} > 0$, for all $x, y \in X$. Show that *d* and *d** are equivalent.

(b) Prove that a subspace Y of a complete metric space X is complete iff it is closed.

Section IV

8. (a) Let (X, *d*) and (Y, *d**) be two metric spaces and let *f*, *g* be two continuous functions of X into Y. Then the set $\{x \in X : f(x) = g(x)\}$ is a closed subset of X.

(b) Prove that every compact metric space is complete.

9. (a) Prove that a metric space is compact iff it has Bolzano-Weierstrass property.

(b) Prove that a metric space (X, *d*) is disconnected iff there exist a non-empty proper subset of X which are both *d*-open and *d*-closed.