

Roll No.

(05/25)

5220

B.A./B.A. (Hons.)/B.Sc. EXAMINATION

(Fourth Semester)

MATHEMATICS

BM-242

Special Foundations and Integral Transforms

Time : Three Hours Maxi. Marks : $\begin{cases} \text{B.Sc. : 40} \\ \text{B.A. : 26} \end{cases}$

Note : Attempt *Five* questions in all, selecting the compulsory question and one question from each Unit. Marks in brackets are for B.A. Students.

(Compulsory Question)

1. (a) Define Beta and Gamma Function. Write the relation between them. 2(2)

- (b) Write the general solution of : $1\frac{1}{2}(1)$

$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + (x^2 - 36)y = 0$$

- (c) Define Laplace transform and its shifting property. $1\frac{1}{2}(1)$

- (d) Define Parseval's identity for Fourier transform. $1\frac{1}{2}(1)$

- (e) Write Rodrigues formula for Legendre and Hermite Polynomial. $1\frac{1}{2}(1)$

Unit I

2. Find the power series solution of the differential

equation $\frac{d^2 y}{dx^2} + x \frac{dy}{dx} + (2x^2 + 1)y = 0$ in powers

of x .

8(5)

3. Solve the equation $x \frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} + \frac{1}{2}xy = 0$ in

terms of Bessel's function.

8(5)

Unit II

4. (a) Express $x^4 + 2x^3 + 2x^2 - x - 3$ in terms of Legendre's polynomials. $4(2\frac{1}{2})$

- (b) Prove that : $4(2\frac{1}{2})$

$$(n+1)p_{n+1}(x) + np_{n-1}(x) = (2n+1)xp_n(x)$$

5. (a) Derive the Hermite's polynomial from Rodrigue's formula. $4(2\frac{1}{2})$

- (b) Prove that Hermite's polynomials are orthogonal over $(-\infty, \infty)$ with respect to the weight function e^{-x^2} . $4(2\frac{1}{2})$

Unit III

6. (a) Find $L[f(t)]$ where $f(t) = |t-1| + |t+1|$,
 $t \geq 0$. 4(2½)
- (b) Find the Laplace transform of the
 function $t^2 e^t \sin 4t$. 4(2½)
7. (a) Find the Laplace inverse of the function

$$\frac{2s-5}{4s^2+25} + \frac{4s-18}{9-s^2}$$
 4(2½)
- (b) Find inverse Laplace transform of

$$\frac{1+s}{(s+2)^2(s-1)^2}$$
 4(2½)

Unit IV

8. (a) Find the Fourier transform of the
 function :

$$f(x) \text{ where } f(x) = \begin{cases} x^2, & |x| < x_0 \\ 0, & |x| > x_0 \end{cases}$$

4(2½)

- (b) State and prove shifting property of
 Fourier transform. 4(2½)

9. (a) State and prove convolution theorem for
 Fourier transform. 4(2½)

- (b) If $f(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| > a \end{cases}$ use the

Parseval's identity to show that : 4(2½)

$$\int_0^{\infty} \frac{\sin^2 ax}{x^2} = \frac{\pi a}{2}$$



- (b) What are thiols ? Give two methods of their preparation. 2
4. (a) Give mechanism of electrophilic substitution in pyrrole. Why is the position-2 more reactive than position-3 in pyrrole ? 3
- (b) Explain the aromatic character of pyrrole on the basis of molecular orbital theory. 2

Section B

5. (a) Write the Claisen condensation reaction to prepare ethyl acetoacetate along with mechanism. What is the driving step for this reaction ? 3
- (b) How can diethyl malonate be used to synthesize the following : 2
- (i) 3-Methylbutanoic acid
- (ii) Acetoacetic acid.

6. (a) Write short notes on solid phase peptide synthesis. 3
- (b) Discuss the following : 2
- (i) Electrophoresis
- (ii) Gabriel phthalimide synthesis of α -amino acids.
7. (a) Giving suitable examples, discuss the mechanism of anionic vinyl polymerization. 3
- (b) Differentiate between Buna-S and Buna-N. 2

