5259

B.A./B.Sc. EXAMINATION

(Sixth Semester)

MATHEMATICS

BM-361

Real and Complex Analysis

Time : Three Hours   Maximum Marks : $\begin{cases} \text{B.Sc.} : 40 \\ \text{B.A.} \;\; : 27 \end{cases}$

Note : Attempt *Five* questions in all, selecting *one* question from each Section. Q. No. 1 is compulsory.

(Compulsory Question)

1. (a) Describe Beta and Gamma functions. 2(1)

   (b) Define Dirichlet's condition.   1½(1)

   (c) Define analytic function and give example of analytic function.   1½(1)

Section III

6. (a) Determine the image or stereographic projection of the following points on the sphere of radius $\frac{1}{2}$ and centre $\left(0,0,\frac{1}{2}\right)$:

   (i) $1 + i$

   (ii) $1 - i$

   (iii) $2 + 3i$.   4(3)

   (b) Show that the function $f(z) = |z|^2$ is continuous everywhere but nowhere differentiable except at the origin.

        4(2½)

7. (a) Show that the function $f(z) = e^{-z^{-4}}$ $(z \neq 0)$ and $f(0) = 0$ is not analytic at $z = 0$ although $C - R$ equations are satisfied at that point.   4(3)

   (b) State and prove sufficient conditions for $f(z)$ to be analytic in a domain D.

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Section IV

8. (a) Let the rectangular region D in the z plane be bounded by $x = 0$, $y = 0$, $x = 1$, $y = 2$. Determine the region D' of the coplane into which D is mapped under the transformation $W = \sqrt{2}e^{i\frac{\pi}{4}}z$.

        4(2½)

   (b) Find the image of $|z - 3i| = 3$ under the mapping $W = \frac{1}{z}$.   4(3)

9. (a) Find the fixed points, normal form and nature of Mobius transformation $W = \frac{3z - 4}{z - 1}$.   4(3)

   (b) Prove that every Mobius transformation maps circles or straight lines into circles or straight lines.   4(2½)

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