Roll No. ....

(05/25)

5181

B.A./B.A.(Hons.)/B.Sc. EXAMINATION

(For Batch 2011 to 2023 Only)

(Second Semester)

MATHEMATICS

BM-123

Vector Calculus

Time : Three Hours *Maximum Marks :* B.Sc. : 40, B.A. : 27

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory.

(Compulsory Question)

1. (a) If :

$$A = \hat{i} + \hat{j} + \hat{k},$$

$$B = \hat{i} - \hat{j} + \hat{k},$$

$$C = 2\hat{i} - \hat{k}$$

then find A × (B × C).

(b) Find the unit normal vector to the surface $x^4 - 3xyz + z^2 + 1 = 0$ at the point (1, 1, 1).

(c) If u, v, w are orthogonal coordinates then prove that $\nabla v \times \nabla w = \frac{\hat{e}_1}{h_2 h_3}$.

(d) State Stoke's theorem.

(e) If $r = x\hat{i} + y\hat{j} + z\hat{k}$, then prove that

$$\nabla.\hat{r} = \frac{2}{r}, \text{ where } r = |r|, \hat{r} = \frac{r}{|r|}.$$

Unit I

2. (a) Prove that the necessary and sufficient condition that :

$$A \times (B \times C) = (A \times B) \times C$$

is A and C are collinear.

(b) Find the constant λ and such that the vectors :

$$2\hat{i} - 4\hat{j} + 5\hat{k},$$

$$\hat{i} - \lambda\hat{j} + \hat{k},$$

$$3\hat{i} + 2\hat{j} - 5\hat{k}$$

are coplanar.

3. (a) A particle moves along the curve $x = t^3 + 1$, $y = t^2$, $z = 2t + 5$, where t is the time. Find the components of velocity and acceleration at time t = 1 in the direction of vector $\hat{i} + \hat{j} + 3\hat{k}$.

(b) Prove that the necessary and sufficient condition for a vector function A of scalar variable t to have constant magnitude is $A.\frac{dA}{dt} = 0$.

Unit II

4. (a) If $r = x\hat{i} + y\hat{j} + z\hat{k}$ and $|r| = r$, then evaluate $\nabla\left(\frac{1}{r}\right)$ and $\nabla \log r$.

(b) Show that :

$$\text{div}\left(\frac{f(r)r}{r}\right) = \frac{1}{r^2}\frac{d}{dr}\left(r^2 f(r)\right).$$

5. (a) Find the directional derivative of :

$$\varphi(x, y, z) = xy + yz + zx$$

at the point (3, 1, 2) in the direction $2\hat{i} + 3\hat{j} + 6\hat{k}$.

(b) If :

$\varphi(x, y, z) = 2x^3y^2z^4$, find $\text{div}(\text{grad}\,\varphi)$.

Unit III

6. (a) Evaluate :

$$\oint_C \left(3x^2 - 8y^2\right)dx + (4y - 6xy)\,dy,$$

where C is the closed curve of the region bounded by $x = 0$, $y = 0$ and $x + y = 1$. Also Verify Green's theorem.

(b) Verify Stoke's theorem for the function $F = x^2\hat{i} + xy\hat{j}$, taken around the square bounded by $x = 0$, $x = a$, $y = 0$, $y = a$ in the plane $z = 0$.

7. Verify Gauss divergence theorem for $F = 4xy\hat{i} + yz\hat{j} - xz\hat{k}$, over the surface S of the cube bounded by the planes $x = 0$, $y = 0$, $z = 0$, $x = 2$, $y = 2$, $z = 2$.

Unit IV

8. (a) Represent the vector $A = xy\hat{i} + 2yz\hat{j} + \left(z^2 - yz\right)\hat{k}$ in cylindri[cal] coordinates.

(b) Prove that :

$$\dot{e}_r = \dot{\theta} e_\theta + \sin\theta \dot{\varphi} e_\varphi, \quad \dot{e}_\theta = -\dot{\theta} e_r + \cos\theta \dot{\varphi} e_\varphi,$$

where, e_r, e_θ and e_φ are unit vectors in spherical coordinates.

9. (a) Transform the function $F = \rho e_\rho + \rho e_\varphi$ from cylindrical coordinates to cartesian coordinates.

(b) Show that the spherical coordinate system is self-reciprocal.

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