



# CLASSICAL MECHANICS



**Mc  
Graw  
Hill**  
Education

N C RANA • P S JOAG

# Contents

Foreword

vii

Preface

ix

## I. Introduction

1

- I.1 What is this chapter about 1
- I.2 What is classical mechanics 2
- I.3 The place of classical mechanics in physics and some definitions 2
- I.4 A brief history of the development and mechanics up to Newton 6
- I.5 Newton's laws of motion 8
- I.6 Limitations of Newton's programme 20
- I.7 Summary 22
- Problems 23

## 1. Constrained Motions in Cartesian Coordinates

31

- 1.0 Introduction 31
- 1.1 Constraints and their classification 32
- 1.2 Examples of constraints 35
- 1.3 Principle of virtual work 38
- 1.4 The basic problem with the constraint forces 40
- 1.5 Lagrange's equations of motion of the first kind 41
- 1.6 Gibbs-Appell's principle of least constraint 45
- 1.7 D'Alembert's principle 46
- 1.8 Some additional remarks 50
- 1.9 Work energy relation for constraint forces of sliding friction 50
- 1.10 Summary 52
- Problems 52

## 2. Lagrangian Formulation in Generalised Coordinates

55

- 2.0 Introduction 55
- 2.1 Change of notation 56
- 2.2 Degrees of freedom 57
- 2.3 Generalised coordinates 59
- 2.4 Lagrange's equations of motion of the second kind 61
- 2.5 Properties of kinetic energy function  $T$  63
- 2.6 Theorem on total energy 66
- 2.7 Some remarks about the Lagrangian 69

- 2.8 Linear generalised potentials 69
- 2.9 Generalised momenta and energy 70
- 2.10 Gauge function for Lagrangian 72
- 2.11 Invariance of the Euler-Lagrange equations of motion under generalised coordinate transformations 73
- 2.12 Cyclic or ignorable coordinates 76
- 2.13 Integrals of motion 77
- 2.14 Concept of symmetry: homogeneity and isotropy 77
- 2.15 Invariance under Galilean transformations 81
- 2.16 Lagrangian for free particle motion 83
- 2.17 Lagrange's equations of motion for nonholonomic systems 85
- 2.18 Lagrange's equations of motion for impulsive forces 88
- 2.19 Summary 91  
Problems 92

### 3. Rotating Frames of Reference

- 3.0 Introduction 96
- 3.1 Inertial forces in the rotating frame 96
- 3.2 Electromagnetic analogy of the inertial forces 100
- 3.3 Effects of coriolis force 101
- 3.4 Foucault's pendulum 108
- 3.5 Velocity and acceleration of a particle with respect to a system having two independent rotations about a common point 110
- 3.6 More general case of two rotations separated by one translation 112
- 3.7 Summary 114  
Problems 115

### 4. Central Force

- 4.0 Introduction 118
- 4.1 Definition and properties of the central force 118
- 4.2 Two-body central force problem 120
- 4.3 Stability of orbits 124
- 4.4 Conditions for closure 125
- 4.5 Integrable power laws of the central force 126
- 4.6 Derivation of force laws from kinematical laws of motion 127
- 4.7 Kepler's problem 131
- 4.8 Actual Geometry of orbits and orbital elements 135
- 4.9 Kepler's equation 137
- 4.10 Construction of an orbit from given set of initial conditions 139
- 4.11 Kepler's problem in velocity space 140
- 4.12 Orbits of artificial satellites 142
- 4.13 Precession of the perihelia of planetary orbits due to small perturbing noninverse square law of force 144
- 4.14 The basic physics of tides 150

4.15	Scattering in a conservative central force field	158	
4.16	Virial theorem	171	
4.17	Summary	175	
	Problems	176	
<b>5.</b>	<b>Hamilton's Equations of Motion</b>		<b>180</b>
5.0	Introduction	180	
5.1	Legendre's dual transformation	181	
5.2	Hamilton's function and Hamilton's equations of motion	183	
5.3	Properties of the Hamiltonian and of Hamilton's equations of motion	184	
5.4	Routhian	185	
5.5	Configuration space, phase space and state space	187	
5.6	Lagrangian and Hamiltonian of relativistic particles and light rays	189	
5.7	Relativistic mass tensors	192	
5.8	Summary	195	
	Problems	196	
<b>6.</b>	<b>Principle of Least Action and Hamilton's Principle</b>		<b>198</b>
6.0	Introduction	198	
6.1	Principle of least action	199	
6.2	Hamilton's principle	206	
6.3	Comparison between Fermat's principle of least action in optics and Maupertuis' principle of least action in mechanics	208	
6.4	Derivation of Euler-Lagrange equations of motion from Hamilton's principle	209	
6.5	Derivation of Hamilton's equations of motion for holonomic systems from Hamilton's principle	210	
6.6	Invariance of Hamilton's principle under generalised coordinate transformation	211	
6.7	Hamilton's principle and characteristic functions	212	
6.8	Noether's theorem	215	
6.9	Lorentz invariance of Hamilton's principal function for the relativistic motion of a free particle	217	
6.10	Significance of Hamilton's principal	218	
6.11	Summary	219	
	Problems	220	
<b>7.</b>	<b>Brachistochrones, Tautochrones and the Cycloid Family</b>		<b>222</b>
7.0	Introduction	222	
7.1	The 'chrone' family of curves	223	
7.2	Brachistochrone for uniform force field	223	
7.3	Cycloid as a tautochrone	225	

7.4	Brachistochrone for spherically symmetric potential field $V(r)$	228
7.5	Brachistochrones and Tautochrones inside a gravitating homogeneous sphere	230
7.6	Tautochronous motion in a centrifugal force field and epicycloids	232
7.7	Summary	233
	Problems	234
<b>8.</b>	<b>Canonical Transformations</b>	<b>236</b>
8.0	Introduction	236
8.1	Background and definition	237
8.2	Generating functions	238
8.3	Properties of canonical transformations	245
8.4	Some examples of canonical transformations	248
8.5	Canonical transformation of the free particle Hamiltonian	252
8.6	Liouville's theorem	254
8.7	Area conservation property of Hamiltonian flows	255
8.8	Summary	257
	Problems	258
<b>9.</b>	<b>The Poisson Bracket</b>	<b>262</b>
9.0	Introduction	262
9.1	Definition	262
9.2	Some useful identities	263
9.3	Elementary PBs	264
9.4	Poisson's theorem	264
9.5	Jacobi–Poisson theorem (or Poisson's second theorem) on PBs	265
9.6	Invariance of PB under canonical transformations	267
9.7	PBs involving angular momentum	268
9.8	Dirac's formulation of the generalised Hamiltonian	270
9.9	Lagrange bracket (LB)	271
9.10	Summary	273
	Problems	273
<b>10.</b>	<b>Hamilton–Jacobi Theory</b>	<b>276</b>
10.0	Introduction	276
10.1	Solution to the time dependent Hamilton–Jacobi equation and Jacobi's theorem	276
10.2	Connection with canonical transformation	279
10.3	How to find the complete integral of the HJ equation	281
10.4	Worked-out examples	283
10.5	Action–Angle variables	292
10.6	Adiabatic invariants	299
10.7	Classical–quantum analogies	302
10.8	Summary	309
	Problems	309

**11. Small Oscillations**

- 11.0 Introduction 311
- 11.1 Types of equilibria and the potential at equilibrium 311
- 11.2 Study of small oscillations using generalised coordinates 317
- 11.3 Forced vibrations and resonance 327
- 11.4 Summary 332
- Problems 333

311

**12. Rigid Body Dynamics**

- 12.0 Introduction 335
- 12.1 Degrees of freedom of a free rigid body 336
- 12.2 Euler's and Chasles' theorems 338
- 12.3 Frames of reference used to describe the motion of a rigid body 345
- 12.4 Kinetic energy of a rotating rigid body 347
- 12.5 Angular momentum 349
- 12.6 Transformations of and theorems on the moment of inertia tensor 350
- 12.7 Examples of the calculation and the experimental measurement of the moment of inertia tensor 358
- 12.8 Angular momentum in laboratory and centre of mass frames 366
- 12.9 Torque and its relation to angular momentum 368
- 12.10 Euler's equation of motion for rigid body 371
- 12.11 Time variation of rotational kinetic energy 372
- 12.12 Rotation of a free rigid body 372
- 12.13 Poinsot's method of geometrical construction 373
- 12.14 Analytical method of Euler for free rotation and the third integral of motion 377
- 12.15 Chandler wobbling of the earth 379
- 12.16 Motion of  $w$  in space for free rotation 381
- 12.17 Why should a freely rotating body precess at all? 384
- 12.18 Steady precession of a uniaxial body (symmetric top) under the action of an external torque 386
- 12.19 The case of arbitrary rotations 390
- 12.20 Addition of two angular velocities 391
- 12.21 Eulerian angles 392
- 12.22 Motion of a heavy symmetric top rotating about fixed point in the body under the action of gravity 396
- 12.23 Detailed study of the motion of a symmetric top 398
- 12.24 Examples of tops and their analogues 411
- 12.25 Forced precession of the earth's axis of rotation 415
- 12.26 Foucault's gyroscope 420
- 12.27 Stability conditions for motions of rigid bodies in rotating frames 423
- 12.28 Dynamics of some games and sports 425
- 12.29 Summary 440
- Problems 441

335

**13. Elasticity**

- 13.0 Introduction 447
- 13.1 Displacement vector and the strain tensor 448
- 13.2 Stress tensor 455
- 13.3 Strain energy 459
- 13.4 Possible forms of free energy and stress tensor for isotropic solids 461
- 13.5 Elastic moduli for isotropic solids 462
- 13.6 Elastic properties of general solids: Hooke's law and stiffness constants 464
- 13.7 Elastic properties of isotropic solids 466
- 13.8 Propagation of elastic waves in isotropic elastic media 469
- 13.9 Summary 473  
Problems 474

**14. Fluid Dynamics**

- 14.0 Introduction 476
- 14.1 A few basic definitions 477
- 14.2 The central problem of fluid dynamics 478
- 14.3 Equation of state 478
- 14.4 Types of time rates of change of quantities 478
- 14.5 Equation of continuity 480
- 14.6 Application to Liouville's theorem 482
- 14.7 Equations of motion 482
- 14.8 Pressure potential 483
- 14.9 External force field 484
- 14.10 Cases of equilibrium fluid distribution in presence of external fields 485
- 14.11 Bernoulli's theorem 486
- 14.12 Applications of Bernoulli's theorem 491
- 14.13 Gravity waves and ripples 496
- 14.14 Two-dimensional steady irrotational flow of incompressible fluids 502
- 14.15 Kelvin's and Helmholtz's theorems 510
- 14.16 Representation of vortices by complex functions 514
- 14.17 Flow of imperfect fluids 516
- 14.18 Summary 521  
Problems 521

**Appendix A1 Coordinate Frames**

- A1.1 Orthogonal coordinate frames 525
- A1.2 Nonorthogonal or oblique coordinate frames 531

**Appendix A2 Vector Calculus**

- A2.1 Introduction to Kronecker delta and Levi-civita symbols 534.

A2.2	Partial differentiation of vectors and scalars	536	
A2.3	Ordinary differentiation of vectors	537	
A2.4	Vector integration	538	
A2.5	Tangent, principal normal and binormal of orbits	540	
A2.6	Kinematics of particle motion	543	
A2.7	Kinematics in spherical polar and other coordinate frames	545	
A2.8	Vectors in orthogonal curvilinear coordinate systems	547	
A2.9	Vectors in general curvilinear coordinates	551	
<b>Appendix A3</b>	<b>Tensors</b>		<b>554</b>
A3.1	Formal concepts of scalars and vectors	554	
A3.2	Tensors	558	
<b>Appendix B</b>	<b>Sample of Short Questions</b>		<b>564</b>
	Class test I	564	
	Class test II	565	
	Class test III	567	
	Class test IV	568	
	Final examination	569	
<b>Appendix C</b>	<b>Hints and Answers to Selected Problems</b>		<b>572</b>
	Introduction	572	
	Chapter 1	575	
	Chapter 2	575	
	Chapter 3	577	
	Chapter 4	577	
	Chapter 5	579	
	Chapter 6	580	
	Chapter 7	580	
	Chapter 8	581	
	Chapter 9	581	
	Chapter 10	582	
	Chapter 11	583	
	Chapter 12	584	
	Chapter 13	585	
	Chapter 14	586	
<b>Appendix D</b>	<b>Physical Constants</b>		<b>588</b>
	<b>Bibliography</b>		<b>590</b>
	<b>Index</b>		<b>594</b>