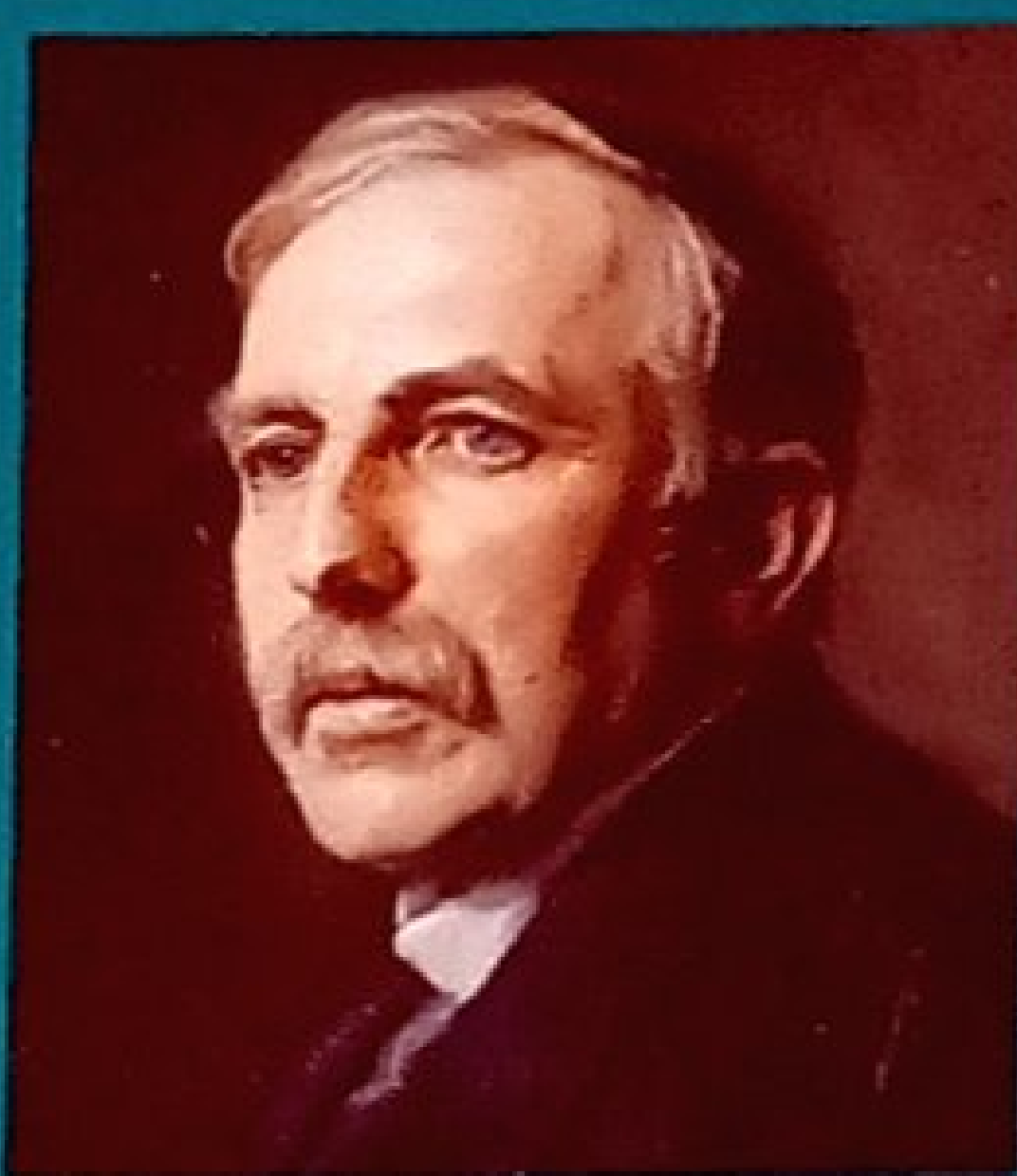




CLASSICAL MECHANICS

GUPTA • KUMAR • SHARMA



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1st. Baron Rutherford of
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New Zealand-born
British Physicist



A Pragati Edition

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