Roll No. ....

(05/25)

61311

Discipline Specific Courses (MD-DSC)

EXAMINATION

(For Batch 2024 & Onwards)

(Second Semester)

ALGEBRA AND NUMBER THEORY

BA/BSC/MD/MAT/2/DSC/102

Time : Three Hours *Maximum Marks* : 70

Note : Attempt *Five* questions in all, selecting *one* question from each Unit I to Unit IV. Q. No. **1** is compulsory. Marks are indicated along with questions.

Compulsory Question

1. (a) Define a skew Hermitian matrix. Give its example. 2

(b) Prove that $|A| = 1$, for a unitary matrix A. 2

(c) Show that the equation $x^6 + x^4 + 2 = 0$ has all its roots imaginary. 2

(d) Using the synthetic division, find the value of $3x^6 + 2x^4 - 14x^2 + x + 1$ when $x = -2$. 2

(e) If a/b and a/c, then $a/(bx + cy)$ for all integral values of x and y. 2

(f) Find the L.C.M. of integers 119, 272. 2

(g) Find x such that $x \equiv 7 (\bmod 5)$. 2

Unit I

2. (a) Every Hermitian matrix A can be written as A = B + iC, where B is real and

symmetric and C is real and skew-symmetric. 7

(b) Reduce the matrix $A = \begin{bmatrix} 1 & -1 & 2 & -1 \\ 4 & 2 & -1 & 2 \\ 2 & 2 & -2 & 0 \end{bmatrix}$ to $[I_3 \ 0]$. Hence find $\rho(A)$. 7

3. (a) Find the value of 'a' if the vectors $\begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$, $\begin{bmatrix} 9 \\ 0 \\ 1 \end{bmatrix}$ are linearly dependent. 7

(b) Verify the Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & 1 & 2 \\ 5 & 3 & 3 \\ -1 & 0 & -2 \end{bmatrix}$ and hence find A^{-1}. 7

Unit II

4. (a) Solve the equation :

$$x^3 - 9x^2 + 14x + 24 = 0,$$

given that two of the roots are in the ratio 3 : 2. 7

(b) Find the condition that the equations $x^3 + px + q = 0$ and $x^3 + rx + s = 0$ shall have a common root. 7

5. (a) Increase by 4 the roots of the equation $3x^5 - 5x^3 + 7 = 0$. 7

(b) If α, β, γ are the roots of the equation $x^3 + ax^2 + bx + c = 0$, form an equation whose roots are $\frac{\beta+\gamma}{\alpha}, \frac{\gamma+\alpha}{\beta}, \frac{\alpha+\beta}{\gamma}$. 7

Unit III

6. (a) Solve the equation $x^3 + x^2 - 16x + 20 = 0$ by Cardon's method. 7

   (b) Solve the equation :

   $$x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$$

   by Ferrari's method. 7

7. (a) Show that $2^{un} - 1$ is divisible by 15. 7

   (b) Show that there are infinitely many primes of the form $4n - 1$. 7

Unit IV

8. (a) Find the remainder when $53^{103} + 103^{53}$ is divided by 39. 7

   (b) Find the general solution and least positive integral solution of $11x + 5y = 79$. 7

9. (a) State and prove Euler's theorem. 7

(b) Show that $28! + 233 \equiv 0 \pmod{899}$ by using Wilson's theorem. 7

10. Solve the congruences $x \equiv 1 \pmod 4$, $x \equiv 3 \pmod 5$ and $x \equiv 2 \pmod 7$ simultaneously. 14

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