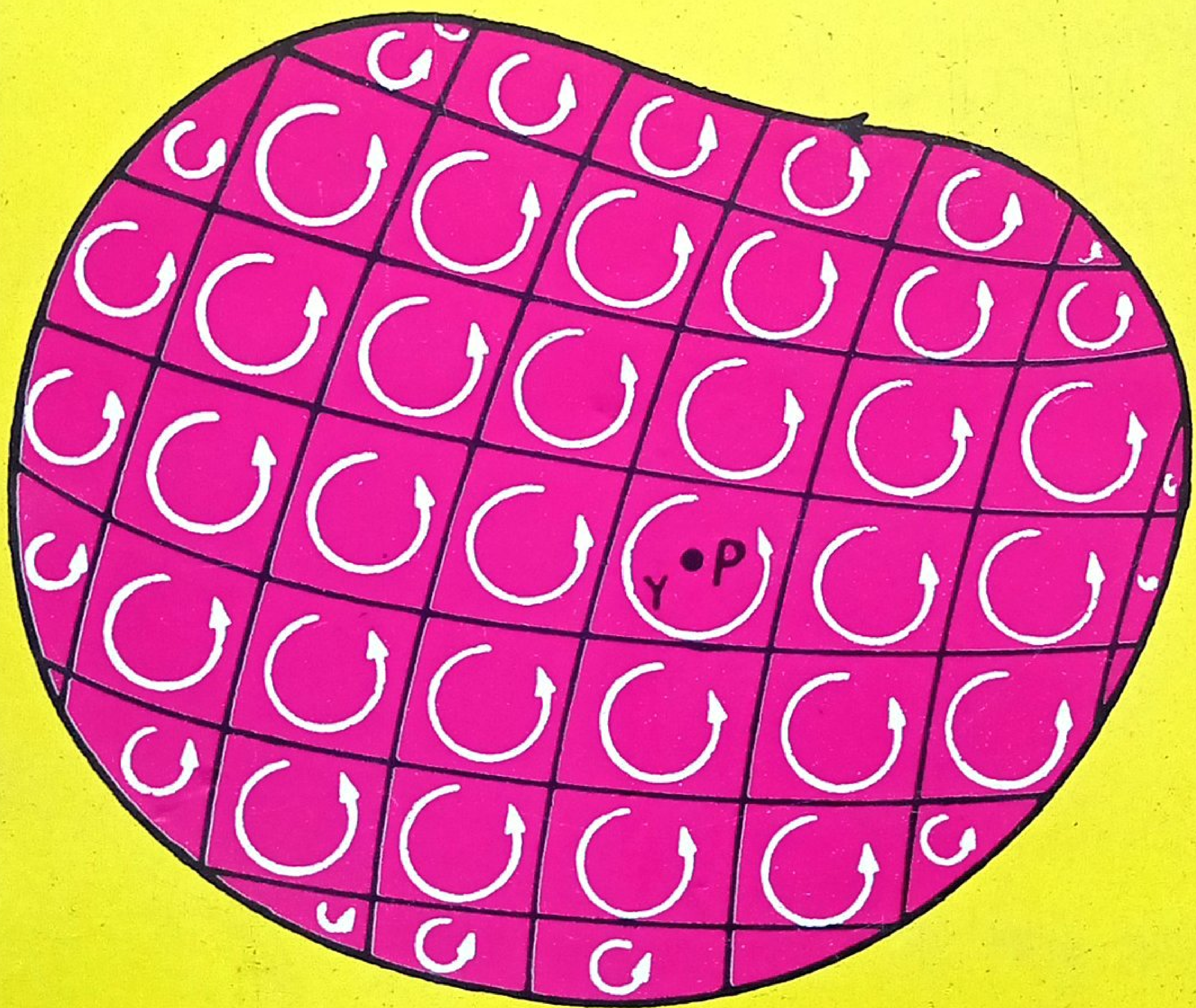


Textbook of  
**Fluid  
Dynamics**

F. Chorlton





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