Lokenath Debnath
Dambaru Bhatta

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} f(\xi)\, e^{-ik\xi} d\xi\right] e^{ikx}\, dk$$

Integral Transforms and Their Applications

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Contents

1 Integral Transforms **1**

1.1 Brief Historical Introduction 1

1.2 Basic Concepts and Definitions 6

2 Fourier Transforms and Their Applications **9**

2.1 Introduction 9

2.2 The Fourier Integral Formulas 10

2.3 Definition of the Fourier Transform and Examples 12

2.4 Fourier Transforms of Generalized Functions 17

2.5 Basic Properties of Fourier Transforms 28

2.6 Poisson's Summation Formula 37

2.7 The Shannon Sampling Theorem 44

2.8 Gibbs' Phenomenon 54

2.9 Heisenberg's Uncertainty Principle 57

2.10 Applications of Fourier Transforms to Ordinary Differential Equations 60

2.11 Solutions of Integral Equations 65

2.12 Solutions of Partial Differential Equations 68

2.13 Fourier Cosine and Sine Transforms with Examples 91

2.14 Properties of Fourier Cosine and Sine Transforms 93

2.15 Applications of Fourier Cosine and Sine Transforms to Partial Differential Equations 96

2.16 Evaluation of Definite Integrals 100

2.17 Applications of Fourier Transforms in Mathematical Statistics 103

2.18 Multiple Fourier Transforms and Their Applications 109

2.19 Exercises 119

3 Laplace Transforms and Their Basic Properties **133**

3.1 Introduction 133

3.2 Definition of the Laplace Transform and Examples 134

3.3 Existence Conditions for the Laplace Transform 139

3.4 Basic Properties of Laplace Transforms 140

3.5 The Convolution Theorem and Properties of Convolution . . 145

3.6 Differentiation and Integration of Laplace Transforms 151

3.7 The Inverse Laplace Transform and Examples 154

3.8 Tauberian Theorems and Watson's Lemma 168

Scanned by TapScanner

3.9 Exercises 173

4 Applications of Laplace Transforms **181**
4.1 Introduction 181
4.2 Solutions of Ordinary Differential Equations 181
4.3 Partial Differential Equations, Initial and Boundary Value Problems 182
4.4 Solutions of Integral Equations 207
4.5 Solutions of Boundary Value Problems 222
4.6 Evaluation of Definite Integrals 225
4.7 Solutions of Difference and Differential-Difference Equations 228
4.8 Applications of the Joint Laplace and Fourier Transform 230
4.9 Summation of Infinite Series 237
4.10 Transfer Function and Impulse Response Function of a Linear System 248
4.11 Exercises 251

5 Fractional Calculus and Its Applications **269**
5.1 Introduction 269
5.2 Historical Comments 270
5.3 Fractional Derivatives and Integrals 272
5.4 Applications of Fractional Calculus 279
5.5 Exercises 282

6 Applications of Integral Transforms to Fractional Differential and Integral Equations **283**
6.1 Introduction 283
6.2 Laplace Transforms of Fractional Integrals and Fractional Derivatives 284
6.3 Fractional Ordinary Differential Equations 287
6.4 Fractional Integral Equations 290
6.5 Initial Value Problems for Fractional Differential Equations . 295
6.6 Green's Functions of Fractional Differential Equations 298
6.7 Fractional Partial Differential Equations 299
6.8 Exercises 312

7 Hankel Transforms and Their Applications **315**
7.1 Introduction 315
7.2 The Hankel Transform and Examples 316
7.3 Operational Properties of the Hankel Transform 319
7.4 Applications of Hankel Transforms to Partial Differential Equations 322
7.5 Exercises 331

Scanned by TapScanner

8 Mellin Transforms and Their Applications **339**
8.1 Introduction 339
8.2 Definition of the Mellin Transform and Examples 340
8.3 Basic Operational Properties of Mellin Transforms 343
8.4 Applications of Mellin Transforms 349
8.5 Mellin Transforms of the Weyl Fractional Integral and the Weyl Fractional Derivative 353
8.6 Application of Mellin Transforms to Summation of Series . . 358
8.7 Generalized Mellin Transforms 361
8.8 Exercises 365

9 Hilbert and Stieltjes Transforms **371**
9.1 Introduction 371
9.2 Definition of the Hilbert Transform and Examples 372
9.3 Basic Properties of Hilbert Transforms 375
9.4 Hilbert Transforms in the Complex Plane 378
9.5 Applications of Hilbert Transforms 380
9.6 Asymptotic Expansions of One-Sided Hilbert Transforms . . . 388
9.7 Definition of the Stieltjes Transform and Examples 391
9.8 Basic Operational Properties of Stieltjes Transforms 394
9.9 Inversion Theorems for Stieltjes Transforms 396
9.10 Applications of Stieltjes Transforms 399
9.11 The Generalized Stieltjes Transform 401
9.12 Basic Properties of the Generalized Stieltjes Transform 403
9.13 Exercises 404

10 Finite Fourier Sine and Cosine Transforms **407**
10.1 Introduction 407
10.2 Definitions of the Finite Fourier Sine and Cosine Transforms and Examples 408
10.3 Basic Properties of Finite Fourier Sine and Cosine Transforms 410
10.4 Applications of Finite Fourier Sine and Cosine Transforms . . 416
10.5 Multiple Finite Fourier Transforms and Their Applications . 422
10.6 Exercises 425

11 Finite Laplace Transforms **429**
11.1 Introduction 429
11.2 Definition of the Finite Laplace Transform and Examples . . 430
11.3 Basic Operational Properties of the Finite Laplace Transform 436
11.4 Applications of Finite Laplace Transforms 439
11.5 Tauberian Theorems 443
11.6 Exercises 443

Scanned by TapScanner

12 Z Transforms 445
12.1 Introduction 445
12.2 Dynamic Linear Systems and Impulse Response 445
12.3 Definition of the Z Transform and Examples 449
12.4 Basic Operational Properties of Z Transforms 453
12.5 The Inverse Z Transform and Examples 459
12.6 Applications of Z Transforms to Finite Difference Equations 463
12.7 Summation of Infinite Series 466
12.8 Exercises 469

13 Finite Hankel Transforms 473
13.1 Introduction 473
13.2 Definition of the Finite Hankel Transform and Examples 473
13.3 Basic Operational Properties 476
13.4 Applications of Finite Hankel Transforms 476
13.5 Exercises 481

14 Legendre Transforms 485
14.1 Introduction 485
14.2 Definition of the Legendre Transform and Examples 486
14.3 Basic Operational Properties of Legendre Transforms 489
14.4 Applications of Legendre Transforms to Boundary Value Problems 497
14.5 Exercises 498

15 Jacobi and Gegenbauer Transforms 501
15.1 Introduction 501
15.2 Definition of the Jacobi Transform and Examples 501
15.3 Basic Operational Properties 504
15.4 Applications of Jacobi Transforms to the Generalized Heat Conduction Problem 505
15.5 The Gegenbauer Transform and Its Basic Operational Properties 507
15.6 Application of the Gegenbauer Transform 510

16 Laguerre Transforms 511
16.1 Introduction 511
16.2 Definition of the Laguerre Transform and Examples 511
16.3 Basic Operational Properties 516
16.4 Applications of Laguerre Transforms 520
16.5 Exercises 523

Scanned by TapScanner

17 Hermite Transforms **525**
17.1 Introduction 525
17.2 Definition of the Hermite Transform and Examples 526
17.3 Basic Operational Properties 529
17.4 Exercises 538

18 The Radon Transform and Its Applications **539**
18.1 Introduction 539
18.2 The Radon Transform 541
18.3 Properties of the Radon Transform 545
18.4 The Radon Transform of Derivatives 550
18.5 Derivatives of the Radon Transform 551
18.6 Convolution Theorem for the Radon Transform 553
18.7 Inverse of the Radon Transform and the Parseval Relation . . 554
18.8 Applications of the Radon Transform 560
18.9 Exercises 561

19 Wavelets and Wavelet Transforms **563**
19.1 Brief Historical Remarks 563
19.2 Continuous Wavelet Transforms 565
19.3 The Discrete Wavelet Transform 573
19.4 Examples of Orthonormal Wavelets 575
19.5 Exercises 584

Appendix A Some Special Functions and Their Properties **587**
A-1 Gamma, Beta, and Error Functions 587
A-2 Bessel and Airy Functions 592
A-3 Legendre and Associated Legendre Functions 598
A-4 Jacobi and Gegenbauer Polynomials 601
A-5 Laguerre and Associated Laguerre Functions 605
A-6 Hermite Polynomials and Weber-Hermite Functions 607
A-7 Mittag Leffler Function 609

Appendix B Tables of Integral Transforms **611**
B-1 Fourier Transforms 611
B-2 Fourier Cosine Transforms 615
B-3 Fourier Sine Transforms 617
B-4 Laplace Transforms 619
B-5 Hankel Transforms 624
B-6 Mellin Transforms 627
B-7 Hilbert Transforms 630
B-8 Stieltjes Transforms 633
B-9 Finite Fourier Cosine Transforms 636
B-10 Finite Fourier Sine Transforms 638
B-11 Finite Laplace Transforms 640

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B-12 Z Transforms 642
B-13 Finite Hankel Transforms 644

Answers and Hints to Selected Exercises 645
2.19 Exercises 645
3.9 Exercises 651
4.11 Exercises 655
6.8 Exercises 662
7.5 Exercises 662
8.8 Exercises 663
9.13 Exercises 664
10.6 Exercises 665
11.6 Exercises 667
12.8 Exercises 667
13.5 Exercises 670
16.5 Exercises 670
17.4 Exercises 670
18.9 Exercises 671
19.5 Exercises 671

Bibliography 673

Index 689

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