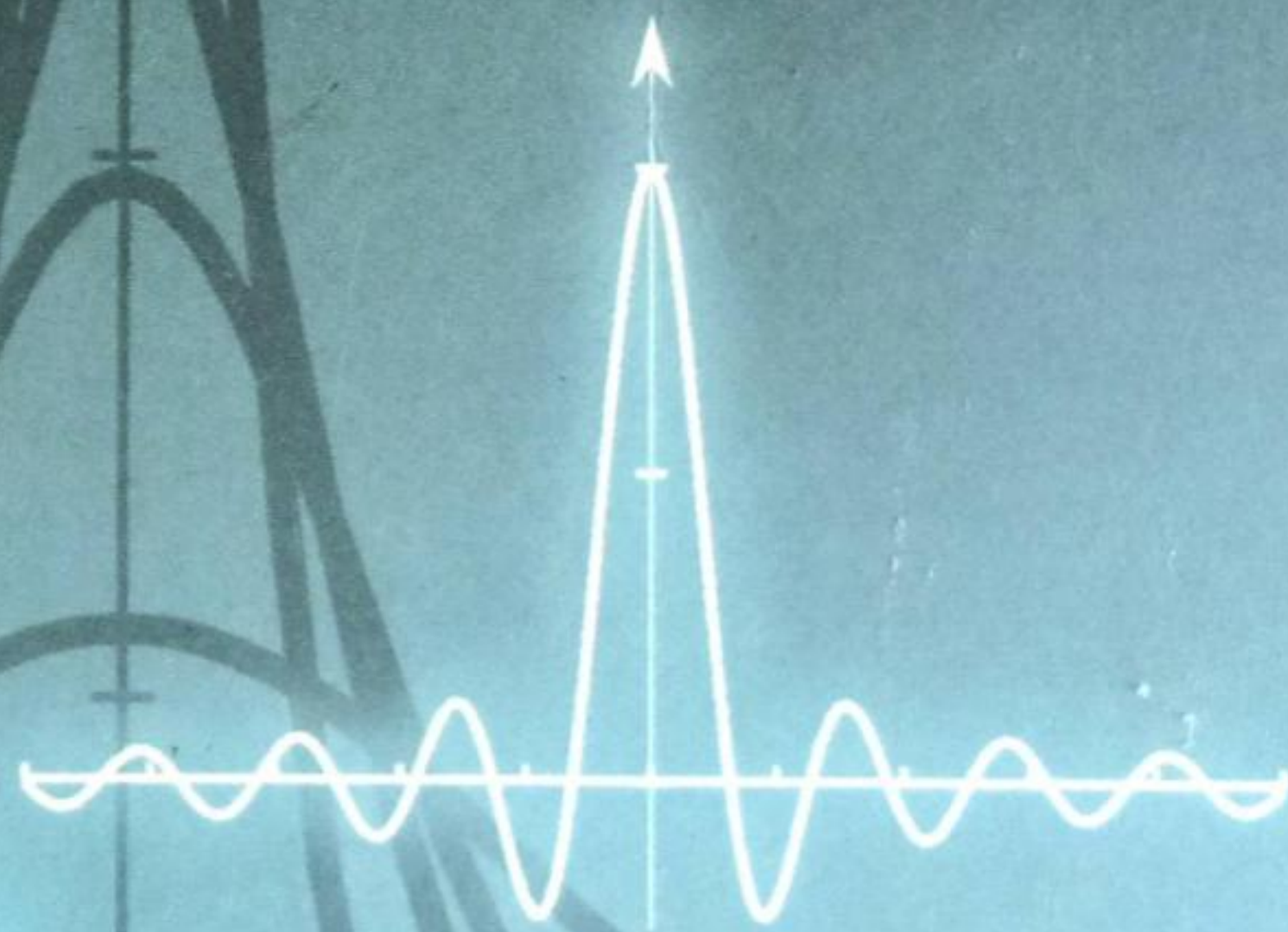


Lokenath Debnath
Dambaru Bhatta

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} f(\xi) e^{-ik\xi} d\xi \right] e^{ikx} dk$$

Integral Transforms and Their Applications

Second Edition



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A CHAPMAN & HALL BOOK

Special Indian Edition

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