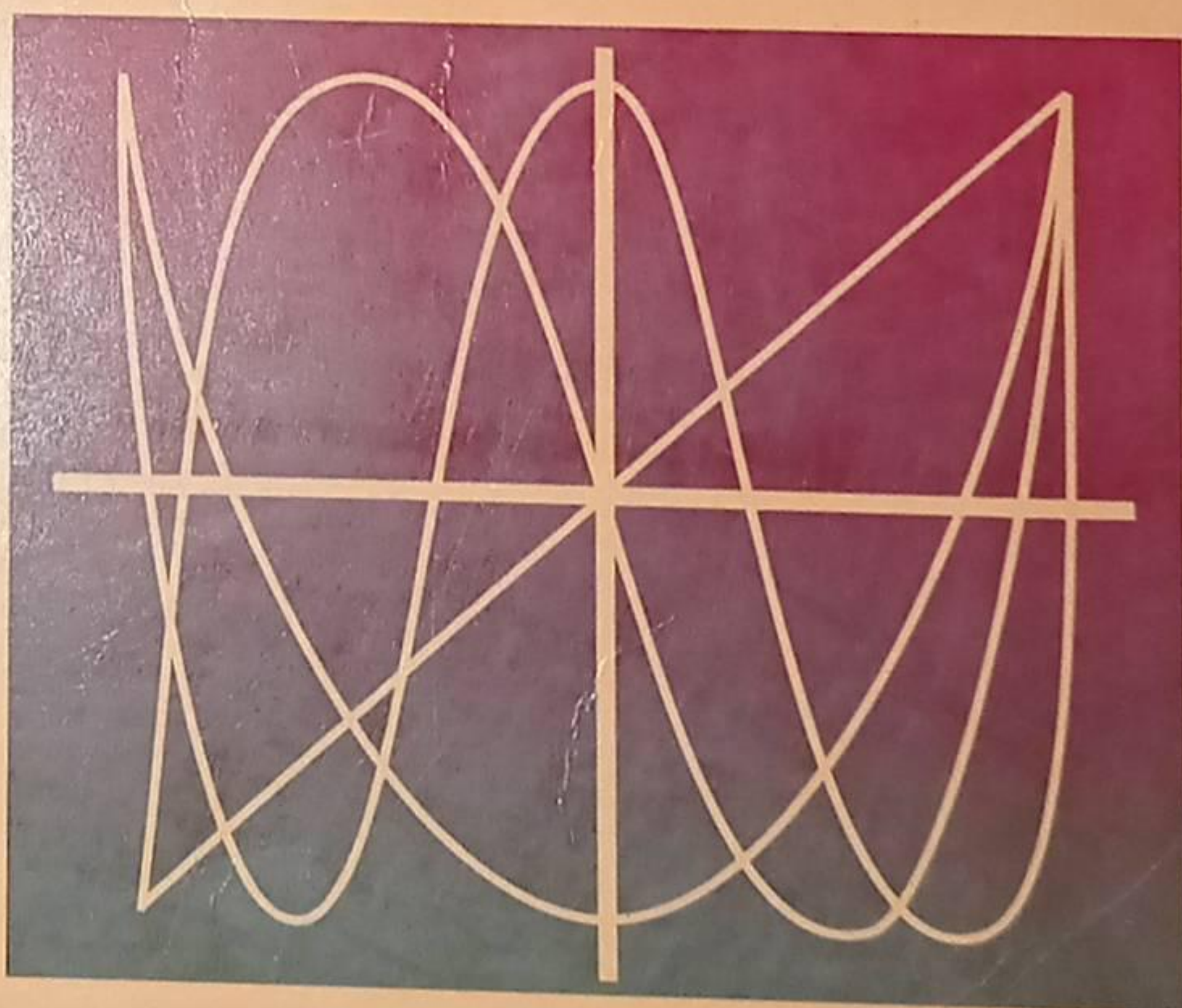


Eastern
Economy
Edition

THIRD EDITION

**INTRODUCTORY
METHODS
OF NUMERICAL
ANALYSIS**



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