Roll No. .....

(05/25)

15214

M.Sc. (2 Year) EXAMINATION

(For Batch 2021 & Onwards)

(Second Semester)

MATHEMATICS

MSc/Math/2/CC9

System of Differential Equations

Time : Three Hours *Maximum Marks : 70*

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. 1 is compulsory.

(Compulsory Question)

1. (a) Define Poincare-Benedixson theorem.

   (b) Explain Saddle point with example.

(c) Define asymptotical stability. Give one example of it.

(d) Define Half path and Limit set.

(e) Define Wronskian of a system. 5×2=10

Unit I

2. (a) If $A=\begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 0 \\ 1 & 4 & -2 \end{bmatrix}$, find the determinant of the fundamental matrix ϕ satisfying $\phi(0) = E$. 7.5

(b) The vector function $\phi_1, \phi_2, \ldots\ldots, \phi_n$ are linearly dependent on I, then prove that Wronskian $W(\phi_1, \phi_2, \ldots\ldots, \phi_n)(t) = 0$ for every $t \in I$. 7.5

3. (a) State and prove Abel-Liouville formula. 7.5

(b) Show that the set of all solutions of the system $x'(t) = A(t)x(t)$, $x(t_0) = x_0$, $t, t_0 \in I$ forms an n-dimensional vector space over the field of complex numbers. 7.5

Unit II

4. Explain method of variation of constants for a non-homogeneous system with constant coefficients. Also find the general solution of $y'' + y = \sec t(t)$. 15

5. (a) State and prove Floquet theory for periodic systems. 7.5

(b) Prove that the solution : 7.5

$$\phi(t) = \Phi(t)\int_{\tau}^{t} \phi^{-1}(s)b(s)ds,\ t \in \mathrm{I}$$

of NH system $x' = \mathrm{A}(t)x + \mathrm{b}$ can be written as :

$$\phi(t) = \psi^{*-1}(t)\int_{\tau}^{t} \psi^{*}(s)b(s)ds$$

where ψ is a fundamental matrix of the adjoint system $x' = -\mathrm{A}^*(t)x$.

Unit III

6. (a) Define Node, center, saddle point and spiral point with example. 7.5

(b) If (0, 0) is the only critical point of the linear system $\frac{dx}{dt} = ax + by,\ \frac{dy}{dt} = cx + dy$ and the roots of its characteristics equation are real, unequal and of the same sign then prove that (0, 0) is a node. 7.5

7. (a) Find the nature and stability of the critical point of the system : 7.5

$$\frac{dx}{dt} = \sin x - 4y,$$

$$\frac{dy}{dt} = \sin 2x - 5y.$$

(b) Find the nature of critical point of the system $\frac{dx}{dt} = ax + by$, $\frac{dy}{dt} = cx + dy$ when the roots of characteristics equation are pure imaginary. 7.5

Unit IV

8. (a) What is Liapunov Stability ? Also define stability of quasi linear system with example. 7.5

(b) Explain Liapunov function, positive definite, positive semidefinite, negative definite, negative semidefinite with examples. Also check $E(x, y) = -x^2$ is positive definite or not. 7.5

9. (a) Show that the system :

$$\frac{dx}{dt} = y + x(1 - x^2 - y^2),$$

$$\frac{dy}{dt} = -x + y(1 - x^2 - y^2)$$

has a limit cycle 10

(b) State Poincare-Bendixson Theorem. 5

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