Roll No. ....

(12/24)

15202

M.Sc. (2 Year) EXAMINATION

(For Batch 2021 & Onwards)

(First Semester)

MATHEMATICS

MSC/Math/1/CC1

Abstract Algebra

Time : Three Hours *Maximum Marks :* 80

Note : Attempt *Five* questions in all, selecting *one* question from each Section. Q. No. 1 (Section I) is compulsory.

Section I

1. (a) Define the centralizer of an element in a group.

   (b) Define a normal series of a group.

(c) What is a composition series of a group ?

(d) Explain the concept of a nil ideal.

(e) Define a direct sum of modules.

Section II

2. State and prove Scheier Refinement Theorem.

3. (a) Define normal series and composition series of a group. Provide an example of a group with both types of series, and discuss their properties.

(b) State and prove the Jordan-Holder Theorem. Does this theorem holds for infinite groups ?

Section III

4. (a) Prove that every finite p-group is solvable.

(b) Define nilpotent group in terms of upper central series. Show that a group G is nilpotent if and only if there is a normal series

$$\{e\} = G_0 \subseteq G_1 \subseteq .... \subseteq G_k = G$$

such that $G_i \triangleleft G$ and

$$G_{i+1}/G_i \subseteq z(G/G_i) \; \forall \; i.$$

5. (a) Describe the properties of Sylow subgroups in the context of nilpotent groups. Use Sylow's Theorems to analyze the structure of Sylow subgroups in a specific nilpotent group.

(b) Illustrate the concept of the derived series with a specific example of a non-abelian group.

Section IV

6. (a) Define submodule and provide an example. Prove that the sum of two submodules is also a submodule.

(b) If A and B are submodules of M, then show that $(A+B)/B \cong A/A \cap B$.

7. Let R be Euclidean Ring, then show that any finitely generated R-module M, is direct sum of finite number of cyclic modules.

Section V

8. (a) Prove that if an R-module satisfies both the ascending and descending chain conditions, then it is both Noetherian and Artinian.

(b) Define primary modules and explain their significance in module theory. Prove that in an Artinian ring, every submodule of a primary module is primary.

9. (a) State and prove the structure theorem for finite Boolean rings.

(b) Prove that the sum and product of nilpotent ideals are also nilpotent.