Roll No. ....

(12/24)

15205

M.Sc. (2 Year) EXAMINATION

(For Batch 2021 & Onwards)

(First Semester)

MATHEMATICS

MSC/Maths/I/CC4

Complex Analysis

Time : Three Hours *Maximum Marks* : 70

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory.

1. (a) Define contour, region, simply and multiply connected region.

(b) Show that :

$$\int_c \frac{dz}{z^n} = \begin{cases} 2\pi i & \text{for } n=1 \\ 0 & \text{for } n \neq 1 \end{cases},$$ where c is a +vely oriented simple closed controur enclosing the origin.

(c) Define power series, radius of convergence and circle of convergence.

(d) What do you mean by a singular point ? Describe essential singularity.

(e) Evaluate $\int_c \frac{z-3}{z^2+2z+5} dz$, where $c : |z+1-i| = 2$ in the positive sense.

2×5=10

Unit I

2. (a) State and prove the necessary conditions for $f(z) = u + iv$ to be analytic in a domain D. 8

(b) Prove that $u = e^{-x}(x \sin y - y \cos y)$ is harmonic. Find the harmonic conjugate of the function u and give the corresponding analytic function. 7

3. (a) Suppose that a function f is continuous in a domain D. If f has an antiderivative F in D then the integral of $f(z)$ along the contours lying entirely in D and extending from any fixed point z_1 to z_2, all have the same value. 7

(b) Find the value of the integral :

$$\int_0^{1+i} (x - y + ix^2) dx$$.

(i) Along the straight line from $z = 0$ to $z = 1 + i$,

(ii) along the real axis from $z = 0$ to $z = 1$ and then along a line parallel to the imaginary axis from $z = 1$ to $z = 1 + i$. 8

Unit II

4. (a) Let $f(z)$ be analytic within and on a positively oriented simple closed contour C and z_0 is any point lying in it. Then prove that :

$$f'(z) = \frac{1}{2\pi i}\int_c \frac{f(z)}{(z-z_0)^2}dz \quad 8$$

(b) If f is an entire and bounded function in the complex plane, then show that $f(z)$ is constant throughout the plane. 7

5. (a) State and prove fundamental theorem of Algebra. 8

(b) State and prove Poisson's integral formula. 7

Unit III

6. (a) Describe the logarithmic function log(z) for Branch, branch cut and branch point. 7

(b) Find the fixed point and the normal form of the following bilinear transformations and classify their nature : 8

(i) $w = \dfrac{3z-4}{z-1}$

(ii) $w = \dfrac{z-1}{z+1}$.

7. (a) Describe the Taylor's series expansion of a function $f(z)$ which is analytic in a circular domain. 7

(b) Expand $f(z) = \dfrac{z^2-1}{(z+2)(z+3)}$ in a Laurent's series valid for the regions :

(i) $|z| < 2$

(ii) $2 < |z| < 3$

(iii) $|z| > 3$. 8

Unit IV

8. (a) Define isolated singularities of a function and give its classification in detail. 8

   (b) State and prove Cauchy residue theorem and hence evaluate the integral $\int_{c} \frac{5z-2}{z(z-1)} dz$, where c is the circle in counter clockwise sense. 7

9. (a) Using the calculus of residue, prove that :

$$\int_{0}^{\infty} \frac{\sin x}{x} dx = \frac{\pi}{2}.$$ 8

   (b) Show that every polynomial of degree n has exactly n roots, using Rouche's theorem. 7