(12/24)

15206

M. Sc. (2 Year) EXAMINATION

(For Batch 2021 & Onwards)

(First Semester)

MATHEMATICS

MSC/MATHS/I/CC5

Ordinary Differential Equations

Time : *Three Hours*   *Maximum Marks* : 70

Note : Attempt *Five* questions in all. Q. No. **1** is compulsory of 10 marks. Attempt *four* more questions (15 marks each) by selecting *one* question from each Unit.

Compulsory Question

1. (a) Define a uniformaly Lipschitz continuous function.

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(b) Define Wronskian for a homogeneous LDE.

(c) Describe Green's formula.

(d) Describe oscillatory and non-oscillatory equations.

(e) Describe Green function.

Unit I

2. (a) State and prove Gronwall's inequality.

(b) State and prove Cauchy-Peano existence theorem.

3. State and prove Picard-Lindelof existence and uniqueness theorem.

Unit II

4. State and prove Abel's identity for nth order linear differential equation.

5. (a) State and prove Lagrange's identity.

(b) Find Wronskian of three independent solution of $\dfrac{d^3x}{dt^3} - \dfrac{d^2x}{dt^2} - \dfrac{dx}{dt} + x = 0$ in $[0, 1]$.

Unit III

6. (a) Give a detailed note on Pruffer's transformation.

(b) Define self adjoint equation of second order. Discuss necessary and sufficient condition for linear second order differential equation to be a self adjoint equation.

7. State and prove Sturm fundamental comparison theorem.

Unit IV

8. (a) What is Orthogonality of functions ? Prove that eigen functions corresponding to distinct eigen values are orthogonal.

(b) Describe regular linear BVP, singular linear BVP, non-linear BVP, Sturm Liouville BVP.

9. (a) Using Green function, solve : **10**

$$y'' + y = x^2, y(0) = y\left(\frac{\pi}{2}\right) = 0$$

(b) Find eigen values and eigen vectors of

$$\frac{d^2y}{dx^2} + \lambda x = 0, \ y(0) = 0, \ y(L) = 0 \text{ where}$$

L > 0. **5**