Roll No. ....

(05/25)

15211

M. Sc. (2 Year) EXAMINATION

(For Batch 2021 & Onwards)

(Second Semester)

MATHEMATICS

MSc/Maths/2/CC6

Advanced Abstract Algebra

Time : Three Hours *Maximum Marks : 70*

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory. All questions carry equal marks.

(Compulsory Question)

1. (a) Prove that G(K, F) is a subfield of AutK.

(b) Show that [K : F] = 1 if and only if K = F.

(c) Define Galois group of polynomials.

(d) Prove that [C : R] is finite.

(e) If T is nilpotent and if $\alpha \in F$ then αT is also nilpotent.

Unit I

2\. (a) If K is a finite extension of F and E is a subfield of K which contains F, then prove that [E : F] |[K : F].

(b) Let K/F be any extension and 'a' $\in K$ is algebraic over F. Let $p(x) \in F[x]$ be the minimal polynomial of 'a'. Then,

$$F[x]/\langle p(x) \rangle \cong F[a] = F(a).$$

3\. (a) Let $f(x) \in F[x]$ be any polynomial of degree n. Then, there exists an extension E of F containing all the roots of $f(x)$ and $[E : F] \leq n!$.

(b) Find the splitting field and its degree for the polynomial $f(x) = x^p - 1$ over Q, where p is a prime.

Unit II

4\. (a) If a polynomial $f(x) \in Z[x]$ can be expressed as a product of two polynomials over Q, the rational field, then show that it can be expressed as a product of two polynomials over Z.

(b) Let E be a finite extension of F, then E is a normal extension of F if and only if E is a splitting field of a polynomial $f(x) \in F[x]$.

5\. (a) Prove that if ch.F = 0, then any algebraic extension of F is always separable extension.

(b) Prove that for every prime p and integer $n \geq 1$, there exist a field having p^n elements.

Unit III

6. (a) Let E/F be a finite extension, then E/F is a Galois if and only if F is the fixed field of the group of all F-automorphisms of E.

(b) Prove that if α is a primitive nth root of unity over k, then $k(\alpha)/k$ is Galois.

7. (a) Prove that if F is a field of characteristic zero, K a normal extension of F including F with an abelian Galois group, $[K : F] = n$ and the polynomial $k_n = X^n - e$ splits completely in P(F), then K is an extension of F by radicals.

(b) State and prove fundamental theorem of algebra.

Unit IV

8. (a) Let $S, T \in A(V)$ are nilpotent transformations then S and T are similar if and only if they have the same set of invariants.

(b) Prove that if M, of dimension m, is cyclic with respect to T, then the dimension of MT^k is $m-k$ for all $k < m$.

9. Find invariant factors, elementary divisors, and the Jordan canonical form of the following matrix $\begin{bmatrix} 5 & \frac{1}{2} & -2 & 4 \\ 0 & 5 & 4 & 4 \\ 0 & 0 & 5 & 3 \\ 0 & 0 & 0 & 4 \end{bmatrix}$.

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