Roll No. .....

(05/24)

15213

M. Sc. (2 Year) EXAMINATION

(For Batch 2021 & Onwards)

(Second Semester)

MATHEMATICS

MSc/MATH/2/CC8

Mechanics of Solids

Time : Three Hours *Maximum Marks :* 70

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory.

Compulsory Question

1. (a) Define tensor and its different types of order.

   (b) Define pure deformation and components of strain tensor.

(c) Define stress vector and stress tensor.

(d) Define generalized Hooke's law.

(e) Define biharmonic function and show that stress and strain components are biharmonic functions, if the body forces are constant.

Unit I

2. (a) If u_{ij} and v_{ij} are components of second order tensor $\bar{U}$ and $\bar{V}$ and w_i are components of vector $\bar{W}$, then

(i) $u_{ij}w_j$ are components of a tensor of order one.

(ii) $u_{ik}v_{ij}$ is a tensor of order two.

(iii) $u_{ij}v_{ij}$ is a scalar.

(b) Prove that ε_{ijk} is a tensor of order 3.

3. (a) Prove that any tensor of second order can be uniquely expressed as a sum of symmetric and skew-symmetric tensors.

(b) Find the eigen values and corresponding eigen vectors of a second order tensor whose matrix representation is :

$$U_{ij} = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

Unit II

4. Prove that the necessary and sufficient condition for an infinitesimal affine transformation :

$$\xi_i = \alpha_{io} + (\alpha_{ij} + \delta_{ij}),$$

to represent a rigid body motion is that the matrix α_{ij} is skew-symmetric.

5. (a) Explain geometrical representation of shearing strain.

(b) Show that principle directions of strain are normal to the quadric surface of Cauchy.

Unit III

6. (a) State and prove Cauchy equation of equilibrium.

   (b) Define stress quadric of Cauchy and show that normal to the quadric surface at the end of a radius vector is parallel to the stress vector acting on plane through P^0 normal to the radius vector.

7. Show that maximum shearing stress is equal to one-half the difference between the greatest and least normal stresses and act on the plane that bisect the angle between the directions of the largest and smallest principal stresses.

Unit IV

8. (a) Explain physical interpretations of Young's modulus and Poisson ratio when body is subjected to simple tension.

   (b) Derive Navier equation of equilibrium for an isotropic elastic solid.

9. Show that total work done by the external forces in changing the configuration of the natural state to the state at time t is equal to the sum of the Kinetic enegy and Strain energy.