Roll No. ....

(05/24)

15215

M. Sc. (2 Year) EXAMINATION

(For Batch 2021 & Onwards)

(Second Semester)

MATHEMATICS

MSc/Math/2/DSC1

Methods of Applied Mathematics

Time : Three Hours *Maximum Marks :* 70

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory.

1. (a) Prove that for cylindrical unit vectors :

$$\frac{d}{dt}\hat{e}_\theta = -\theta^\bullet \hat{e}_r .$$

(b) What do you mean by covariant components of a vector ?

(c) If $F(s)$ is Fourier transform of function $f(t)$, then find Fourier transform of $f(t)\cos at$.

(d) Define Hankel transform of a function.

(e) Find the extremal of functional :

$$\int_{x_2}^{x_1}\left(\frac{y'^2}{x^3}\right)dx.$$ 5×2=10

Unit I

2. (a) Obtain expression for gradient of a scalar in orthogonal curvilinear co-ordinates and deduce it in cylindrical co-ordinates. 7

(b) Obtain expression for velocity and acceleration of a particle in spherical co-ordinates. 8

3. (a) Represent the vector $\vec{A} = y\hat{i} + 2x\hat{j} + z\hat{k}$ in cylindrical co-ordinates. 8

(b) If (u_1, u_2, u_3) are general co-ordinates, prove that $\frac{\partial \vec{r}}{\partial u_1}, \frac{\partial \vec{r}}{\partial u_2}, \frac{\partial \vec{r}}{\partial u_3}$ and ∇u_1, ∇u_2, ∇u_3 are reciprocal system of vectors. 7

Unit II

4. (a) Find Fourier cosine transform of $f(t) = e^{-t^2}$. 8

(b) Find Fourier transform of :

$$f(t) = \begin{cases} t, & |t| \le a \\ 0, & |t| > a \end{cases}.$$ 7

5. (a) Using Fourier transforms, find solution of $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = e^{-|t|}$. 10

(b) Prove that the Fourier transform of the product of two functions is the convolution of their Fourier transforms. 5

Unit III

6. (a) Find Hankel transform $H_n\left[\frac{e^{-ax}}{x}\right]$. 7

(b) Obatain $H_n\left[\frac{\sin ax}{x}\right]$. 8

7. Prove that $H_n\left[\frac{1}{x}J_{n+1}(ax)\right]=\frac{\xi^n}{a^{n+1}}H(a-\xi)$,

where symbols have their usual meanings. 15

Unit IV

8. (a) Show that the shortest curve joining two fixed points is a straight line. 7

(b) Find the extremal of functional $I=\int_0^{\pi}\left(y'^2-y^2\right)dx$ under the conditions $y(0)=0,\ y(\pi)=1$ and subject to the constraint $\int_0^{\pi} y\,dx=1$. 8

9. (a) Find the extremal of functional :

$$I=\int_{x_0}^{x_1}\left(y''^2-2y'^2+y^2-2y\sin x\right)dx$$ 7

(b) Using Pitz's method, find approximate solution of the problem :

$$I=\int_0^1\left(xy'^2+\frac{y^2}{x}+4xy\right)dx = \text{minimum}$$

with $y(0)=0,\ y(1)=1$. 8