

Roll No. ....

(05/24)

**15214**

**M. Sc. (2 Year) EXAMINATION**

(For Batch 2021 & Onwards)

(Second Semester)

**MATHEMATICS**

**MSc/Math/2/CC9**

**Systems of Differential Equations**

*Time : Three Hours*

*Maximum Marks : 70*

**Note :** Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. 1 is compulsory.

**Compulsory Question**

1. (a) Define Fundamental set and Fundamental Matrix. 2
- (b) What is the significance of Floquet theory for periodic system ? 2

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- (c) Define any two types of critical points. 2
- (d) Define Liapunov function with example. 2
- (e) Define adjoint system. 2

### Unit I

2. (a) Describe the reduction of order of a Homogenous system. 10
- (b) If  $\phi$  is fundamental matrix for Linear Homogenous system  $X' = A(t)X$  then  $\psi$  is a fundamental matrix for  $X' = -A^*(t)X$  iff  $\psi^* \phi = C$  where  $C$  is a constant non-singular matrix. 5
3. (a) Let the vector function  $\phi_1, \phi_2, \dots, \phi_n$  are linearly dependent on  $I$  the Wronskian  $W(\phi_1, \phi_2, \dots, \phi_n) = 0$  for every  $t \in I$ . 7

- (b) Find the fundamental matrix of the system

$$X' = A(t)X \text{ if } X = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} \text{ and}$$

$$A = \begin{bmatrix} -t & t & 0 \\ 0 & -t & 0 \\ 0 & 0 & t \end{bmatrix}.$$

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### Unit II

4. (a) Let  $f(t)$  be a periodic function with period  $w$ . Then the solution of  $X'(t) = A(t)x + f(t)$  is periodic of period  $w$  iff  $X(0) = X(w)$ . 8
- (b) Compute the fundamental matrix of Linear Homogenous system with constant coefficient. 7
5. Explain method of variation of constant for non-Homogenous system. 15

### Unit III

6. Examine the critical points of the system

$\frac{dx}{dt} = y; \frac{dy}{dt} = x^2 - 4x + \lambda$ , where  $\lambda$  is a parameter. Also find the critical value of parameter  $\lambda$ . 15

7. Discuss the nature of stability of Critical point

if the roots of the system  $\frac{dx}{dt} = ax + by;$

$\frac{dy}{dt} = cx + dy$  are conjugate complex with real part non-zero. 15

### Unit IV

8. (a) Determine the type and stability of critical

point  $(0, 0)$  of  $\frac{dx}{dt} = (y+1)^2 - \cos x,$

$\frac{dy}{dt} = \sin(x+y).$  10

- (b) Define index of critical point and its result. 5

9. Define limit cycle with example. 15