Roll No. .....

(12/24)

# 15222

## M. Sc. (2 Year) EXAMINATION

(For Batch 2021 & Onwards)

(Third Semester)

MATHEMATICS

MSc/Maths/3/CC11

Fluid Mechanics

*Time : Three Hours* *Maximum Marks : 70*

**Note** : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory.

1. (i) Describe the Eulerian method to describe fluid motion.

   (ii) Determine the acceleration when the velocity distribution is given by $\vec{q} = \hat{i}\,Axy^2t + \hat{j}\,Bx^2yt + \hat{k}\,Cxyz$, where A, B and C are constants.

   (iii) From the vector form of Euler's equation of motion, derive the equation in spherical coordinates.

   (iv) Find the image of a three dimensional source with regard to a plane.

   (v) Describe two-dimensional motion of a fluid. 5×2=10

**Unit I**

2. (i) Derive the equation of continuity for fluid motion in Cartesian coordinates. 8

   (ii) If $\sigma(s)$ is the cross-sectional area of a stream filament, prove that the equation of continuity is $\frac{\partial}{\partial t}(\rho\sigma)+\frac{\partial}{\partial s}(\rho\sigma q)=0$, where $\delta s$ is an element of arc of the filament and $q$ is the fluid speed. 7

3. (i) Describe stream line, path line and streak line. Hence find the path lines and the streak lines, when the velocity field at a point is $\vec{q} = \left(\frac{x}{t}, y, 0\right)$. 8

(ii) Show that $\left(\frac{x^2}{a^2}\right)\tan^2 t + \left(\frac{y^2}{b^2}\right)\cot^2 t = 1$ is a possible form for the bounding surface of a liquid. 7

**Unit II**

4. (i) A homogeneous liquid is in motion in a vertical plane within a curved tube of uniform small bore under the action of gravity. Describe the motion of the fluid. 7

(ii) Derive equations of motion under impulsive forces in Cartesian form. 8

5. (i) A quantity of liquid occupies a length $2l$ of a straight tube of uniform small bore under the action of a force to a point in the tube varying as a distance from that point. Determine the pressure at any point. 7

(ii) State and prove Kelvin's circulation theorem. 8

**Unit III**

6. (i) Describe the motion for liquid streaming past a fixed sphere in an infinite mass of liquid. Also derive the equation of the lines of flow relative to the sphere. 8

(ii) A sphere of radius '$a$' is surrounded by a concentric sphere of radius '$b$', the space between the spheres is being filled with liquid at rest. The inner sphere is given a velocity U and the outer sphere a velocity V in the same direction. Find the initial motion of the liquid. 7

7. (i) Describe three dimensional doublets and hence find the velocity potential due to a three dimensional doublet. 6

(ii) Find the image of a three dimensional source with regard to a sphere. 9

**Unit IV**

8. (i) Describe stream function in two dimensional motion, and give its physical significance. 8

(ii) Define complex potential and show that the curves of constant velocity potential and constant stream functions cut orthogonally at their points of intersection. 7

9. (i) Find the image of a source with regard to a circle. 6

(ii) State and prove Blasius theorem. 9