Roll No. ....

(12/24)

15223

M.Sc. (2 Year) EXAMINATION

(For Batch 2021 & Onwards)

(Third Semester)

MATHEMATICS

MSc/Maths/3/DSC4

Integral Equations

Time : Three Hours *Maximum Marks* : 70

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory.

1. (a) What do you mean by Fredholm Integral Equation of first kind and second kind ? Also define homogenous Fredholm Integral equation. 2

(b) Define Symmetric kernel and Degenerate Kernel. 2

(c) What do you mean by Singular Integral equation and Convolution type integral equation ? 2

(d) Verify that the given function $u(x) = \frac{1}{2}$ is the solution of the integral equation : 2

$$\int_0^x \frac{u(t)}{\sqrt{x-t}} dt = \sqrt{x}$$

(e) State Plemelj Formula. 2

Unit I

2. (a) Solve the integral equation $u(x) = x - \int_0^x (x-t)\, u(t)dt$ taking $u_0(x) = 0$ using successive approximation method. 7.5

(b) With the help of resolvent kernel find the solution of the following integral equation : 7.5

$$u(x) = 1 + x^2 + \int_0^x \frac{1+x^2}{1+t^2} u(t)\, dt$$

3. (a) Solve the following integral equation by using Laplace transform method : 7.5

$$u(x) = x + \frac{1}{6}\int_0^x (x-t)^3\, u(t)dt$$

(b) Solve the following Volterra integral equation of first kind by changing it to Volterra integral equation of second kind : 7.5

$$\int_0^x (2+x^2-t^2)\, u(t)dt = x^2$$

Unit II

4. (a) Solve the integral equation : **7.5**

$$u(x) = \frac{5x}{6} + \frac{1}{2}\int_0^1 x\,t\,u(t)dt$$

by the method of successive approximation.

(b) Solve the integral equation with degenerate kernel : **7.5**

$$u(x) = \cos x + \lambda\int_0^{\pi} \sin(x - t)u(t)dt$$

5. (a) Consider $u(x) = 1 + \lambda\int_0^1 (1-3xt)\,u(t)dt$. Evaluate the resolvent kernel and for what values of λ the solution does not exist. Obtain solution of the given integral equation. **7.5**

(b) Using Fredholm determinants, find the resolvent kernel and hence solve the integral equation : **7.5**

$$u(x) = e^x + \lambda\int_0^1 2\,e^{x+t}\,u(t)dt$$

Unit III

6. (a) Solve the singular integral equation : **7.5**

$$ax + bx^2 = \int_x^x \frac{g(t)dt}{(x-t)^{\frac{1}{2}}}$$

(b) Define Cauchy Type integral and derive Poincare-Bertrand transformation formula. **7.5**

7. (a) Describe the solution of the Hilbert-type integral equation of the first kind. **7.5**

(b) Solve the singular integral equation : 7.5

$$\mu g(x) = f(x) - \frac{v}{2\pi} P \int_{-\infty}^{2\pi} g(t) \cot \frac{t-x}{2} dt$$

Unit IV

8. (a) Find the Green's function for the boundary value problem using method of variation of parameters : 7.5

$$\frac{d^2 y}{dx^2} - y = 0; \; y(0) = 0 = y(1)$$

(b) Obtain Green's function of boundary value problem : 7.5

$$\frac{d^2 y}{dx^2} = 0, \; y(0) = y(1) = 0$$

by Using its basic four properties.

9. Using Green's function basic four properties, reduce the following boundary value problem into integral equation and hence solve it. 15

$$\frac{d^2 y}{dx^2} - y = x; \; y(0) = 0 = y(1)$$