Roll No. ....

(12/24)

15221

M. Sc. (2 Year) EXAMINATION

(For Batch 2021 & Onwards)

(Third Semester)

MATHEMATICS

MSc/Maths/3/CC10

Topology

Time : Three Hours *Maximum Marks* : 70

Note : Attempt *Five* questions in all, selecting *one* question from each Section. Q. No. **1** is compulsory.

1. (a) Show that closed subspace of a compact space is compact.

   (b) If $x \notin F$, where F is a closed subset of a topological space (X, T), then show that there exists an open set G such that $x \in G \subseteq F^C$.

(c) If E is a subset of a subspace (X^*, T^*) of a topological space (X, T), then show that $C^*(E) = X^* \cap C(E)$.

(d) Let :

$$X = \{1, 2, 3\}$$

and $\beta = \{(1, 2), (2, 3), X\}$.

Show that β is not a base for topology on X.

(e) Define Projection Mapping with example.

5×2=10

Section I

2. (a) Define a topology T on a non-empty set X. If X is an uncountable set and T be the family of all complements of countable sets together with Ø, then show that T is a topology on X. What would happen to T if X is a countable set ? 7.5

(b) Let $X = \{a, b, c\}$ and $T = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$. Then find $d(\{a\})$ and $d(\{a, b\})$. 7.5

3. (a) Let C^* be a closure operator defined on a set X. Let F be the family of all subsets F of X for which $C^*(F) = F$ and let T be a family of all complements of membes of F. Show that T is a topology for X and if C is the closure operator defined by the topology T, then $C^*(E) = C(E)$ for all subsets $E \subseteq X$. 7.5

(b) State and prove Lindelof's theorem. 7.5

Section II

4. (a) Let β and β′ be bases for the topologies T and T′ respectively on X. Show that the following are equivalent :

(i) T′ is finer that T;

(ii) For each $x \in X$ and each basis element $B \in \beta$ such that $x \in B$ there is a member $B' \in \beta'$ such that $x \in B' \subset B$. 7.5

(b) State and prove the pasting lemma about continuous functions. 7.5

5. (a) Let (X, T) be a discrete topological space. (Y, U) be any topological space. Is every function $f : X \to Y$ necessarily continuous ? Prove or disapprove. 7.5

(b) Show that $\prod_\lambda X_\lambda$ is T_2 iff each space X_λ is T_2, where $\{X_\lambda\}_{\lambda \in \Lambda}$ is any family of topological spaces. 7.5

Section III

6. (a) Prove that a convergent sequence has a unique limit in a Hausdorff space. Is the converse true ? Justify. 7.5

(b) Let S be a subbase for a topological space X. Then, prove that X is completely regular iff for each $V \subseteq S$ for each $x \in V$, there exists a continuous function $f : X \to [0, 1]$ such that :

$$f(x) = 0, f(y) = 1$$

for all $y \notin V$. 7.5

7. (a) Define Normal space. Is hereditary property holds in Normal space ? 5

(b) Show that a topological space (X, T) is normal if and only if for each two disjoint closed subsets F_1 and F_2 in X and closed interval $[a, b]$ of reals, there exists a continuous mapping $f : X \to [a, b]$ such that $f(F_1) = \{a\}$ and $f(F_2) = \{b\}$. 10

Section IV

8. (a) Prove that union of a collection of connected sets having a point in common is connected. 7.5

(b) Show that a topological space X is connected iff there exists no non-empty proper subset which is both open and closed. 7.5

9. (a) Prove that every closed subset of a compact space is compact. Is every compact subset closed ? Justify. 7.5

(b) Let (X, T) be a topological space such that every countable open covering of X is reducible to a finite subcover then prove that X is countably compact. 7.5