Roll No. ....

(05/25)

15231

M. Sc. (2 Year) EXAMINATION

(For Batch 2021 & Onwards)

(Fourth Semester)

MATHEMATICS

MSc/Maths/4/CC12

Functional Analysis

Time : Three Hours *Maximum Marks : 70*

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory. All questions carry equal marks.

Compulsory Question

1. (i) Prove that norm is a continuous function.

   (ii) State F. Riesz Lemma.

   (iii) Show that spaces l_1 and l_∞ are not reflexive space.

(iv) Define adjoint operator and norm of adjoint operator.

(v) State Pythagorean and Apollonius' theorem.

(vi) An orthonormal set S in Hilbert space is complete if for any $x \in H$ is such that $x \perp H$ implies $x = 0$.

(vii) Define normal and positive operator.

Unit I

2. Prove that linear spaces l_p is a Banach spaces under the norm $\|x\|_p = \left(\sum_{i=1}^{\infty} |x_i^p|\right)^{\frac{1}{p}}$ such that $\sum_{i=1}^{\infty} |x_i^p|$ is convergent.

3. Let T be a bounded linear transformation from a normed linear space N_1 to N_2 :

Put $a = \sup\{\|T(x)\| : x \in N_1, \|x\| = 1\}$

$$b = \sup\left\{\frac{\|T(x)\|}{\|x\|} : x \in N_1, \|x\| \neq 0\right\}$$

$$c = \inf\{k : k \geq 0, \|T(x)\| \leq k\|x\| : \text{for all } x \in N_1\}$$

Then $\|T\| = a = b = c$

Also deduce that $\|T(x)\| \leq \|T\|\|x\|$ for all $x \in N_1$.

Unit II

4. State and prove Riesz Representation theorem for bounded linear functionals on C[a, b] space.

5. (a) State and prove uniform bounded principal.

(b) Prove that each reflexive space is Banach space but not conversely.

Unit III

6. State and prove open mapping theorem.

7. (a) State and prove Pythagorean theorem and Schwartz inequality.

(b) State and prove projection theorem in Hilbert space.

Unit IV

8. (a) Define orthonormal set. State and prove Bessel's inequality for countable orthonormal sets in Hilbert spaces.

   (b) State and prove Parseval's inequality.

9. (a) State and prove Riesz representation theorem for bounded linear functional on Hilbert space.

   (b) Discuss self adjoint operator, unitary operator, normal operator and positive operators.

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