Roll No. ....

(05/24)

15236

M.Sc. (2 Year) EXAMINATION

(For Batch 2021 & Onwards)

(Fourth Semester)

MATHEMATICS

MSc/Maths/4/DSC16

Boundary Value Problems

Time : Three Hours *Maximum Marks* : 70

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory.

Compulsory Question

1. (a) Define Newtonian potential.

(b) Define Fourier transform and its shifting property.

(c) Define boundary value problem with an example.

(d) Solve the integral equation :

$$\sin s = \frac{1}{\pi}\int_{-\infty}^{+\infty}\left[\frac{g(t)}{t-s}\right]dt .$$

(e) Define Low-Reynolds number. 2×5=10

Unit I

2. (a) Reduce the boundary value problem :

$$y' + \sqrt{y} = 0, y(0) = 0, \quad y'(1) + \sqrt{2}y(1) = 0$$

to Fredholm integral equation. 7½

(b) Solve the boundary value problem :

$\frac{d^2y}{dx^2} = F(x),\ y(0) = 0,\ y(e) = 0$ using Green function. 7½

3. (a) Find Modified Green's function for the system :

$$y'' - \sqrt{y} = 0, y(0) = y(1), y'(0) = y'(1) .$$ 7½

(b) Convert the self adjoint initial value problem :

$$\frac{d}{ds}\left(p\frac{dy}{ds}\right) + qy = F(s)$$

$y(a) = 0, y'(a) = 0$ into an integral equation. 7½

Unit II

4. (a) Define single layer and double layer potentials in detail. 7½

(b) Discuss interior Dirichlet problem. 7½

5. (a) State and prove Poisson's integral formula. 7½

(b) Define exterior Neumann problem. 7½

Unit III

6. (a) State and prove convolution theorem. 7½

(b) Explain three part boundary value problem. 7½

7. (a) Find the electrostatic potential due to a spherical cap. 7½

(b) Solve the integral equation :

$$\int_0^\infty F(x)\cos px\, dx = \begin{cases} 1-p, & 0 \le p \le 1 \\ 0 \;, & p > 1 \end{cases}$$

by using Fourier transform. 7½

Unit IV

8. (a) Discuss steady Stokes flow in an unbounded medium. 7½

(b) Discuss boundary effects on stokes flow. 7½

9. Solve the problem of the diffraction of a plane wave by a soft sphere, taking spherical polar co-ordinates. 14